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# Temporal evolution of solute dispersion in three-dimensional porous rocks

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# Abstract

We study the temporal evolution of solute dispersion in three-dimensional porous rocks of different heterogeneity and pore structure. To this end, we perform direct numerical simulations of pore-scale flow and transport in a sand-like medium, which exhibits mild heterogeneity, and a Berea sandstone, which is characterized by strong heterogeneity as measured by the variance of the logarithm of the flow velocity. Solute dispersion is quantified by effective and ensemble dispersion coefficients. The former is a measure for the typical width of the plume, the latter for the deformation, that is, the spread of the mixing front. Both dispersion coefficients evolve from the molecular diffusion coefficients toward a common finite asymptotic value. Their evolution is governed by the interplay between diffusion, pore-scale velocity fluctuations and the medium structure, which determine the characteristic diffusion and advection time scales. Dispersion in the sand-like medium evolves on the transverse diffusion time across a characteristic streamtube diameter, which is the mechanism by which pore-scale flow variability is sampled by the solute. Dispersion in the Berea sandstone in contrast is governed by both the diffusion time across a typical streamtube, and the diffusion time along a pore conduit. These insights shed light on the evolution of mixing fronts in porous rocks, with implications for the understanding and modeling of transport phenomena of microbes and reactive solutes in porous media.

# Temporal evolution of solute dispersion in three-dimensional porous rocks

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# Key Points:

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8	• Pore-scale simulations of temporal evolution of solute dispersion in three-dimensional
9	porous rocks
10	• Systematic study of effective and ensemble dispersion coefficients as measures for
11	solute spreading and mixing
12	• Time evolution of dispersion coefficients is determined by medium structure, pore-

• Time evolution of dispersion coefficients is determined by medium structure, porescale flow heterogeneity and diffusion

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#### 14 Abstract

We study the temporal evolution of solute dispersion in three-dimensional porous rocks 15 of different heterogeneity and pore structure. To this end, we perform direct numerical 16 simulations of pore-scale flow and transport in a sand-like medium, which exhibits mild 17 heterogeneity, and a Berea sandstone, which is characterized by strong heterogeneity as 18 measured by the variance of the logarithm of the flow velocity. Solute dispersion is quan-19 tified by effective and ensemble dispersion coefficients. The former is a measure for the 20 typical width of the plume, the latter for the deformation, that is, the spread of the mix-21 ing front. Both dispersion coefficients evolve from the molecular diffusion coefficients to-22 ward a common finite asymptotic value. Their evolution is governed by the interplay be-23 tween diffusion, pore-scale velocity fluctuations and the medium structure, which deter-24 mine the characteristic diffusion and advection time scales. Dispersion in the sand-like 25 medium evolves on the transverse diffusion time across a characteristic streamtube di-26 ameter, which is the mechanism by which pore-scale flow variability is sampled by the 27 solute. Dispersion in the Berea sandstone in contrast is governed by both the diffusion 28 time across a typical streamtube, and the diffusion time along a pore conduit. These in-29 sights shed light on the evolution of mixing fronts in porous rocks, with implications for 30 the understanding and modeling of transport phenomena of microbes and reactive so-31 lutes in porous media. 32

#### **1 Introduction**

The transport of solutes in porous media is driven by the phenomenon of disper-34 sion, which results from the interplay between advective spreading and diffusion. The 35 former is triggered by the spatial variability of the fluid speed which is controlled by the 36 geometry of the connected pore network (Datta et al., 2013; Alim et al., 2017; Valocchi 37 et al., 2018; Puyguiraud et al., 2021) while the later is ubiquitously controlled by the con-38 centration gradients. The heterogeneity of the porous medium that triggers the flow speed 39 distribution is therefore a primary parameter that controls dispersion from pre-asymptotic 40 to Fickian regime (Dentz et al., 2004; Sherman et al., 2021). Transport in porous me-41 dia is considered in many fields of academic and industrial applications from materials 42 science, engineering and medicine to groundwater hydrology, environmental technolo-43 gies and petroleum engineering, and at many scales from microfluidic applications to ground-44 water management. Beside being necessary for understanding and predicting the spread-45 ing of chemicals such as pollutants or bionutrients, modeling dispersion is required also 46 to understand and predict solute-solute and solute-minerals reactions that can produce 47 new solute species and trigger mineral dissolution and precipitation features, for instance. 48

Dispersion in porous media has been extensively studied from the pore to the re-49 gional scale for decades (Saffman, 1959; Whitaker, 1967; Gelhar & Axness, 1983; Dagan, 50 1990; Dentz et al., 2023). Here we focus on hydrodynamic dispersion due velocity fluc-51 tuations caused by the heterogeneity of the pore space. A main challenge concerns how 52 continuum-scale solute transport can be modeled by macroscopic parameters, such as 53 the dispersion coefficient, that can be inferred experimentally, by using direct pore scale 54 simulations or upscaling methods such as volume averaging or stochastic modeling (Brenner, 55 1980; Ahmadi et al., 1998; Koch & Brady, 1985; Scheven, 2013; Bijeljic & Blunt, 2006; 56 Le Borgne et al., 2011; Souzy et al., 2020; Puyguiraud et al., 2021). Similar challenges 57 are encountered for reactive transport that is controlled by the time resolved distribu-58 tion of the solutes and their mixing. If the reaction thermodynamics and kinetics are known, 59 then the goal is to be able to model the local reaction rate from knowing dispersion prop-60 erties (Battiato et al., 2009; Battiato & Tartakovsky, 2011). However, it is well known 61 that the advection-dispersion equation parameterized by constant asymptotic dispersion 62 coefficients are not suited to evaluate the effective reaction rates, because it assumes full 63 mixing whereas incomplete mixing is the rule during the pre-asymptotic (non-Fickian) 64 dispersion regimes (Rolle et al., 2009; Le Borgne et al., 2010; Dentz et al., 2011; Le Borgne 65

et al., 2011; Puyguiraud et al., 2021). Nevertheless, diffusion and transverse mixing tend
to homogenize concentration and full mixing can be expected in the asymptotic regime,
as long as the characteristic length of heterogeneity is finite. Clearly, the convergence
rate toward asymptotic dispersion and full mixing depend on the medium heterogeneity, but characterizing the relationship is still challenging and requires investigating both
mixing and spreading mechanisms at all scales.

Solute dispersion and its pre-asymptotic behavior have been analyzed in terms of 72 breakthrough curves, the time evolution of the spatial variance of concentration or par-73 74 ticle distributions, or directly from particle velocities, using experiments and direct numerical pore scale simulations (Hulin & Plona, 1989; Khrapitchev & Callaghan, 2003; 75 Bijeljic et al., 2004; Gouze et al., 2021; Puyguiraud et al., 2021; Gouze et al., 2023). These 76 studies, accounting for the heterogeneity as a whole, show that the pore structure shapes 77 the evolution of dispersion during the pre-asymptotic regime and then determine the asymp-78 totic value. Hulin and Plona (1989) and Khrapitchev and Callaghan (2003) study the 79 reversibility of pore-scale dispersion upon flow reversal, which addresses the issue of un-80 der which conditions hydrodynamic dispersion describes solute mixing or advective so-81 lute spreading. As mentioned above, the fundamental mechanisms of hydrodynamic dis-82 persion are pore-scale velocity fluctuations and diffusion. The former mechanism is re-83 versible in the Stokes regime, which holds for typical applications in groundwater resources. 84 Irreversibility, or actual solute mixing is induced by the interaction of spatial velocity 85 fluctuations and molecular diffusion (Dentz et al., 2023). Consider for example a solute 86 that evolves from an extended areal source. At early times, the solute front deforms due 87 to velocity variability within the source distribution, which leads to a complex concen-88 tration distribution, which nevertheless is partially reversible. Hydrodynamic dispersion 89 coefficients that are defined in terms of the spatial variance of the global solute distri-90 bution, measure at pre-asymptotic this advective spreading rather than actual solute mix-91 ing. 92

This issue was recognized by Kitanidis (1988) in the context of solute dispersion 93 in heterogeneous porous formations, and Bouchaud and Georges (1990) in the context of random walks in quenched disordered media. These authors propose to define disper-95 sion coefficients from the second-centered moments of the solute or particle distributions 96 that evolve from a point-like initial condition. In the absence of local scale dispersion 97 or molecular diffusion, these dispersion coefficients are exactly zero. In the following, we 98 refer to this concept as *effective dispersion*. The dispersion concept based on the spa-99 tial variance of the solute concentration evolving from an extended areal or line source, 100 is called *ensemble dispersion* in the following. As outlined above, at preasymptotic times 101 ensemble dispersion measures advective solute spreading rather than mixing. In fact, it 102 measures the center of mass fluctuations of the partial plume that evolves from the point 103 injections that constitute the spatially extended initial distribution (Bouchaud & Georges, 104 1990). Several authors studied these dispersion concept in the context of mixing and dis-105 persion in porous media on the continuum scale characterized by spatially variable hy-106 draulic conductivity (Attinger et al., 1999; Dentz et al., 2000; Fiori, 2001; Fiori & Da-107 gan, 2000; Vanderborght, 2001; Dentz & de Barros, 2015; De Barros et al., 2015; de Bar-108 ros & Dentz, 2016). Dentz et al. (2000) analyzed the time evolution of the effective and 109 ensemble dispersion coefficients. They showed that the time resolved ensemble disper-110 sion coefficient is usually larger than the effective dispersion until the effective disper-111 sion growth rate increases due transverse local dispersion and diffusion and eventually 112 converges with the ensemble dispersion coefficient. This increase of the effective disper-113 sion value denotes the convergence of average local mixing toward macroscopic mixing 114 that accounts for heterogeneity as a whole. Because it is a quantitative way to discrim-115 inate mixing from spreading, the notion of effective dispersion has been discussed and 116 used by several authors for the modeling of experimental and numerical reactive trans-117 port data (Cirpka, 2002; Jose et al., 2004; Perez et al., 2019, 2020; Puyguiraud et al., 2020). 118 As discussed above, most works that analyze effective and ensemble dispersion to quan-119

tify the impact of spatial heterogeneity on solute mixing and spreading consider continuum scale fluctuations of the hydraulic conductivity. To the best of our knowledge, the
concept of effective dispersion has not been studied for transport in three-dimensional
porous media despite its potential to explain the overestimation of pore-scale mixing and
reaction by constant asymptotic hydrodynamic dispersion coefficients (Kapoor et al., 1998;
Gramling et al., 2002; Perez et al., 2019).

In the present communication we investigate in detail the temporal evolution of mix-126 ing and spreading mechanisms occurring in porous media, in order to evaluate the dif-127 ferent regimes in relation with the porous media structure. To this end, we perform three-128 dimensional direct numerical simulations of pore-scale flow and solute transport in a sand-129 pack medium and in a Berea sandstone of distinctly different heterogeneity levels, that 130 can be measured, for instance, by the variance the logarithm of the flow velocity distri-131 bution. Solute dispersion is quantified by the temporal evolution of the effective and of 132 the ensemble dispersion coefficients. This paper is organized as follows: the methodol-133 ogy used to calculate flow and transport and measure dispersion are presented in Sec-134 tion 2. In Section 3, we present the analyze of the dispersion behavior in the sand pack 135 and Berea samples and discuss how these information can help us depicting the differ-136 ent dispersion stages in relation with the porous media structure. Section 4 presents the 137 conclusions of the study. 138

# 139 2 Methodology

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#### 2.1 Pore-scale flow and transport

Flow in three-dimensional porous media, described as dual solid-void structures, is described by the Stokes equation together with the continuity equation (Leal, 2007),

$$\nabla^2 \mathbf{u}(\mathbf{x}) = -\frac{1}{\mu} \nabla p(\mathbf{x}), \qquad \nabla \cdot \mathbf{u}(\mathbf{x}) = 0, \qquad (1)$$

where  $\mu$  is the dynamic viscosity,  $\mathbf{u}(\mathbf{x})$  is the Eulerian velocity and  $p(\mathbf{x})$  is the fluid pressure at position  $\mathbf{x} = (x_1, x_2, x_3)$ . Here, flow is driven by the macroscopic pressure gradient, which is aligned with the *x*-axis of the coordinated system. Zero-flux boundary conditions are set at the solid-void interface and at the lateral domain boundaries.

Transport of solutes is described by the advection-diffusion equation (ADE) for the solute concentration  $c(\mathbf{x}, t)$ 

$$\frac{\partial c(\mathbf{x},t)}{\partial t} + \nabla \cdot [\mathbf{u}(\mathbf{x}) - D\nabla] \ c(\mathbf{x},t) = 0, \tag{2}$$

where  $c(\mathbf{x}, t)$  is the solute concentration at position  $\mathbf{x}$  and time t, and D is the molecular diffusion coefficient. The advection-diffusion equation (2) is equivalent to the Langevin equation (Risken, 1996)

$$\frac{\mathrm{d}\mathbf{x}(t)}{\mathrm{d}t} = \mathbf{u}[\mathbf{x}(t)] + \sqrt{2D}\boldsymbol{\xi}(t), \tag{3}$$

where  $\boldsymbol{\xi}(t)$  is a Gaussian white noise with mean  $\langle \xi_i \rangle = 0$  and covariance  $\langle \xi_j(t)\xi_k(t) \rangle = \delta_{jk}\delta(t-t'); \ \delta_{jk}$  is the Kronecker delta.

The average pore length  $\ell_0$ , the mean streamwise flow velocity  $\langle v \rangle = \langle |v(\mathbf{x})| \rangle$  and the diffusion coefficient D set the advection time  $\tau_v = \ell_0 / \langle v \rangle$  and the characteristic diffusion time  $\tau_D = \ell_0^2 / D$ . The two time scales are compared by the Péclet number  $Pe = \tau_D / \tau_v = \langle v \rangle \ell_0 / D$ .

## <sup>164</sup> 2.2 Mixing versus spreading

In this section, we discuss plume mixing versus spreading in terms of effective and
 ensemble dispersion coefficients. Then, we pose an approximate model based on the con cept of effective dispersion to upscale pore-scale mixing to the continuum scale.

We analyze the mixing and dispersion of a solute by considering the concentration distribution  $c(\mathbf{x}, t)$  for the normalized plane source

$$c(\mathbf{x}, t = 0) = \rho(\mathbf{x}) = \phi^{-1}\delta(x_1)\frac{\mathbb{I}(\mathbf{x} \in \Omega_f)}{wh},$$
(4)

where  $\Omega_f$  denotes the fluid domain and  $\mathbb{I}(\cdot)$  is the indicator function, which is one if its argument is true and zero else. w and h denote the width and height of the medium and  $\phi$  is porosity. The injection plane is large enough such that

$$\int_{\Omega} d\mathbf{x} \rho(\mathbf{x}) = \phi, \tag{5}$$

where  $\Omega$  denotes the bulk domain, that is, the union of fluid domain and solid domain. The solute distribution can be decomposed into partial plumes  $g(\mathbf{x}, t | \mathbf{x}')$  that satisfy Eq. (2)

178 for the initial conditions

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$$g(\mathbf{x}, t = 0 | \mathbf{x}') = \delta(\mathbf{x} - \mathbf{x}') \mathbb{I}(\mathbf{x}' \in \Omega_f).$$
(6)

181 Then, we can write the concentration distribution  $c(\mathbf{x}, t)$  as

$$c(\mathbf{x},t) = \int_{\Omega} d\mathbf{x}' \rho(\mathbf{x}') g(\mathbf{x},t|\mathbf{x}').$$
(7)

Note that  $g(\mathbf{x}, t|y', z')$  is the Green function of the transport problem. In the following, we define a surrogate model for the Green function using the concept of effective dispersion.

# 2.2.1 Effective and ensemble dispersion coefficients

In order to define effective and ensemble dispersion coefficients, we consider the moments of the Green function  $g(\mathbf{x}, t | \mathbf{x}')$  and the concentration distribution  $c(\mathbf{x}, t)$ . The first and second moments of  $g(\mathbf{x}, t | \mathbf{x}')$  are defined by

$$m_i(t; \mathbf{x}') = \int d\mathbf{x} x_i g(\mathbf{x}, t | \mathbf{x}'), \tag{8}$$

$$m_{ij}(t; \mathbf{x}') = \int d\mathbf{x} x_i x_j g(\mathbf{x}, t | \mathbf{x}').$$
(9)

The first moments  $m_i(t; \mathbf{x}')$  determine the center of mass position of  $g(\mathbf{x}, t | \mathbf{x}')$ . The second centered moments

$$\kappa_{ij}(t;\mathbf{x}') = m_{ij}^{(2)}(t;\mathbf{x}') - m_i^{(1)}(t;\mathbf{x}')m_j^{(1)}(t;\mathbf{x}')$$
(10)

are measures for the spatial extension of the Green function. The average of  $\kappa_{ij}(t; \mathbf{x}')$ over all Green functions defines the effective second centered moment

$$\kappa_{ij}^{\text{eff}}(t) = \int d\mathbf{x}' \rho(\mathbf{x}') \kappa_{ij}(t; \mathbf{x}').$$
(11)

It is a measure for the average width of the Green function. The temporal rate of growth of  $\kappa_{ij}^{\text{eff}}(t)$  is given by the effective dispersion coefficients

$$D_{ij}^{\text{eff}}(t) = \frac{1}{2} \frac{d}{dt} \kappa_{ij}^{e}(t), \qquad (12)$$

The effective dispersion coefficient measures the rate of growth of the spatial variance of a concentration distribution that evolves from a point-like initial condition.

In full analogy, we define the first and second moments of  $c(\mathbf{x}, t)$  as

$$m_i(t) = \int d\mathbf{x} x_i c(\mathbf{x}, t) = \int d\mathbf{x}' \rho(\mathbf{x}') m_i(t; \mathbf{x}'), \qquad (13)$$

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 $m_{ij}(t) = \int d\mathbf{x} x_i x_j c(\mathbf{x}, t) = \int d\mathbf{x}' \rho(\mathbf{x}') m_{ij}(t; \mathbf{x}').$ (14)

As per the second equality signs, the moments are determined by taking ensemble averages over the moments of the set of Green functions and as such are named the ensemble moments in the following. The second centered ensemble moments are defined by

$$\kappa_{ij}^{\text{ens}}(t) = m_{ij}(t) - m_i(t)m_j(t).$$
 (15)

They are measures for the spatial extension of the concentration distribution, or equivalently for the ensemble of Green functions. The temporal rate of growth of the second centered ensemble moments is measured by the ensemble dispersion coefficients

$$D_{ij}^{\rm ens}(t) = \frac{1}{2} \frac{d}{dt} \kappa_{ij}^{\rm ens}(t).$$
(16)

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228 229 The difference between the ensemble and effective variances,

$$\delta\kappa_{ij}^{m}(t) = \int d\mathbf{x}'\rho(\mathbf{x}') \left[m_{i}^{(1)}(t;\mathbf{x}') - m_{i}^{(1)}(t)\right] \left[m_{j}^{(1)}(t;\mathbf{x}') - m_{j}^{(1)}(t)\right],\tag{17}$$

quantifies the variance of the center of mass fluctuations of the Green functions that constitute the solute plume. Along the same lines, the difference between the ensemble and
effective dispersion coefficients measures the dispersion of the center of mass positions
of the Green functions that constitute the solute plume

$$\delta D_{ij}^m(t) = \frac{1}{2} \frac{d}{dt} \delta \kappa_{ij}^m(t).$$
(18)

In the following, we study the effective and ensemble dispersion coefficients as well as the center of mass fluctuations in streamwise direction, that is, for i = j = 1.

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# 2.3 Numerical simulations

In the following, we describe the studied porous media, the numerical solution of the pore-scale flow problem and of the transport problem using random walk particle tracking.

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# 2.3.1 Porous media and fluid flow

We study two three-dimensional porous media of different complexity, (i) a Berea 237 sandstone sample and (ii) a sand pack sample illustrated in Figure 1 The Berea sample 238 displays a complex pore structure with a porosity of  $\phi = 0.18$ , see also (Puyguiraud et 239 al., 2021). This type of porous rock is considered to be a pertinent large-scale homoge-240 neous proxy of high permeability sedimentary reservoirs (Churcher et al., 1991). The sand 241 pack sample has a high porosity of  $\phi = 0.37$  with a more regular structure of the pore 242 space. The sand-pack image (Sand Pack LV60C) was obtained from the Imperial Col-243 lege image repository (Imperial College Consortium on Pore-scale Imaging and Modelling, 244 2014). It is a compact packing of irregular quartz grains of variable size that is a proxy 245 of sub-surface aquifers (Di Palma et al., 2019). The difference between the two porous 246 medium samples can be illustrated by the distribution of flow speeds (Alhashmi et al., 247 2016) shown in Figure 1. The flow heterogeneity is measured by the variance  $\sigma_f^2$  of the 248



**Figure 1.** Eulerian velocity pdfs for the sand pack (blue circles) and the Berea sandstone (red squares). Inlay: The three-dimensional pore geometry of (left) the sand pack sample (5mm<sup>3</sup>) and of (right) the Berea sandstone (1mm<sup>3</sup>). The grey and blue colors represent the pore space and the solid phase, respectively.

natural logarithm  $f = \ln v$  of the flow speed v. For the Berea sandstone sample, we obtain  $\sigma_f^2 = 10$ , for the sand pack sample  $\sigma_f^2 = 2$ , that is, the Berea sample is significantly more heterogeneous. The characteristic pore length scale is  $\ell_0 = 1.5 \times 10^{-6}$  m both for the Berea and sand pack samples.

<sup>253</sup> Both pore geometries are based on X-Ray microtomography images. The geome-<sup>254</sup> tries are meshed using regular hexahedron cells (voxels). This type of mesh has two ma-<sup>255</sup> jor advantages. Firstly, it perfectly fits the voxels of the X-Ray tomography images, and <sup>256</sup> secondly, it allows for a simple and computationally efficient velocity interpolation scheme, <sup>257</sup> which is required for the transport simulation based on random walk particle tracking (Mostaghimi <sup>258</sup> et al., 2012). Each of the images is decomposed in 900<sup>3</sup> voxels of length  $l_m = 1.060 \cdot$ <sup>259</sup>  $10^{-6}$ m for the Berea sandstone and  $l_m = 5.001 \cdot 10^{-6}$ m for the sand pack.

Fluid flow in the pore space is solved numerically using the SIMPLE algorithm implemented in OpenFOAM (Weller et al., 1998). Pressure boundary conditions are set at the inlet (x=0) and outlet  $(x = 900l_m)$  of the domains. No-slip boundary conditions are prescribed at the void-solid interface and at the lateral boundaries of the domain. Once the solver has converged, the flow velocities are extracted at the centers of the interfaces of the mesh (that is, at the six faces of each of the regular hexahedra that form the mesh) in the normal direction to the face.

The ratio between the mean flow speed  $\langle v \rangle$  and the mean flow velocity  $\langle u \rangle$  in streamwise direction defines the advective tortuosity  $\chi = \langle v \rangle / \langle u \rangle$ . For the Berea sample, we find  $\chi = 1.64$ , and for the sand pack  $\chi = 1.32$ . Since for Stokes flow, the flow velocities scale with the pressure gradient, the flow field is determined for a unit gradient and then scaled for the Péclet scenario under consideration. For example, for Pe = 200, the mean flow speeds are  $\langle v \rangle = 2.67 \times 10^{-3}$  m/s. The mean streamwise velocities can be obtained from the respective tortuosity values.

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# 2.3.2 Random walk particle tracking

Solute transport is modeled using random walk particle tracking (Noetinger et al.,
2016). The numerical simulation is based on the discretized version of the Langevin equation (3),

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$$\mathbf{x}(t + \Delta t) = \mathbf{x}(t) + \mathbf{u}[\mathbf{x}(t)]\Delta t + \sqrt{2D\Delta t}\boldsymbol{\zeta}(t), \tag{19}$$

where  $\boldsymbol{\zeta} = (\zeta_1, \zeta_2, \zeta_3)$ . The  $\zeta_i$  are independent random variables that are uniformly dis-280 tributed in  $\left[-\sqrt{3},\sqrt{3}\right]$ . The central limit theorem ensures that the sum of these uniform 281 random variables is Gaussian distributed with zero mean and unit variance. The par-282 ticle velocities  $\mathbf{u}[\mathbf{x}(t)]$  are interpolated from the velocities at the voxel faces using the 283 algorithm of Mostaghimi et al. (2012), which implements a quadratic interpolation in 284 the void voxels that are in contact with the solid and thus guarantees an accurate rep-285 resentation of the flow field in the vicinity of the solid-void interface. The time step is 286 variable and chosen such that the particle displacement at a given step is shorter than 287 or equal to the side length of a voxel. The time step varies from  $\Delta t = 10^{-8}$  s at early 288 times to get an accurate resolution of the moments to  $\Delta t = 10^{-3}$ s at late times to en-289 sure faster simulations. The diffusion coefficient is set to  $D = 10^{-9} \text{ m}^2/\text{s}$ . 290

To investigate the effective and ensemble dispersion coefficients,  $1.5 \times 10^7$  particles are uniformly placed at a plane perpendicular to the mean flow direction, see Figure 2 for the Berea sandstone. A similar setup is used for the sand-pack. We consider this scenario for Pe = 200 and Pe = 2000.

# <sup>295</sup> **3** Dispersion behavior

In this section, we analyze the dispersion behavior in the sand pack and Berea samples. Figure 2 displays three snapshots of the concentration distribution for the Berea



Figure 2. Snapshots of the conservative simulation for the Berea sandstone for Pe = 2000 at three different times  $t = 0.15\tau_v$ ,  $t = 0.8\tau_v$  and  $t = 5\tau_v$ . The density of particles represents the concentration.

sandstone at Pe = 2000. The concentration distribution is heterogeneous and characterized by fast solute transport along preferential flow paths and retention in slow flowing regions. In the following, we discuss the evolution of the mean displacement, and the longitudinal effective and ensemble dispersion coefficients defined in Section 2.2 for the sand pack and the Berea sandstone samples. In the following figures, time is non-dimensionalized by the advection time  $\tau_v$ .

# 3.1 Center of mass

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Figure 3 shows the evolution of the streamwise center of mass position  $m_1(t)$  of the 305 global solute distribution  $c(\mathbf{x},t)$  in the top panels. The bottom panels show the rate of 306 change  $\delta D_{11}^m(t)$  of the variance of the center of mass positions  $m_1(t|\mathbf{x}')$  of partial plumes 307  $g(\mathbf{x},t|\mathbf{x}')$  defined by (18). The center of mass of the global plume moves with the mean 308 flow velocity  $\langle u \rangle$ , while the center of mass velocities of the partial plumes evolve from 309 the velocities at the respective injection points toward the mean flow velocity. At short 310 times  $t \ll \tau_v$ , that is, travel distances shorter than the average pore size, the center of 311 mass velocities are approximately constant, which implies  $m_1(t; \mathbf{x}') = u_1(\mathbf{x}')t$  and there-312 fore 313

$$\delta D_{11}^m(t) = \sigma_0^2 t, \tag{20}$$

where  $\sigma_0^2$  denotes the initial velocity variability. The initial particle velocities persist until the plume starts sampling the flow field by transverse diffusion across streamlines, and by advection along the streamlines. This ballistic early time regime is observed for both the sand pack and Berea samples.

3.1.1

# 3.1.1 Sand pack sample

The evolution of  $\delta D_{11}^m(t)$  for the sand pack sample is characterized by two regimes. 321 The early time ballistic regime, and a sharp decay after a maximum that is assumed on 322 the advective time scale  $\tau_v$ . This is at first counter-intuitive because transverse diffusion 323 is the only mechanisms that makes the partial plume sample the flow heterogeneity such 324 that the differences between the center of mass positions of different partial plumes de-325 crease. Thus, one would expect that the relevant time scale is set by the characteristic 326 pore length and diffusion, that is, by the diffusion time  $\tau_D$ . Sampling occurs indeed by 327 diffusion in transverse direction. However, the distance  $\ell_c$  to sample a new velocity de-328 pends on the flow rate because streamtubes in low velocity regions are wider than in high 329 velocity regions. Since the flow rate is constant in a streamtube,  $Q_c = \ell_c^2 \langle v \rangle$ , with  $Q_c$ 330



Figure 3. Temporal evolution of the center of mass position of the (black solid line) global plume, and (orange dashed lines) selected partial plumes for the sand-pack with (top left) Pe = 200 and (top right) Pe = 2000, and the Berea sample with (bottom left) Pe = 200 and (bottom right) Pe = 2000. The dashed vertical lines denote (black) the advection time scale  $\tau_v$ , (yellow and orange) the respective diffusion time scales  $\tau_D$ .

a characteristic flow rate, the decorrelation length becomes  $\ell_c = \sqrt{Q_c/\langle v \rangle}$ . Thus, the time scale at which particles decorrelate is

$$\tau_c = \frac{\ell_c^2}{D} = \frac{Q_c}{D\ell_0} \tau_v. \tag{21}$$

From Figure 3, we observe that  $\tau_c \approx \tau_v$ , which means that the characteristic flow rate is  $Q_c \approx D\ell_0$ .

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# 3.1.2 Berea sandstone sample

For the Berea sample, we observe three different regimes for  $\delta D_{11}^m(t)$ . The early time 338 regime is ballistic as discussed above. The start of the second regime is marked by the 339 advective time scale  $\tau_v$  as observed for the sand pack. Here, however,  $\delta D_{11}^m(t)$  does not 340 assume a maximum on the advective time scale and then decays, but keeps increasing 341 until the diffusion time  $\tau_D$ , where it reaches maximum and then shows a rapid decay. 342 The behavior in the second time regime is characterized by the transverse velocity sam-343 pling of particles that are initialized at moderate to high flow velocities on the one hand 344 and the persistence of particles in low velocity conducts on the other hand, which gives 345 rise to the observed sub-linear increase of  $\delta D_{11}^m(t)$ . These low velocities are eliminated 346 on the time scale  $\tau_D$ , which sets the maximum transition time along a conduct. In other 347 words, transition times of particles that move a low velocities along a conduct are cut-348 off at the diffusion time scale (Puyguiraud et al., 2021). 349

In summary, the evolution of the center of mass fluctuations is marked by the ad-350 vection time scale for the sand pack sample, and by the advection and diffusion time scales 351 for the Berea sample. The fact that the intermediate regime is not present for the sand 352 pack sample can be explained by the spatial medium structures of the two samples shown 353 in Figure 1. The structure of the Berea sample can be seen as a connected network of 354 conducts, while the sand pack is more a connected network of pore bodies. These dif-355 ferences are also reflected in the evolutions of the effective and ensemble dispersion co-356 efficients discussed in the next section. 357

#### 3.2 Ensemble and effective dispersion

Figures 4 and 5 show the evolution of the effective and ensemble dispersion coef-359 ficients for the sand pack and Berea samples. One observes a marked difference between 360 the ensemble and effective dispersion coefficients at short and intermediate times. At early 361 times  $t < \tau_0 = D/\langle v \rangle^2 = P e^{-1} \tau_v$ , diffusion dominates over advection, and both the 362 ensemble and effective dispersion coefficients are equal to the molecular diffusion coef-363 ficient D. For  $\tau_0 < t < \tau_v$ , advection starts dominating over diffusion. As outlined in 364 the previous section, particles are transported at their initial velocities that persist over 365 the characteristic length scale  $\ell_0$ . Thus, the ensemble dispersion coefficients evolve bal-366 367 listically in this regime

$$D_{11}^{\rm ens}(t) = \sigma_0^2 t, \tag{22}$$

where  $\sigma_0^2$  is the initial velocity variance. It behaves in the same way as  $\Delta D_{11}^m(t)$ , see Eq. (20).

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This effect of the center of mass fluctuations between partial plumes is removed by 372 the definition of the effective dispersion coefficients as the average dispersion coefficient 373 of the partial plumes. For  $\tau_0 < t < \tau_v$ , a partial plume is translated by its initial ve-374 locity. As its size increases by diffusion, the plume gets sheared by the transverse veloc-375 ity contrast. Therefore, the effective dispersion coefficients  $D_{11}^{\text{eff}}(t)$  first remain at the value 376 of the molecular diffusion coefficient and then increase steeply due to shear dispersion. 377 Figures 4b and 5b show that the increase of the effective dispersion coefficients occurs 378 for high Pe at earlier non-dimensional times than for low Pe. This indicates that the 379 shear rate does not scale linearly with  $\langle u \rangle$ . In fact, a typical shear rate can be written 380 381 as

$$\gamma = \frac{\langle v \rangle}{\ell_{\gamma}},\tag{23}$$

where  $\ell_{\gamma}$  is the scale of transverse velocity contrast. The latter is proportional to the typical streamtube size. That is, as  $\ell_{\gamma}^2 \langle v \rangle = \text{constant}$ , we have  $\ell_{\gamma} \sim \langle v \rangle^{-1/2}$ . The characteristic shear length scale decreases with increasing flow rate, and thus the shear rate scales as  $\gamma \sim \langle u \rangle^{3/2}$ . Thus, the characteristic shear time scale  $\tau_{\gamma} = \gamma^{-1} \propto \tau_v / \langle v \rangle^{1/2}$ . This dependence explains the differences in the time behaviors of the effective dispersion coefficients for different *Pe*.

The early time ballistic and shear dispersion behaviors for  $t < \tau_v$  are observed for both the sand pack and Berea samples. For  $t > \tau_v$  the dispersion behaviors are different.

# 3.2.1 Sand pack sample

Figures 4a–d show the evolution of the ensemble and effective dispersion coefficients for the sand pack sample. For times  $t > \tau_v$ , that is for mean travel distances larger than the average pore size, particles start sampling different flow velocities along their trajectories, and the ballistic behavior for the ensemble dispersion coefficients breaks down, see Figure 4a.



Figure 4. Dispersion coefficients of the sand pack. Top panels: (Black solid lines) Ensemble and (blue solid lines) effective dispersion coefficients for (a) Pe = 200 and (b) Pe = 2000. Bottom panels: (c) Ensemble dispersion coefficients for (red solid line) Pe = 2000 and (orange solid line) Pe = 200 for the sand pack, and (d) corresponding effective dispersion coefficients. The vertical dashed lines denote the decorrelation time scale  $\tau_c = \tau_v$ . The horizontal dash-dotted lines denote the asymptotic short time and long time values.

For purely advective transport, the ensemble dispersion coefficients continue growing non-linearly with time, which can be traced back to the broad distribution of transition time across pores (Puyguiraud et al., 2019). At finite Pe, the ensemble dispersion coefficients first follow the purely advective behavior and eventually cross over toward their asymptotic value on the time scale. The effective dispersion coefficients shown in Figure 4 cross over toward their asymptotic values, also on the time scale  $\tau_v$ . As shown in Figures 4c and d, they converge with  $D_{11}^{ens}(t)$ .

As mentioned in Section 3.1, these behaviors are at first sight counter-intuitive be-406 cause we expect the deviation from the purely advective behavior observed for  $D_{11}^{\text{ens}}(t)$ and the convergence of  $D_{11}^{\text{eff}}(t)$  toward  $D_{11}^{\text{ens}}(t)$  to be governed by diffusion. For ensem-408 ble dispersion, diffusion is the mechanism that decorrelates subsequent (low) velocities 409 in time and thus leads to the separation of  $D_{11}^{ens}(t)$  from the (anomalous) purely advec-410 tive behavior. Similarly, the mechanism by which the effective dispersion coefficients con-411 verge toward the ensemble dispersion coefficients is due to decorrelation of the particles 412 that start from the same point, which is due to diffusion in transverse direction. Thus 413 one would expect that the dispersion coefficients evolve on the diffusion time scale  $\tau_D$ . 414

As discussed in Section 3.1.1, the decorrelation mechanism is indeed transverse diffusion across a length scale that is related to a typical streamtube width. Thus, the decorrelation time  $\tau_c$  is given by Eq. (21), which is proportional to  $\tau_v$ . This observation explains the temporal evolution of the ensemble and effective dispersion coefficients for  $t < \tau_v$ .

# 3.2.2 Berea sandstone sample

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Figures 5a-d show the evolution of the ensemble and effective dispersion coefficients 421 for the Berea sandstone sample. As seen in Figure 5a, the initial ballistic behavior for 422 the ensemble dispersion coefficients breaks down on the time scale  $\tau_v$  when particles start 423 sampling different flow velocities along their trajectories. For purely advective transport, we observe anomalous dispersion characterized by a super-linear growth of the ensem-425 ble dispersion coefficients, which can be traced back to broad distributions of advective 426 particle transition times (Puyguiraud et al., 2019). Unlike for the sand pack, here the 427 cross-over toward the constant asymptotic long time values occurs on the diffusion time 428 scale  $\tau_D$ . As discussed in Section 3.1.2, here the temporal decorrelation of low velocities 429 is due to diffusion along pore channels with the characteristic time scale  $\tau_D$  (Puyguiraud 430 et al., 2021). Similary, the convergence of the effective dispersion coefficient shown in Fig-431 ure 5b occurs on the time scale  $\tau_D$ . 432

The cross-over of the effective to the ensemble dispersion coefficients shown in Fig-433 ures 5c and d occurs on the decorrelation time scale  $\tau_c$ , see Eq. (21). This time scale is 434 set by transverse diffusion across streamtubes, which is the mechanisms by which par-435 ticles that originate at the same initial position start decorrelating and sampling differ-436 ent flow velocities. The independent sampling of flow velocities along trajectories between 437 different particles is the ensemble mechanism of dispersion as measured by the ensem-438 ble dispersion coefficients, and therefore effective and ensemble dispersion converge on 439 the scale  $\tau_c$ . 440

# 441 4 Conclusions

We investigate solute dispersion in three-dimensional porous rocks using detailed numerical simulations of pore-scale flow and transport. We consider a sand-like medium, and a Berea sandstone sample. The two media have quite distinct pore structure, which manifests in distinct pore-scale flow variability. The latter is quantified by the distribution of Eulerian flow speeds. The degree of flow heterogeneity is measured by the variance of the logarithm of the flow speed, which is significantly higher for the Berea sam-



**Figure 5.** Dispersion coefficients for the Berea sandstone sample. Top panels: (a) Ensemble dispersion coefficients for (red solid line) Pe = 2000 and (orange solid line) Pe = 200, and (b) corresponding effective dispersion coefficients. The vertical dashed lines denote the corresponding diffusion time scale  $\tau_D = \tau_v Pe$ . Bottom panels: (Black solid lines) Ensemble and (blue solid lines) effective dispersion coefficients for (a) Pe = 200 and (b) Pe = 2000. The vertical black dashed lines denote the decorrelation time scale  $\tau_c = \tau_v$ , the blue dashed lines the respective diffusion time scales. The horizontal dash-dotted lines denote the asymptotic short time and long time values.

ple than for the sand pack sample. Solute dispersion is quantified by effective and en-448 semble dispersion coefficients. The former is defined in terms of the spatial average of 449 the second-centered moments of the partial plumes (Green functions) that constitute the 450 global solute distribution. Ensemble dispersion coefficients are defined in terms of the 451 second centered moments of the global solute plume. Thus, the effective dispersion co-452 efficients can be seen as a measure for the typical width of a mixing front, while the en-453 semble dispersion coefficients are a measure for its deformation due to the flow variabil-454 ity within the initial plume. The mechanisms that cause hydrodynamic dispersion are 455 pore-scale flow variability and molecular diffusion, and govern the evolution of both the 456 effective and ensemble dispersion coefficients. They eventually converge toward the same 457 asymptotic value, which quantifies the impact of spatial heterogeneity on large-scale mix-458 ing. 459

The early time behavior of the ensemble coefficient is ballistic as a result of the spa-460 tial persistence of flow velocities in the initial plume. The effective coefficients on the other 461 hand are significantly smaller than their ensemble counterparts. Their early time evo-462 lution is dominated by shear dispersion, which results from the velocity gradients within the partial plumes, whose lateral extent initially increases by diffusion. The two disper-464 sion coefficients start converging when the lateral extent of the partial plumes is large 465 enough for the efficient sampling of the flow heterogeneity, and it is here, where disper-466 sion in the sand pack and Berea sandstone behave differently. For the sand pack, the evo-467 lution of effective dispersion is marked by the characteristic diffusion time across a stream-468 tube, which sets the time for both convergence to ensemble dispersion and its asymp-469 totic behavior. For the Berea sandstone, this time scale marks the time for convergence 470 of effective and ensemble dispersion, which, however, still evolve non-linearly with time 471 until they assume their asymptotic long time value on the time scale for diffusion over 472 a typical pore length. These behaviors can be traced back to the network-like medium 473 structure in case of the Berea sample, and the strong connectivity of pores in the sand 474 pack. Thus, the evolution of solute dispersion reflects the medium structure, which de-475 termines the microscopic mass transfer mechanisms. While the behavior of ensemble dis-476 persion can be captured by travel-time based approaches like the continuous time ran-477 dom walk in terms of flow variability and medium structure, it is still elusive how to quan-478 tify effective dispersion in these terms. 479

We argue that it is first important to realize that solute dispersion evolves in time, 480 and on time scales that are relevant for the understanding of transport phenomena of 481 reactive solutes and microbes, for example. Second, it is important to realize that there 482 is a conceptual and quantitative difference between solute spreading, as quantified by 483 ensemble dispersion, and solute mixing, which is represented here by effective dispersion 484 because it measures the typical rate of growth of the width of a partial plume that evolves 485 from a point-like injection. The temporal evolution of effective dispersion from molec-486 ular diffusion to asymptotic hydrodynamic dispersion sheds light on the evolution of mix-487 ing fronts in porous media, and may explain phenomena of incomplete mixing observed 488 for fast chemical reactions in porous media. 489

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<sup>495</sup> played in the figures can be downloaded at http://hdl.handle.net/10261/331188.

# 496 References

<sup>497</sup> Ahmadi, A., Quintard, M., & Whitaker, S. (1998, September). Transport in chem-

<ul> <li>sources, 22(1), 59-86. doi: 10.1016/s0309-1708(97)00032-8</li> <li>Alhashni, Z., Blunt, M., &amp; Bijeljic, B. (2016). The impact of pore structure heterogeneity, transport, and reaction conditions on fluid-fluid reaction rate studied on images of pore space. Transport in Porvus Media, 115(2), 215-237.</li> <li>Alim, K., Paras, S., Weitz, D. A., &amp; Brenner, M. P. (2017, Oct). Local pore size correlations determine flow distributions in porous media. <i>Phys. Rev. Lett.</i>, 119, 144501. doi: 10.1103/PhysRevLett.119.144501</li> <li>Attinger, S., Dentz, M., Kinzelbach, H., &amp; Kinzelbach, W. (1999). Temporal behavior of a solute cloud in a chemically heterogeneous porous medium. J. Fluid Mech., 386, 77-104.</li> <li>Battiato, I., &amp; Tartakovsky, D. M. (2011, MAR 1). Applicability regimes for macroscopic models of reactive transport in porous media [Article]. Journal of Contaminant Hydrology, 120-21 (SI), 18-26. doi: 10.1016/j.jconhyd.2010.05.005</li> <li>Battiato, I., Tartakovsky, D. M., Tartakovsky, A. M., &amp; Scheibe, T. (2009). On breakdown of macroscopic models of mixing-controlled heterogeneous reactions in porous media. Advances in water resources, 32(11), 1664-1673.</li> <li>Bijeljic, B., &amp; Blunt, M. J. (2006). Pore-scale modeling and continuous time random walk analysis of dispersion in porous media. Water Resources Research, 42(1). doi: 10.1029/2005WR004578</li> <li>Bijeljic, B., Muggeridge, A. H., &amp; Blunt, M. J. (2004). Pore-scale modeling of longitudinal dispersion. Water Resources Research, 49(1).</li> <li>Bouchaud, JP., &amp; Georges, A. (1990). Anomalous diffusion in disordered media: statistical mechanisms, models and physical applications. Physics reports, 19(54-6), 127-293.</li> <li>Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. Phys. Rev. Lett., 111, 064501. Retrieved from https:// 1108, 45.4, 261-382.</li> <li>Dagan, G. (1990). Transport in heterogeneous formations: Spatial moments, ergodicity, and effective disper</li></ul>	498	ically and mechanically heterogeneous porous media. Advances in Water Re-
<ul> <li>Alhashmi, Z., Blunt, M., &amp; Bijcljic, B. (2016). The impact of pore structure hetero- geneity, transport, and reaction conditions on fluid-fluid reaction rate studied on images of pore space. Transport in Porous Media, 115(2), 215–237.</li> <li>Alim, K., Parsa, S., Weitz, D. A., &amp; Brenner, M. P. (2017, Oct). Local pore size cor- relations determine flow distributions in porous media. <i>Phys. Rev. Lett.</i>, 119, 144501. Retrieved from https://link.aps.org/doi/10.1103/PhysRevLett .119.144501 doi: 10.1103/PhysRevLett.119.144501</li> <li>Attinger, S., Dentz, M., Kinzelbach, H., &amp; Kinzelbach, W. (1999). Temporal behavior of a solute cloud in a chemically heterogeneous porous medium. J. Fluid Mech., 386, 77–104.</li> <li>Battiato, I., &amp; Tartakovsky, D. M. (2011, MAR 1). Applicability regimes for macro- scopic models of reactive transport in porous media [Article]. Journal of Con- tarinant Hydrology, 120-21 (SI), 18-26. doi: 10.1016/j.jconhyd.2010.05.005</li> <li>Battiato, I., Tartakovsky, D. M., Tartakovsky, A. M., &amp; Scheibe, T. (2009). On breakdown of macroscopic models of mixing-controlled heterogeneous reactions in porous media. Advances in water resources, 32(11), 1664–1673.</li> <li>Bijeljic, B., Muggeridge, A. H., &amp; Blunt, M. J. (2004). Pore-scale modeling of longi- tudinal dispersion. Water Resources Research, 40(11).</li> <li>Bonchaud, JP., &amp; Goczyes, A. (1990). Anomalous diffusion in disordered me- dia: statistical mechanisms, models and physical applications. Physics reports, 195(4-5), 127–293.</li> <li>Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. Proc. Roy. Soc. A, 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baket colomite, and indiana limestone. In Spe international sympo- sium on oilfield chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calcula- tions on smoothed fields. J. Contan. Hydrol., 58</li></ul>	499	sources, $22(1)$ , 59–86. doi: $10.1016/s0309-1708(97)00032-8$
<ul> <li>geneity, transport, and reaction conditions on fluid-fluid reaction rate studied on images of pore space. Transport in Porous Media, 115(2), 215–237.</li> <li>Alim, K., Parsa, S., Weitz, D. A., &amp; Brenner, M. P. (2017, Oct). Local pore size cor- relations determine flow distributions in porous media. <i>Phys. Rev. Lett.</i>, 119, 144501. Retrieved from https://link.ags.org/doi/10.1103/PhysRevLett .119.144501 doi: 10.1103/PhysRevLett.119.144501</li> <li>Attinger, S., Dentz, M., Kinzelbach, H., &amp; Kinzelbach, W. (1999). Temporal be- havior of a solute cloud in a chemically heterogeneous porous medium. <i>J. Fluid Mech.</i>, 386, 77–104.</li> <li>Battiato, I., &amp; Tartakovsky, D. M. (2011, MAR 1). Applicability regimes for macro- scopic models of reactive transport in porous media [Article]. <i>Journal of Con- taminant Hydrology</i>, 120-21 (SI), 18-26. doi: 10.1016/j.jcomhyd.2010.05.005</li> <li>Battiato, I., Tartakovsky, D. M., Tartakovsky, A. M., &amp; Scheibe, T. (2009). On breakdown of macroscopic models of mixing-controlled heterogeneous reactions in porous media. Advances in water resources, 32(11), 1664–1673.</li> <li>Bijeljic, B., &amp; Blunt, M. J. (2006). Pore-scale modeling and continuous time random walk analysis of dispersion in porous media. <i>Water Resources Research</i>, 42(1). doi: 10.1022/005WR004578</li> <li>Bijeljic, B., Muggeridge, A. H., &amp; Blunt, M. J. (2004). Pore-scale modeling of longi- tudinal dispersion. <i>Water Resources Research</i>, 40(11).</li> <li>Bouchaud, JP., &amp; Georges, A. (1990). Anomalous diffusion in disordered me- dia: statistical mechanisms, models and physical applications. <i>Physics reports</i>, 195(4-5), 127–203.</li> <li>Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. <i>Proc. Roy. Soc. A</i>, 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana limestone. In <i>Spe international sympo- sium on oilfield chemistry</i>.</li> <li>Cirp</li></ul>	500	Alhashmi, Z., Blunt, M., & Bijeljic, B. (2016). The impact of pore structure hetero-
<ul> <li>an images of pore space. Transport in Porous Media, 115(2), 215–237.</li> <li>Alim, K., Paras, S., Weitz, D. A., &amp; Brenner, M. P. (2017, Oct). Local pore size correlations determine flow distributions in porous media. <i>Phys. Rev. Lett.</i>, 119, 144501.</li> <li>Retrieved from https://link.aps.org/doi/10.1103/PhysRevLett .119.144501</li> <li>Attinger, S., Dentz, M., Kinzelbach, H., &amp; Kinzelbach, W. (1999). Temporal behavior of a solute cloud in a chemically heterogeneous porous media. <i>J. Fluid Mech.</i>, 386, 77–104.</li> <li>Battiato, I., &amp; Tartakovsky, D. M. (2011, MAR 1). Applicability regimes for macroscopic models of reactive transport in porous media [Article]. Journal of Contaminant Hydrology, 120–21(SI), 18–26. doi: 10.1016/j.jconhyd.2010.05.005</li> <li>Battiato, I., Tartakovsky, D. M., Tartakovsky, A. M., &amp; Scheiber, T. (2009). On breakdown of macroscopic models of mixing-controlled heterogeneous reactions in porous media. Advances in water resources, 32(11), 1664–1673.</li> <li>Bijeljie, B., &amp; Blunt, M. J. (2006). Pore-scale modeling and continuous time random walk analysis of dispersion in porous media. Water Resources Research, 42(1). doi: 10.1029/2005WR004578</li> <li>Bijeljie, B., Muggeridge, A. H., &amp; Blunt, M. J. (2004). Pore-scale modeling of longitudinal dispersion. Water Resources Research, 40(11).</li> <li>Bouchaud, JP., &amp; Georges, A. (1990). Anomalous diffusion in disordered media: statistical mechanisms, models and physical applications. Physics reports, 195(4-5), 127–293.</li> <li>Birenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. Proc. Roy. Soc. A, 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berca sandstone, baker dolomite, and indiana limestone. In Spc international symposium on oilfield chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calculations on smoothed fields. J. Contam. Hydrol., 58</li></ul>	501	geneity, transport, and reaction conditions on fluid-fluid reaction rate studied
<ul> <li>Ahm, K., Parsa, S., Weitz, D. A., &amp; Brenner, M. P. (2017, Oct). Local pore size correlations determine flow distributions in porous media. <i>Phys. Rev. Lett.</i>, 119, 144501. Retrieved from https://link.aps.org/doi/10.1103/PhysRevLett .119.144501</li> <li>Attinger, S., Dentz, M., Kinzelbach, H., &amp; Kinzelbach, W. (1999). Temporal behavior of a solute cloud in a chemically heterogeneous porous medium. <i>J. Fluid Mech.</i>, 386, 77–104.</li> <li>Battiato, I., &amp; Tartakovsky, D. M. (2011, MAR 1). Applicability regimes for macroscopic models of reactive transport in porous media [Article]. <i>Journal of Contanianat Hydrology</i>, 130–21(St), 18–26. doi: 10.1016/j.joonlyd.2010.05.005</li> <li>Battiato, I., Tartakovsky, D. M., Tartakovsky, A. M., &amp; Scheibe, T. (2009). On breakdown of macroscopic models of mixing-controlled heterogeneous reactions in porous media. <i>Advances in water resources</i>, 32(11), 1664–1673.</li> <li>Bijeljic, B., &amp; Blunt, M. J. (2006). Pore-scale modeling and continuous time random walk analysis of dispersion in porous media. <i>Water Resources Research</i>, 42(1). doi: 10.1029/2005WR004578</li> <li>Bijeljic, R., Muggeridge, A. H., &amp; Blunt, M. J. (2004). Pore-scale modeling of longitudinal dispersion. <i>Water Resources Research</i>, 40(11).</li> <li>Bouchaud, JP., &amp; Georges, A. (1990). Anomalous diffusion in disordered media: statistical mechanisms, models and physical applications. <i>Physics reports</i>, 195(4-5), 127–293.</li> <li>Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. <i>S. S., Chang. J. J. &amp; Schramm</i>, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana linestone. In <i>Spe international symposium on oilfield chemistry</i>.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calculations on smoothed fields. <i>J. Contam. Hydrol.</i>, 58(3-4), 261–282.</li> <li>Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective disper</li></ul>	502	on images of pore space. Transport in Porous Media, 115(2), 215–237.
<ul> <li>Freiations determine now distributions in porous media. Phys. Rev. Lett., 119, 144501.</li> <li>Attinger, S., Dentz, M., Kinzelbach, H., &amp; Kinzelbach, W. (1999). Temporal behavior of a solute cloud in a chemically heterogeneous porous medium. J. Fluid Mech., 386, 77–104.</li> <li>Battiato, I., &amp; Tartakovsky, D. M. (2011, MAR 1). Applicability regimes for macroscopic models of reactive transport in porous media [Article]. Journal of Contaminant Hydrology, 120-21(SI), 18-26. doi: 10.1016/j.jconhyd.2010.05.005</li> <li>Battiato, I., Tartakovsky, D. M., Tartakovsky, A. M., &amp; Scheibe, T. (2009). On breakdown of macroscopic models of mixing-controlled heterogeneous treations in porous media. Advances in water resources, 32(11), 1664-1673.</li> <li>Bijeljic, B., &amp; Bluut, M. J. (2006). Pore-scale modeling and continuous time random walk analysis of dispersion in porous media. Water Resources Research, 42(1). doi: 10.1029/2005WH004578</li> <li>Bijeljic, B., Muggridge, A. H., &amp; Blunt, M. J. (2004). Pore-scale modeling of longitudinal dispersion. Water Resources Research, 40(11).</li> <li>Bouchaud, JP., &amp; Cocrogs, A. (1990). Anomalous diffusion in disordered media: statistical mechanisms, models and physical applications. Physics reports, 195(4-5), 127-293.</li> <li>Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. Proc. Roy. Soc. A, 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana limestone. In Spe international symposium on oiffeld chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calculations on smoothed fields. J. Contam. Hydrol., 58(4-4), 261-282.</li> <li>Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281-1290.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., Weitz,</li></ul>	503	Alim, K., Parsa, S., Weitz, D. A., & Brenner, M. P. (2017, Oct). Local pore size cor-
<ul> <li>144001. Retrieved from https://link.aps.org/doi/10.1103/PhysRevLett</li> <li>1419.144501 doi: 10.1103/PhysRevLett.119.144501</li> <li>Attinger, S., Dentz, M., Kinzelbach, H., &amp; Kinzelbach, W. (1999). Temporal behavior of a solute cloud in a chemically heterogeneous porous medium. J. Fluid Mech., 386, 77–104.</li> <li>Battiato, I., &amp; Tartakovsky, D. M. (2011, MAR 1). Applicability regimes for macroscopic models of reactive transport in porous media [Article]. Journal of Contaminant Hydrology, 120-21 (Sl), 18-26. doi: 10.1016/j.jconhyd.2010.05.005</li> <li>Battiato, I., Tartakovsky, D. M., Tartakovsky, A. M., &amp; Scheibe, T. (2009). On breakdown of macroscopic models of mixing-controlled heterogeneous reactions in porous media. Advances in water resources, 32(11), 1664–1673.</li> <li>Bjeljic, B., &amp; Blunt, M. J. (2006). Pore-scale modeling and continuous time random walk analysis of dispersion in porous media. Water Resources Research, 42(1). doi: 10.1029/2005WR004578</li> <li>Bjeljic, B., Muggeridge, A. H., &amp; Blunt, M. J. (2004). Pore-scale modeling of longitudinal dispersion. Water Resources Research, 40(11).</li> <li>Bouchaud, JP., &amp; Georges, A. (1990). Anomalous diffusion in disordered media: statistical mechanisms, models and physical applications. Physics reports, 195(4-5), 127–293.</li> <li>Brenmer, H. (1980). Dispersion resulting from flow through spatially periodic porous media. Proc. Roy. Soc. A, 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baket dolomite, and indiana limestone. In Spc international symposium on oilfield chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calculations on smoothed fields. J. Contam. Hydrol., 58(3-4), 261-282.</li> <li>Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion coefficionts in heterogeneous porous media. Journal of contaminath h</li></ul>	504	relations determine flow distributions in porous media. <i>Phys. Rev. Lett.</i> , 119,
<ul> <li>Attinger, S., Dentz, M., Kinzelbach, H., &amp; Kinzelbach, W. (1999). Temporal behavior of a solute cloud in a chemically heterogeneous porous medium. J. Fluid Mech., 386, 77–104.</li> <li>Battiato, I., &amp; Tartakovsky, D. M. (2011, MAR 1). Applicability regimes for macroscopic models of reactive transport in porous media [Article]. Journal of Contaminant Hydrology, 120-21 (SI), 18-26. doi: 10.1016/j.jconhyd.2010.05.005</li> <li>Battiato, I., Tartakovsky, D. M., Tartakovsky, A. M., &amp; Scheibe, T. (2009). On breakdown of macroscopic models of mixing-controlled heterogeneous reactions in porous media. Advances in water resources, 32 (11), 1664–1673.</li> <li>Bijeljic, B., &amp; Blunt, M. J. (2006). Pore-scale modeling and continuous time random walk analysis of dispersion in porous media. Water Resources Research, 42 (1). doi: 10.1029/2005WR004578</li> <li>Bijeljic, B., Muggeridge, A. H., &amp; Blunt, M. J. (2004). Pore-scale modeling of longitudinal dispersion. Water Resources Research, 40 (11).</li> <li>Bouchaud, JP., &amp; Georges, A. (1990). Anomalous diffusion in disordered media: statistical mechanisms, models and physical applications. Physics reports, 195 (4-5), 127–293.</li> <li>Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. Proc. Roy. Soc. A, 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana limestone. In Spe international symposium on adjfield chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calculations on smoothed fields. J. Contam. Hydrol., 58 (34.), 261-282.</li> <li>Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281–129.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug). Spatial fluctuations of fluid velocities in flow through a three-dimens</li></ul>	505	144501. Retrieved from https://link.aps.org/doi/10.1103/PhysRevLett
<ul> <li>Attinger, S., Dehtz, M., Khizeioach, H., &amp; Khizeioach, W. (1999). Temporal behavior of a solute cloud in a chemically heterogeneous porous medium. J. Fluid Mech., 386, 77–104.</li> <li>Battiato, I., &amp; Tartakovsky, D. M. (2011, MAR 1). Applicability regimes for macroscopic models of reactive transport in porous media [Article]. Journal of Contaminant Hydrology, 120-21(SI), 18-26. doi: 10.1016/j.jconhyd.2010.05.005</li> <li>Battiato, I., Tartakovsky, D. M., Tartakovsky, A. M., &amp; Scheibe, T. (2009). On breakdown of macroscopic models of mixing-controlled heterogeneous reactions in porous media. Advances in water resources, 32(11), 1664–1673.</li> <li>Bijeljic, B., &amp; Blunt, M. J. (2006). Pore-scale modeling and continuous time random walk analysis of dispersion in porous media. Water Resources Research, 42(1). doi: 10.1029/2005WR004578</li> <li>Bijeljic, B., Muggeridge, A. H., &amp; Blunt, M. J. (2004). Pore-scale modeling of longitudinal dispersion. Water Resources Research, 40(11).</li> <li>Bouchaud, JP., &amp; Georges, A. (1990). Anomalous diffusion in disordered media: statistical mechanisms, models and physical applications. Physics reports, 195(4-5), 127–233.</li> <li>Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. Proc. Roy. Soc. A, 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana limestone. In Spe international symposium on oilfield chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calculations on smoothed fields. J. Contam. Hydrol., 58(3-4), 261-282.</li> <li>Dagan, G. (1990). Transport in heterogeneous prous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281–1290.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug). Spatial fluctuations of fluid velocities in flow through a three-dimensional</li></ul>	506	119.144501 doi: 10.1103/PhysRevLett.119.144501
<ul> <li><sup>536</sup> havior of a source cloud in a chemically neterogeneous porous medium. J. Pada Mech., 386, 77–104.</li> <li><sup>537</sup> Battiato, I., &amp; Tartakovsky, D. M. (2011, MAR 1). Applicability regimes for macroscopic models of reactive transport in porous media [Article]. Journal of Contaminant Hydrology, 120-21(5), 18-26. doi: 10.1016/j.jconhyd.2010.05.005</li> <li><sup>538</sup> Battiato, I., Tartakovsky, D. M., Tartakovsky, A. M., &amp; Scheibe, T. (2009). On breakdown of macroscopic models of mixing-controlled heterogeneous reactions in porous media. Advances in water resources, 32(11), 1664–1673.</li> <li><sup>537</sup> Bijeljic, B., &amp; Blunt, M. J. (2006). Pore-scale modeling and continuous time random walk analysis of dispersion in porous media. Water Resources Research, 42(1). doi: 10.1029/2005WR004578</li> <li><sup>538</sup> Bijeljic, B., Muggeridge, A. H., &amp; Blunt, M. J. (2004). Pore-scale modeling of longitudinal dispersion. Water Resources Research, 40(11).</li> <li><sup>539</sup> Bouchaud, JP., &amp; Georges, A. (1990). Anomalous diffusion in disordered media: statistical mechanisms, models and physical applications. Physics reports, 195(4-5), 127–293.</li> <li><sup>539</sup> Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. Proc. Ray. Soc. A, 297, 81-133.</li> <li><sup>530</sup> Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana limestone. In Spe international symposium on oiffeld chemistry.</li> <li><sup>531</sup> Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calculations on smoothed fields. J. Contam. Hydrol., 58(3-4), 261-282.</li> <li><sup>532</sup> Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281–1290.</li> <li><sup>534</sup> Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug). Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium</li></ul>	507	Attinger, S., Dentz, M., Kinzelbach, H., & Kinzelbach, W. (1999). Temporal be-
<ul> <li>Battia, J., &amp; Tratakovsky, D. M. (2011, MAR 1). Applicability regimes for macroscopic models of reactive transport in porous media [Article]. Journal of Contaminant Hydrology, 120-21(SI), 18-26. doi: 10.016/j.jconhyd.2010.05.005</li> <li>Battiato, I., Tartakovsky, D. M., Tartakovsky, A. M., &amp; Scheibe, T. (2009). On breakdown of macroscopic models of mixing-controlled heterogeneous reactions in porous media. Advances in water resources, 32(11), 1664–1673.</li> <li>Bijeljic, B., &amp; Blunt, M. J. (2006). Pore-scale modeling and continuous time random walk analysis of dispersion in porous media. Water Resources Research, 42(1). doi: 10.1029/2005WR004578</li> <li>Bijeljic, B., Muggeridge, A. H., &amp; Blunt, M. J. (2004). Pore-scale modeling of longitudinal dispersion. Water Resources Research, 40(11).</li> <li>Bouchaud, JP., &amp; Georges, A. (1990). Anomalous diffusion in disordered media: statistical mechanisms, models and physical applications. Physics reports, 195(4-5), 127–293.</li> <li>Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. Proc. Roy. Soc. A, 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana linestone. In Spe international symposium on oilfield chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calculations on smoothed fields. J. Contam. Hydrol., 58(3-4), 261-282.</li> <li>Dagan, C. (1990). Transport in heterogeneous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281–1290.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug). Spatial fluctuations of fluid velocities in flow through a three-dimensional porous media. Journal of contaminant hydrology, 175, 72–83.</li> <li>de Barros, F., Piori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-rea</li></ul>	508	$M_{ach} = 286 - 77 - 104$
<ul> <li>Databalo, J., &amp; Infrancisky, D. M. (2011, MIRT). Applicability regimes of machesistic transport in porous media [Article]. Journal of Contaminant Hydrology, 120-21 (S1), 18-26. doi: 10.1016/j.jconhyd.2010.05.005</li> <li>Battiato, I., Tartakovsky, D. M., Tartakovsky, A. M., &amp; Scheibe, T. (2009). On breakdown of macroscopic models of mixing-controlled heterogeneous reactions in porous media. Advances in water resources, 32(11), 1664-1673.</li> <li>Bijeljic, B., &amp; Blunt, M. J. (2006). Pore-scale modeling and continuous time random walk analysis of dispersion in porous media. Water Resources Research, 42(1). doi: 10.1029/2005WR004578</li> <li>Bijeljic, B., Muggeridge, A. H., &amp; Blunt, M. J. (2004). Pore-scale modeling of longitudinal dispersion. Water Resources Research, 40(11).</li> <li>Bouchaud, JP., &amp; Georges, A. (1990). Anomalous diffusion in disordered media: statistical mechanisms, models and physical applications. Physics reports, 195(4-5), 127-293.</li> <li>Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. Proc. Roy. Soc. A, 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana limestone. In Spe international symposium on oilfield chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calculations on smoothed fields. J. Contam. Hydrol., 58(3-4), 261-282.</li> <li>Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281-1290.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug). Spatial fluctuations of fluid velocities in flow through a three-dimensional porous media. Journal of contaminant hydrology, 175, 72-83.</li> <li>de Barros, F., Ford, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solut</li></ul>	509	Battiato I & Tartakovsky D M (2011 MAR 1) Applicability regimes for macro-
<ul> <li>barbor hockey reasons and provide memory protects in the protect of the process of</li></ul>	510	scopic models of reactive transport in porous media [Article] Journal of Con-
<ul> <li>Battiato, I., Tartakovsky, D. M., Tartakovsky, A. M., &amp; Scheibe, T. (2009). On breakdown of macroscopic models of mixing-controlled heterogeneous reactions in porous media. Advances in water resources, 32(11), 1664–1673.</li> <li>Bijeljic, B., &amp; Blunt, M. J. (2006). Pore-scale modeling and continuous time random walk analysis of dispersion in porous media. Water Resources Research, 42(1). doi: 10.1029/2005WR004578</li> <li>Bijeljic, B., Muggeridge, A. H., &amp; Blunt, M. J. (2004). Pore-scale modeling of longi- tudinal dispersion. Water Resources Research, 40(11).</li> <li>Bouchaud, JP., &amp; Georges, A. (1990). Anomalous diffusion in disordered me- dia: statistical mechanisms, models and physical applications. Physics reports, 195(4-5), 127–293.</li> <li>Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. Proc. Roy. Soc. A, 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana limestone. In Spe international sympo- sium on oilfiel chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calcula- tions on smoothed fields. J. Contam. Hydrol., 58(3-4), 261-282.</li> <li>Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281– 1290.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug). Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. Phys. Rev. Lett., 111, 064501. Retrieved from https:// link.aps.org/doi/10.1103/PhysRevLett.111.064501 doi: 10.1103/ PhysRevLett.111.064501</li> <li>De Barros, F., Fori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. Journal of contaminant hydrology, 175, 72–83.</li> <li>de Barros, F. P., &amp; Dentz, M.</li></ul>	512	taminant Hudrology 120-21 (SI) 18-26 doi: 10.1016/j.iconhyd.2010.05.005
<ul> <li>bertakiown of macroscopic models of mixing-controlled heterogeneous reactions in porous media. Advances in water resources, 32(11), 1664–1673.</li> <li>Bijeljic, B., &amp; Blunt, M. J. (2006). Pore-scale modeling and continuous time random walk analysis of dispersion in porous media. Water Resources Research, 42(1). doi: 10.1029/2005WR004578</li> <li>Bijeljic, B., Muggeridge, A. H., &amp; Blunt, M. J. (2004). Pore-scale modeling of longi- tudinal dispersion. Water Resources Research, 40(11).</li> <li>Bouchaud, JP., &amp; Georges, A. (1990). Anomalous diffusion in disordered me- dia: statistical mechanisms, models and physical applications. Physics reports, 195(4-5), 127–293.</li> <li>Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. Proc. Roy. Soc. A, 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana limestone. In Spe international sympo- sium on oilfield chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calcula- tions on smoothed fields. J. Contam. Hydrol., 58(3-4), 261-282.</li> <li>Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281– 1290.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug). Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. Phys. Rev. Lett., 111.064501 doi: 10.1103/ PhysRevLett.111.064501 doi: 10.1103/ PhysRevLett.111.064501</li> <li>De Barros, F., Fiori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. Journal of contaminant hydrology, 175, 72–83.</li> <li>de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective ver- sus ensemble dispersion and its uncertainty. Advances i</li></ul>	512	Battiato I Tartakovsky D M Tartakovsky A M & Scheibe T (2009) On
<ul> <li>in porous media. Advances in water resources, 32(11), 1664–1673.</li> <li>Bijeljic, B., &amp; Blunt, M. J. (2006). Pore-scale modeling and continuous time random walk analysis of dispersion in porous media. Water Resources Research, 42(1). doi: 10.1029/2005WR004578</li> <li>Bijeljic, B., Muggeridge, A. H., &amp; Blunt, M. J. (2004). Pore-scale modeling of longitudinal dispersion. Water Resources Research, 40(11).</li> <li>Bouchaud, JP., &amp; Georges, A. (1990). Anomalous diffusion in disordered media: statistical mechanisms, models and physical applications. Physics reports, 195(4-5), 127–293.</li> <li>Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. Proc. Roy. Soc. A. 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana limestone. In Spe international symposium on oiffeld chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calculations on smoothed fields. J. Contam. Hydrol., 58(3-4), 261-282.</li> <li>Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281–1290.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug). Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. Phys. Rev. Lett., 111, 064501. Retrieved from https://link.aps.org/doi/10.1103/PhysRevLett.111.064501 doi: 10.1103/PhysRevLett.111.064501 doi: 10.1103/PhysRevLett.111.064501 doi: 10.1103/PhysRevLett.111.064501 doi: 10.1103/PhysRevLett.111.064501 doi: 10.1103/PhysRevLett.2010. Nature Resources for solute transport. Adv. Water Resources in Water Resources, 91, 11-22.</li> <li>Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of solute transport in heterogeneous media: transition from anomalous to normal transport. Adv. Water Resource, 27(2), 155</li></ul>	514	breakdown of macroscopic models of mixing-controlled heterogeneous reactions
<ul> <li>Bijeljic, B., &amp; Blunt, M. J. (2006). Pore-scale modeling and continuous time random walk analysis of dispersion in porous media. Water Resources Research, 42(1). doi: 10.1029/2005WR004578</li> <li>Bijeljic, B., Muggeridge, A. H., &amp; Blunt, M. J. (2004). Pore-scale modeling of longi- tudinal dispersion. Water Resources Research, 40(11).</li> <li>Bouchaud, JP., &amp; Georges, A. (1990). Anomalous diffusion in disordered me- dia: statistical mechanisms, models and physical applications. Physics reports, 195(4-5), 127-293.</li> <li>Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. Proc. Roy. Soc. A, 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana limestone. In Spe international sympo- sium on oilfield chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calcula- tions on smoothed fields. J. Contam. Hydrol., 58(3-4), 261-282.</li> <li>Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281- 1290.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug). Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. Phys. Rev. Lett., 111, 064501. Retrieved from https:// link.aps.org/doi/10.1103/PhysRevLett.111.064501 doi: 10.1103/ PhysRevLett.111.064501</li> <li>De Barros, F., Fiori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. Journal of contaminant hydrology, 175, 72-83.</li> <li>de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective ver- sus ensemble dispersion and its uncertainty. Advances in Water Resources, 91, 11-22.</li> <li>Dentz, M., Cotris, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of s</li></ul>	515	in porous media. Advances in water resources, 32(11), 1664–1673.
<ul> <li>walk analysis of dispersion in porous media. Water Resources Research, 42(1). doi: 10.1029/2005WR004578</li> <li>Bijeljic, B., Muggeridge, A. H., &amp; Blunt, M. J. (2004). Pore-scale modeling of longi- tudinal dispersion. Water Resources Research, 40(11).</li> <li>Bouchaud, JP., &amp; Georges, A. (1990). Anomalous diffusion in disordered me- dia: statistical mechanisms, models and physical applications. Physics reports, 195(4-5), 127-293.</li> <li>Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. Proc. Roy. Soc. A, 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana limestone. In Spe international sympo- sium on oilfield chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calcula- tions on smoothed fields. J. Contam. Hydrol., 58(3-4), 261-282.</li> <li>Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281- 1290.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug). Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. Phys. Rev. Lett., 111, 064501 doi: 10.1103/ PhysRevLett.111.064501</li> <li>De Barros, F., Foird, A., Boos, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. Journal of contaminant hydrology, 175, 72-83.</li> <li>de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective ver- sus ensemble dispersion and its uncertainty. Advances in Water Resources, 91, 11-22.</li> <li>Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of so- lute transport in heterogeneous media: transition from anomalous to normal transport. Adv. Water Resour., 27(2), 155-173.</li> <li>Dentz, M., Hidalgo,</li></ul>	516	Bijeljic, B., & Blunt, M. J. (2006). Pore-scale modeling and continuous time random
<ul> <li>doi: 10.1029/2005WR004578</li> <li>Bijeljic, B., Muggeridge, A. H., &amp; Blunt, M. J. (2004). Pore-scale modeling of longi- tudinal dispersion. Water Resources Research, 40(11).</li> <li>Bouchaud, JP., &amp; Georges, A. (1990). Anomalous diffusion in disordered me- dia: statistical mechanisms, models and physical applications. Physics reports, 195(4-5), 127-293.</li> <li>Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. Proc. Roy. Soc. A, 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana limestone. In Spe international sympo- sium on oilfield chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calcula- tions on smoothed fields. J. Contam. Hydrol., 58(3-4), 261-282.</li> <li>Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281- 1290.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug). Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. Phys. Rev. Lett., 111, 064501. Retrieved from https:// 1ink.aps.org/doi/10.1103/PhysRevLett.111.064501 doi: 10.1103/ PhysRevLett.111.064501</li> <li>De Barros, F., Fiori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. Journal of contaminant hydrology, 175, 72-83.</li> <li>de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective ver- sus ensemble dispersion and its uncertainty. Advances in Water Resources, 91, 11-22.</li> <li>Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of so- lute transport. Adv. Water Resourc., 27(2), 155-173.</li> <li>Dentz, M., de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogen</li></ul>	517	walk analysis of dispersion in porous media. Water Resources Research, $42(1)$ .
<ul> <li>Bijeljic, B., Muggeridge, A. H., &amp; Blunt, M. J. (2004). Pore-scale modeling of longitudinal dispersion. Water Resources Research, 40(11).</li> <li>Bouchaud, JP., &amp; Georges, A. (1990). Anomalous diffusion in disordered media: statistical mechanisms, models and physical applications. Physics reports, 195(4-5), 127-293.</li> <li>Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. Proc. Roy. Soc. A, 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana limestone. In Spe international symposium on oilfield chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calculations on smoothed fields. J. Contam. Hydrol., 58(3-4), 261-282.</li> <li>Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281-1290.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug). Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. Phys. Rev. Lett., 111, 064501. Retrieved from https://link.aps.org/doi/10.1103/PhysRevLett.111.064501 doi: 10.1103/PhysRevLett.111.064501 doi: 10.1103/PhysRevLett.111.064501</li> <li>De Barros, F., Fiori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. Journal of contaminant hydrology, 175, 72-83.</li> <li>de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective versus ensemble dispersion and its uncertainty. Advances in Water Resources, 91, 11-22.</li> <li>Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of solute transport in heterogeneous media: transition from anomalous to normal transport. Adv. Water Resour., 27(2), 155-173.</li> <li>Dentz, M., &amp; de Barros, F. (2015). Mixing-</li></ul>	518	doi: 10.1029/2005WR004578
<ul> <li>tudinal dispersion. Water Resources Research, 40(11).</li> <li>Bouchaud, JP., &amp; Georges, A. (1990). Anomalous diffusion in disordered me- dia: statistical mechanisms, models and physical applications. Physics reports, 195(4-5), 127-293.</li> <li>Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. Proc. Roy. Soc. A, 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana limestone. In Spe international sympo- sium on oilfield chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calcula- tions on smoothed fields. J. Contam. Hydrol., 58(3-4), 261-282.</li> <li>Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281– 1290.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug). Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. Phys. Rev. Lett., 111, 064501. Retrieved from https:// link.aps.org/doi/10.1103/PhysRevLett.111.064501 doi: 10.1103/ PhysRevLett.111.064501</li> <li>De Barros, F., Fiori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. Journal of contaminant hydrology, 175, 72–83.</li> <li>de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective ver- sus ensemble dispersion and its uncertainty. Advances in Water Resources, 91, 11-22.</li> <li>Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of so- lute transport in heterogeneous media: transition from anomalous to normal transport. Adv. Water Resour., 27(2), 155-173.</li> <li>Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. J. Fluid Mech., 777, 178-195.</li> <li< td=""><td>519</td><td>Bijeljic, B., Muggeridge, A. H., &amp; Blunt, M. J. (2004). Pore-scale modeling of longi-</td></li<></ul>	519	Bijeljic, B., Muggeridge, A. H., & Blunt, M. J. (2004). Pore-scale modeling of longi-
<ul> <li>Bouchaud, JP., &amp; Georges, A. (1990). Anomalous diffusion in disordered me- dia: statistical mechanisms, models and physical applications. <i>Physics reports</i>, 195(4-5), 127-293.</li> <li>Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. <i>Proc. Roy. Soc. A</i>, 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana limestone. In <i>Spe international sympo- sium on oilfield chemistry</i>.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calcula- tions on smoothed fields. <i>J. Contam. Hydrol.</i>, 58(3-4), 261-282.</li> <li>Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. <i>Water Resources Research</i>, 26(6), 1281– 1290.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug). Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. <i>Phys. Rev. Lett.</i>, 111, 064501 doi: 10.1103/ PhysRevLett.111.064501 doi: 10.1103/ PhysRevLett.111.</li></ul>	520	tudinal dispersion. Water Resources Research, $40(11)$ .
<ul> <li>dia: statistical mechanisms, models and physical applications. Physics reports, 195(4-5), 127–293.</li> <li>Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. Proc. Roy. Soc. A, 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana limestone. In Spe international symposium on oilfield chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calculations on smoothed fields. J. Contam. Hydrol., 58(3-4), 261-282.</li> <li>Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281–1290.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug). Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. Phys. Rev. Lett., 111, 064501. Retrieved from https://link.aps.org/doi/10.1103/PhysRevLett.111.064501 doi: 10.1103/PhysRevLett.111.064501 doi: 10.1103/PhysRevLett.111.064501</li> <li>De Barros, F., Fiori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous prorus media. Journal of contaminant hydrology, 175, 72–83.</li> <li>de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective versus ensemble dispersion and its uncertainty. Advances in Water Resources, 91, 11–22.</li> <li>Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of solute transport. Adv. Water Resour., 27(2), 155-173.</li> <li>Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. J. Fluid Mech., 777, 178–195.</li> <li>Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts and approaches across scales. Transport in Porous Media, 146, 5–53. doi: 10.1007/s11242-022-01852-x</li> <!--</td--><td>521</td><td>Bouchaud, JP., &amp; Georges, A. (1990). Anomalous diffusion in disordered me-</td></ul>	521	Bouchaud, JP., & Georges, A. (1990). Anomalous diffusion in disordered me-
<ul> <li>195(4-5), 127-293.</li> <li>Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. Proc. Roy. Soc. A, 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana limestone. In Spe international sympo- sium on oilfield chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calcula- tions on smoothed fields. J. Contam. Hydrol., 58(3-4), 261-282.</li> <li>Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281– 1290.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug).</li> <li>Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. Phys. Rev. Lett., 111, 064501 doi: 10.1103/ PhysRevLett.111.064501 doi: 10.1103/ PhysRevLett.111.064501 doi: 10.1103/ PhysRevLett.111.064501</li> <li>De Barros, F., Fiori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. Journal of contaminant hydrology, 175, 72–83.</li> <li>de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective ver- sus ensemble dispersion and its uncertainty. Advances in Water Resources, 91, 11–22.</li> <li>Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of so- lute transport. Adv. Water Resour., 27(2), 155-173.</li> <li>Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. J. Fluid Mech., 777, 178–195.</li> <li>Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts and approaches across scales. Transport in Porous Media, 146, 5–53. doi: 10 .1007/s11242-022-01852-x</li> </ul>	522	dia: statistical mechanisms, models and physical applications. Physics reports,
<ul> <li>Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous media. Proc. Roy. Soc. A, 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana limestone. In Spe international sympo- sium on oilfield chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calcula- tions on smoothed fields. J. Contam. Hydrol., 58 (3-4), 261-282.</li> <li>Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26 (6), 1281– 1290.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug).</li> <li>Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. Phys. Rev. Lett., 111, 064501. Retrieved from https:// link.aps.org/doi/10.1103/PhysRevLett.111.064501 doi: 10.1103/ PhysRevLett.111.064501</li> <li>De Barros, F., Fiori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. Journal of contaminant hydrology, 175, 72–83.</li> <li>de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective ver- sus ensemble dispersion and its uncertainty. Advances in Water Resources, 91, 11–22.</li> <li>Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of so- lute transport in heterogeneous media: transition from anomalous to normal transport. Adv. Water Resour., 27(2), 155-173.</li> <li>Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. J. Fluid Mech., 777, 178–195.</li> <li>Dentz, M., K de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. J. Fluid Mech., 777, 178–195.</li> <li>Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts and appro</li></ul>	523	195(4-5), 127-293.
<ul> <li>media. Proc. Roy. Soc. A, 297, 81-133.</li> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana limestone. In Spe international symposium on oilfield chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calculations on smoothed fields. J. Contam. Hydrol., 58(3-4), 261-282.</li> <li>Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281–1290.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug). Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. Phys. Rev. Lett., 111, 064501. Retrieved from https://link.aps.org/doi/10.1103/PhysRevLett.111.064501 doi: 10.1103/PhysRevLett.111.064501</li> <li>De Barros, F., Fiori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. Journal of contaminant hydrology, 175, 72–83.</li> <li>de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective versus ensemble dispersion and its uncertainty. Advances in Water Resources, 91, 11–22.</li> <li>Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of solute transport in heterogeneous media: transition from anomalous to normal transport. Adv. Water Resour., 27(2), 155-173.</li> <li>Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. J. Fluid Mech., 777, 178–195.</li> <li>Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts and approaches across scales. Transport in Porous Media, 146, 5–53. doi: 10.1007/s11242-022-01852-x</li> </ul>	524	Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous
<ul> <li>Churcher, P., French, P., Shaw, J., &amp; Schramm, L. (1991). Rock properties of berea sandstone, baker dolomite, and indiana limestone. In Spe international symposium on oilfield chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calculations on smoothed fields. J. Contam. Hydrol., 58(3-4), 261-282.</li> <li>Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281–1290.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug). Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. Phys. Rev. Lett., 111, 064501. Retrieved from https://link.aps.org/doi/10.1103/PhysRevLett.111.064501 doi: 10.1103/PhysRevLett.111.064501 doi: 10.1103/PhysRevLett.111.064501</li> <li>De Barros, F., Fiori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. Journal of contaminant hydrology, 175, 72–83.</li> <li>de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective versus ensemble dispersion and its uncertainty. Advances in Water Resources, 91, 11–22.</li> <li>Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of solute transport. Adv. Water Resour., 27(2), 155-173.</li> <li>Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. J. Fluid Mech., 777, 178–195.</li> <li>Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts and approaches across scales. Transport in Porous Media, 146, 5–53. doi: 10.1007/s11242-022-01852-x</li> </ul>	525	media. Proc. Roy. Soc. A, 297, 81-133.
<ul> <li>sandstone, baker dolomite, and indiana limestone. In Spe international symposium on oilfield chemistry.</li> <li>Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calculations on smoothed fields. J. Contam. Hydrol., 58(3-4), 261-282.</li> <li>Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281–1290.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug).</li> <li>Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. Phys. Rev. Lett., 111, 064501. Retrieved from https://link.aps.org/doi/10.1103/PhysRevLett.111.064501</li> <li>De Barros, F., Fiori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. Journal of contaminant hydrology, 175, 72–83.</li> <li>de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective versus ensemble dispersion and its uncertainty. Advances in Water Resources, 91, 11–22.</li> <li>Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of solute transport in heterogeneous media: transition from anomalous to normal transport. Adv. Water Resour., 27(2), 155-173.</li> <li>Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. J. Fluid Mech., 777, 178–195.</li> <li>Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts and approaches across scales. Transport in Porous Media, 146, 5–53. doi: 10.1007/s11242-022-01852-x</li> </ul>	526	Churcher, P., French, P., Shaw, J., & Schramm, L. (1991). Rock properties of berea
<ul> <li><sup>528</sup> Sum on onpetitive chemistry.</li> <li><sup>529</sup> Cirpka, O. A. (2002). Choice of dispersion coefficients in reactive transport calculations on smoothed fields. J. Contam. Hydrol., 58 (3-4), 261-282.</li> <li><sup>531</sup> Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26 (6), 1281–1290.</li> <li><sup>534</sup> Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug). Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. Phys. Rev. Lett., 111, 064501. Retrieved from https://link.aps.org/doi/10.1103/PhysRevLett.111.064501 doi: 10.1103/PhysRevLett.111.064501</li> <li><sup>535</sup> De Barros, F., Fiori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. Journal of contaminant hydrology, 175, 72–83.</li> <li><sup>546</sup> de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective versus ensemble dispersion and its uncertainty. Advances in Water Resources, 91, 11–22.</li> <li><sup>547</sup> Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of solute transport in heterogeneous media: transition from anomalous to normal transport. Adv. Water Resour., 27(2), 155-173.</li> <li><sup>548</sup> Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. J. Fluid Mech., 777, 178–195.</li> <li><sup>549</sup> Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts and approaches across scales. Transport in Porous Media, 146, 5–53. doi: 10</li> <li><sup>540</sup> .1007/s11242-022-01852-x</li> </ul>	527	sandstone, baker dolomite, and indiana limestone. In Spe international sympo-
<ul> <li><sup>539</sup> Chipka, O. A. (2002). Choice of dispersion coefficients in reactive transport calculations on smoothed fields. J. Contam. Hydrol., 58(3-4), 261-282.</li> <li><sup>531</sup> Dagan, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281–1290.</li> <li><sup>534</sup> Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug). Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. Phys. Rev. Lett., 111, 064501. Retrieved from https://link.aps.org/doi/10.1103/PhysRevLett.111.064501 doi: 10.1103/PhysRevLett.111.064501</li> <li><sup>536</sup> De Barros, F., Fiori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. Journal of contaminant hydrology, 175, 72–83.</li> <li><sup>547</sup> de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective versus ensemble dispersion and its uncertainty. Advances in Water Resources, 91, 11–22.</li> <li><sup>548</sup> Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of solute transport in heterogeneous media: transition from anomalous to normal transport. Adv. Water Resour., 27(2), 155-173.</li> <li><sup>549</sup> Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. J. Fluid Mech., 777, 178–195.</li> <li><sup>540</sup> Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts and approaches across scales. Transport in Porous Media, 146, 5–53. doi: 10.1007/s11242-022-01852-x</li> </ul>	528	sium on outpeta chemistry. Cimbre $O_{\rm A}$ (2002) Choice of digneration coefficients in prestive transport coloule
<ul> <li><sup>530</sup> Data, G. (1990). Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281– 1290.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug).</li> <li>Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. Phys. Rev. Lett., 111, 064501. Retrieved from https:// link.aps.org/doi/10.1103/PhysRevLett.111.064501 doi: 10.1103/ PhysRevLett.111.064501</li> <li>De Barros, F., Fiori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. Journal of contaminant hydrology, 175, 72–83.</li> <li>de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective ver- sus ensemble dispersion and its uncertainty. Advances in Water Resources, 91, 11–22.</li> <li>Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of so- lute transport in heterogeneous media: transition from anomalous to normal transport. Adv. Water Resour., 27(2), 155-173.</li> <li>Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. J. Fluid Mech., 777, 178–195.</li> <li>Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts and approaches across scales. Transport in Porous Media, 146, 5–53. doi: 10 .1007/s11242-022-01852-x</li> </ul>	529	tions on smoothed fields $I$ Contam Hudrol 58(3.4) 261 282
<ul> <li><sup>531</sup> Dagail, C. (1550). Transport in neterogeneous porous formations. Spatial monicules, ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281– 1290.</li> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug).</li> <li>Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. Phys. Rev. Lett., 111, 064501. Retrieved from https:// link.aps.org/doi/10.1103/PhysRevLett.111.064501 doi: 10.1103/ PhysRevLett.111.064501</li> <li>De Barros, F., Fiori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. Journal of contaminant hydrology, 175, 72–83.</li> <li>de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective ver- sus ensemble dispersion and its uncertainty. Advances in Water Resources, 91, 11–22.</li> <li>Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of so- lute transport in heterogeneous media: transition from anomalous to normal transport. Adv. Water Resour., 27(2), 155-173.</li> <li>Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. J. Fluid Mech., 777, 178–195.</li> <li>Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts and approaches across scales. Transport in Porous Media, 146, 5–53. doi: 10 .1007/s11242-022-01852-x</li> </ul>	530	Degen $C_{-}(1000)$ Transport in heterogeneous porous formations: Spatial moments
<ul> <li><sup>112</sup> Data, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug).</li> <li><sup>11290</sup> Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. <i>Phys. Rev. Lett.</i>, <i>111</i>, 064501. Retrieved from https://link.aps.org/doi/10.1103/PhysRevLett.111.064501 doi: 10.1103/</li> <li><sup>111</sup> De Barros, F., Fiori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. <i>Journal of contaminant hydrology</i>, <i>175</i>, 72–83.</li> <li><sup>111</sup> de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective versus ensemble dispersion and its uncertainty. <i>Advances in Water Resources</i>, <i>91</i>, 11–22.</li> <li><sup>111</sup> Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of solute transport in heterogeneous media: transition from anomalous to normal transport. <i>Adv. Water Resour.</i>, <i>27</i>(2), 155-173.</li> <li><sup>111</sup> Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. <i>J. Fluid Mech.</i>, <i>777</i>, 178–195.</li> <li><sup>111</sup> Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts and approaches across scales. <i>Transport in Porous Media</i>, <i>146</i>, 5–53. doi: 10 .1007/s11242-022-01852-x</li> </ul>	531	ergodicity and effective dispersion Water Resources Research 26(6) 1281-
<ul> <li>Datta, S. S., Chiang, H., Ramakrishnan, T. S., &amp; Weitz, D. A. (2013, Aug).</li> <li>Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. <i>Phys. Rev. Lett.</i>, <i>111</i>, 064501. Retrieved from https://link.aps.org/doi/10.1103/PhysRevLett.111.064501 doi: 10.1103/PhysRevLett.111.064501</li> <li>De Barros, F., Fiori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. <i>Journal of contaminant hydrology</i>, <i>175</i>, 72–83.</li> <li>de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective versus ensemble dispersion and its uncertainty. <i>Advances in Water Resources</i>, <i>91</i>, 11–22.</li> <li>Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of solute transport in heterogeneous media: transition from anomalous to normal transport. <i>Adv. Water Resour.</i>, <i>27</i>(2), 155-173.</li> <li>Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. <i>J. Fluid Mech.</i>, <i>777</i>, 178–195.</li> <li>Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts and approaches across scales. <i>Transport in Porous Media</i>, <i>146</i>, 5–53. doi: 10 .1007/s11242-022-01852-x</li> </ul>	532	
<ul> <li>Spatial fluctuations of fluid velocities in flow through a three-dimensional porous medium. <i>Phys. Rev. Lett.</i>, 111, 064501. Retrieved from https:// link.aps.org/doi/10.1103/PhysRevLett.111.064501 doi: 10.1103/ PhysRevLett.111.064501</li> <li>De Barros, F., Fiori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. <i>Journal of contaminant hydrology</i>, 175, 72–83.</li> <li>de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective ver- sus ensemble dispersion and its uncertainty. <i>Advances in Water Resources</i>, 91, 11–22.</li> <li>Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of so- lute transport in heterogeneous media: transition from anomalous to normal transport. <i>Adv. Water Resour.</i>, 27(2), 155-173.</li> <li>Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. <i>J. Fluid Mech.</i>, 777, 178–195.</li> <li>Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts and approaches across scales. <i>Transport in Porous Media</i>, 146, 5–53. doi: 10 .1007/s11242-022-01852-x</li> </ul>	534	Datta, S. S., Chiang, H., Ramakrishnan, T. S., & Weitz, D. A. (2013, Aug).
<ul> <li>porous medium. Phys. Rev. Lett., 111, 064501. Retrieved from https:// link.aps.org/doi/10.1103/PhysRevLett.111.064501 doi: 10.1103/ PhysRevLett.111.064501</li> <li>De Barros, F., Fiori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. Journal of contaminant hydrology, 175, 72–83.</li> <li>de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective ver- sus ensemble dispersion and its uncertainty. Advances in Water Resources, 91, 11–22.</li> <li>Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of so- lute transport in heterogeneous media: transition from anomalous to normal transport. Adv. Water Resour., 27(2), 155-173.</li> <li>Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. J. Fluid Mech., 777, 178–195.</li> <li>Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts and approaches across scales. Transport in Porous Media, 146, 5–53. doi: 10 .1007/s11242-022-01852-x</li> </ul>	535	Spatial fluctuations of fluid velocities in flow through a three-dimensional
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<ul> <li>PhysRevLett.111.064501</li> <li>De Barros, F., Fiori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. Journal of contaminant hydrology, 175, 72–83.</li> <li>de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective ver- sus ensemble dispersion and its uncertainty. Advances in Water Resources, 91, 11–22.</li> <li>Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of so- lute transport in heterogeneous media: transition from anomalous to normal transport. Adv. Water Resour., 27(2), 155-173.</li> <li>Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. J. Fluid Mech., 777, 178–195.</li> <li>Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts and approaches across scales. Transport in Porous Media, 146, 5–53. doi: 10 .1007/s11242-022-01852-x</li> </ul>	537	link.aps.org/doi/10.1103/PhysRevLett.111.064501 doi: 10.1103/
<ul> <li><sup>539</sup> De Barros, F., Fiori, A., Boso, F., &amp; Bellin, A. (2015). A theoretical framework for <sup>540</sup> modeling dilution enhancement of non-reactive solutes in heterogeneous porous <sup>541</sup> media. Journal of contaminant hydrology, 175, 72–83.</li> <li><sup>542</sup> de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective ver- <sup>543</sup> sus ensemble dispersion and its uncertainty. Advances in Water Resources, 91, <sup>544</sup> 11–22.</li> <li><sup>545</sup> Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of so- <sup>546</sup> lute transport in heterogeneous media: transition from anomalous to normal <sup>547</sup> transport. Adv. Water Resour., 27(2), 155-173.</li> <li><sup>548</sup> Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport <sup>549</sup> in heterogeneous flows. J. Fluid Mech., 777, 178–195.</li> <li><sup>550</sup> Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts <sup>541</sup> and approaches across scales. Transport in Porous Media, 146, 5–53. doi: 10 <sup>542</sup> .1007/s11242-022-01852-x</li> </ul>	538	PhysRevLett.111.064501
<ul> <li>modeling dilution enhancement of non-reactive solutes in heterogeneous porous media. Journal of contaminant hydrology, 175, 72-83.</li> <li>de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective ver- sus ensemble dispersion and its uncertainty. Advances in Water Resources, 91, 11-22.</li> <li>Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of so- lute transport in heterogeneous media: transition from anomalous to normal transport. Adv. Water Resour., 27(2), 155-173.</li> <li>Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. J. Fluid Mech., 777, 178-195.</li> <li>Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts and approaches across scales. Transport in Porous Media, 146, 5-53. doi: 10 .1007/s11242-022-01852-x</li> </ul>	539	De Barros, F., Fiori, A., Boso, F., & Bellin, A. (2015). A theoretical framework for
<ul> <li>media. Journal of contaminant hydrology, 175, 72–83.</li> <li>de Barros, F. P., &amp; Dentz, M. (2016). Pictures of blockscale transport: Effective versus ensemble dispersion and its uncertainty. Advances in Water Resources, 91, 11–22.</li> <li>Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of solute transport in heterogeneous media: transition from anomalous to normal transport. Adv. Water Resour., 27(2), 155-173.</li> <li>Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. J. Fluid Mech., 777, 178–195.</li> <li>Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts and approaches across scales. Transport in Porous Media, 146, 5–53. doi: 10.1007/s11242-022-01852-x</li> </ul>	540	modeling dilution enhancement of non-reactive solutes in heterogeneous porous
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<ul> <li>sus ensemble dispersion and its uncertainty. Advances in Water Resources, 91, 11–22.</li> <li>Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of so- lute transport in heterogeneous media: transition from anomalous to normal transport. Adv. Water Resour., 27(2), 155-173.</li> <li>Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. J. Fluid Mech., 777, 178–195.</li> <li>Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts and approaches across scales. Transport in Porous Media, 146, 5–53. doi: 10 .1007/s11242-022-01852-x</li> </ul>	542	de Barros, F. P., & Dentz, M. (2016). Pictures of blockscale transport: Effective ver-
<ul> <li><sup>544</sup> 11-22.</li> <li><sup>545</sup> Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of so- lute transport in heterogeneous media: transition from anomalous to normal transport. Adv. Water Resour., 27(2), 155-173.</li> <li><sup>548</sup> Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. J. Fluid Mech., 777, 178-195.</li> <li><sup>550</sup> Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts and approaches across scales. Transport in Porous Media, 146, 5-53. doi: 10 .1007/s11242-022-01852-x</li> </ul>	543	sus ensemble dispersion and its uncertainty. Advances in Water Resources, 91,
<ul> <li>Dentz, M., Cortis, A., Scher, H., &amp; Berkowitz, B. (2004). Time behavior of so- lute transport in heterogeneous media: transition from anomalous to normal transport. Adv. Water Resour., 27(2), 155-173.</li> <li>Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. J. Fluid Mech., 777, 178-195.</li> <li>Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts and approaches across scales. Transport in Porous Media, 146, 5-53. doi: 10 .1007/s11242-022-01852-x</li> </ul>	544	11-22.
<ul> <li><sup>546</sup> lute transport in heterogeneous media: transition from anomalous to normal transport. Adv. Water Resour., 27(2), 155-173.</li> <li><sup>548</sup> Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport in heterogeneous flows. J. Fluid Mech., 777, 178–195.</li> <li><sup>550</sup> Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts and approaches across scales. Transport in Porous Media, 146, 5–53. doi: 10 .1007/s11242-022-01852-x</li> </ul>	545	Dentz, M., Cortis, A., Scher, H., & Berkowitz, B. (2004). Time behavior of so-
<ul> <li><sup>547</sup> transport. Auv. water Resour., 27(2), 155-175.</li> <li><sup>548</sup> Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport <sup>549</sup> in heterogeneous flows. J. Fluid Mech., 777, 178-195.</li> <li><sup>550</sup> Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts <sup>551</sup> and approaches across scales. Transport in Porous Media, 146, 5-53. doi: 10 <sup>552</sup> .1007/s11242-022-01852-x</li> </ul>	546	intertransport in neterogeneous media: transition from anomalous to normal transport. Adv. Water Records $\frac{\partial 7}{2}$ , 155,172
<ul> <li><sup>548</sup> Dentz, M., &amp; de Barros, F. (2015). Mixing-scale dependent dispersion for transport</li> <li><sup>549</sup> in heterogeneous flows. J. Fluid Mech., 777, 178–195.</li> <li><sup>550</sup> Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts</li> <li><sup>551</sup> and approaches across scales. Transport in Porous Media, 146, 5–53. doi: 10</li> <li><sup>552</sup> .1007/s11242-022-01852-x</li> </ul>	547	transport. Aut. Water Resource, $ZI(2)$ , 130-176.
<ul> <li><sup>549</sup> In neterogeneous nows. <i>J. Prata Mech.</i>, <i>177</i>, 173–155.</li> <li><sup>550</sup> Dentz, M., Hidalgo, J. J., &amp; Lester, D. (2023). Mixing in porous media: Concepts</li> <li><sup>551</sup> and approaches across scales. <i>Transport in Porous Media</i>, <i>146</i>, 5–53. doi: 10</li> <li><sup>552</sup> .1007/s11242-022-01852-x</li> </ul>	548	in heterogeneous flows I Fluid Mech $\gamma\gamma\gamma$ 178–105
and approaches across scales. <i>Transport in Porous Media</i> , 146, 5–53. doi: 10 .1007/s11242-022-01852-x	549	Dentz M Hidalgo I J & Lester D (2023) Mixing in norous media: Concepts
552 .1007/s11242-022-01852-x	550	and approaches across scales. Transport in Porous Media. 1/6, 5–53 doi: 10
,	552	.1007/s11242-022-01852-x

553	Dentz, M., Kinzelbach, H., Attinger, S., & Kinzelbach, W. (2000). Temporal behav-
554	ior of a solute cloud in a heterogeneous porous medium: 1. point-like injection.
555	Water Resources Research, 36(12), 3591-3604. Retrieved from https://
556	agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2000WR900162 doi: https://doi.org/10.1029/2000WR900162
557	Dentz M. Le Borgne T. Englert A. & Bijelije B. (2011). Mixing spreading and
558	reaction in heterogeneous media: A brief review Journal of contaminant hu-
555	drology 190 1–17
500	Di Palma P. R. Guvannon N. Parmigiani A. Huber C. Haße F. & Romano
501	E (2019) Impact of synthetic porous medium geometric properties on solute
563	transport using direct 3d pore-scale simulations. <i>Geofluids</i> . 2019.
564	Fiori, A. (2001). On the influence of local dispersion in solute transport through
565	formations with evolving scales of heterogeneity. Water Resources Research,
566	37(2), 235–242.
567	Fiori, A., & Dagan, G. (2000, September). Concentration fluctuations in aquifer
568	transport: a rigorous first-order solution and applications. J. of Cont. Hydrol.,
569	45(1-2), 139-163.
570	Gelhar, L. W., & Axness, C. L. (1983). Three-dimensional stochastic analysis of
571	macrodispersion in aquifers. Water Resour. Res., $19(1)$ , 161–180.
572	Gouze, P., Puyguiraud, A., Porcher, T., & Dentz, M. (2021). Modeling longi-
573	tudinal dispersion in variable porosity porous media: Control of velocity
574	distribution and microstructures. Frontiers in Water, 3. Retrieved from
575	https://www.frontiersin.org/articles/10.3389/frwa.2021.766338 doi:
576	10.3389/frwa.2021.766338
577	Gouze, P., Puyguiraud, A., Roubinet, D., & Dentz, M. (2023). Pore-scale transport
578	in rocks of different complexity modeled by random walk methods. <i>Transport</i>
579	in Porous Media, 146, 39—158. doi: 10.1007/s11242-021-01675-2
580	Gramling, C. M., Harvey, C. F., & Meigs, L. C. (2002, jun). Reactive transport in
581	porous media: a comparison of model prediction with laboratory visualiza-
582	tion. Environmental Science & Technology, 36(11), 2508–2514. Retrieved from
583	http://dx.doi.org/10.1021/es0157144 doi: 10.1021/es0157144
584	Hulin, J. P., & Plona, T. J. (1989, August). "Echo" tracer dispersion in porous me-
585	dia. Physics of Fluids A: Fluid Dynamics, 1(8), 1341–1347. Retrieved 2023-07-
586	13, from https://doi.org/10.1063/1.85/309 doi: 10.1063/1.85/309
587	Imperial College Consortium on Pore-scale Imaging and Modelling. (2014, 10).
588	LV60C sandpack (Iech. Rep.). Retrieved from https://iigshare.com/
589	articles/dataset/LV60C_sandpack/1189272 doi: 10.0084/119.ngsnare
590	1109272.V1
591	Jose, S. C., Ramman, M. A., & Cirpka, O. A. (2004). Large-scale sandbox
592	modia Water Resources Research $10(12)$ Betrieved from https:///
593	arribula. Water nesources neseurch, 40(12). Retrieved from https://
594	agupubs.onrineribrary.wriey.com/doi/abs/10.1029/2004wn000000 doi.
595	Kapoor V Jafvert C T $\mu$ Jun D A (1008) Experimental study of a bimology
596	lar reaction in Poisouillo flow. Water recourses research 2/(8) 1007-2004
597	Khranitchev $\Delta = \&$ Callaghan P.T. (2003 Sentember) Reversible and ir-
598	reversible dispersion in a porcus medium Physics of Fluids 15(9) 2649-
599	2660 Retrieved 2023-03-10 from https://aip.scitation.org/doi/
601	10.1063/1.1596914 (Publisher: American Institute of Physics) doi:
602	10 1063/1 1596914
603	Kitanidis, P. K. (1988). Prediction by the method of moments of transport in a het-
604	erogeneous formation. J. Hudrol. 102(1-4), 453–473.
605	Koch, D. L., & Brady, J. F. (1985, May). Dispersion in fixed beds. <i>Journal of</i>
606	Fluid Mechanics, 154, 399–427. Retrieved from https://doi.org/10.1017/
607	s0022112085001598 doi: 10.1017/s0022112085001598

608	Leal, L. G. (2007). Advanced transport phenomena: Fluid mechanics and con-
609	vective transport processes. Cambridge University Press. doi: 10.1017/ CBO9780511800245
611	Le Borgne T Bolster D Dentz M Anna P & Tartakovsky A (2011) Effec-
612	tive pore-scale dispersion upscaling with a correlated continuous time random
613	walk approach. Water Resources Research, $47(12)$ . Retrieved from https://
614	agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2011WR010457 doi:
615	10.1029/2011WR010457
616	Le Borgne T. Dentz, M. Bolster, D. Carrera, J. de Dreuzy, J. B. & Davy, P.
617	(2010, dec). Non-Fickian mixing: Temporal evolution of the scalar dissipation
618	rate in heterogeneous porous media. Advances in Water Resources, 33(12).
619	1468–1475. doi: 10.1016/j.advwatres.2010.08.006
620	Mostaghimi, P., Bijelijc, B., & Blunt, M. J. (2012, 09). Simulation of flow and dis-
621	persion on pore-space images. SPE Journal, 17(04), 1131-1141. doi: 10.2118/
622	135261-PA
623	Noetinger, B., Roubinet, D., Russian, A., Le Borgne, T., Delay, F., Dentz, M.,
624	Gouze, P. (2016). Random walk methods for modeling hydrodynamic trans-
625	port in porous and fractured media from pore to reservoir scale. Transport in
626	Porous Media, 1-41. doi: 10.1007/s11242-016-0693-z
627	Perez, L. J., Hidalgo, J. J., & Dentz, M. (2019). Upscaling of mixing-limited bi-
628	molecular chemical reactions in poiseuille flow. Water Resources Research,
629	55(1), 249-269. Retrieved from https://agupubs.onlinelibrary.wiley
630	.com/doi/abs/10.1029/2018WR022730 doi: https://doi.org/10.1029/
631	2018WR022730
632	Perez, L. J., Hidalgo, J. J., Puyguiraud, A., Jiménez-Martínez, J., & Dentz, M.
633	(2020). Assessment and prediction of pore-scale reactive mixing from ex-
634	perimental conservative transport data. $Water Resources Research, 56(6),$
635	e2019WR026452.
636	Puyguiraud, A., Gouze, P., & Dentz, M. (2019). Upscaling of anomalous pore-scale
636 637	Puyguiraud, A., Gouze, P., & Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.
636 637 638	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. <i>Transport in Porous Media</i>, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the</li> </ul>
636 637 638 639	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126,</li> </ul>
636 637 638 639 640	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> </ul>
636 637 638 639 640 641	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective</li> </ul>
636 637 638 639 640 641 642	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reac-</li> </ul>
636 637 638 639 640 641 642 643	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://</li> </ul>
636 637 638 639 640 641 642 643 644	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi:</li> </ul>
636 637 638 639 640 641 642 643 644 644	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> </ul>
636 637 638 639 640 641 642 643 644 645 646	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837-855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> </ul>
636 637 638 639 640 641 642 643 644 645 646 646	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009).</li> </ul>
636 637 638 640 641 642 643 644 645 646 647 648	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009). Enhancement of dilution and transverse reactive mixing in porous media: Ex-</li> </ul>
636 637 638 639 640 641 642 643 644 645 646 647 648 649	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009). Enhancement of dilution and transverse reactive mixing in porous media: Experiments and model-based interpretation. Journal of contaminant hydrology, 4120-000.</li> </ul>
636 637 638 639 640 641 642 643 644 645 644 645 646 647 648 649 650	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009). Enhancement of dilution and transverse reactive mixing in porous media: Experiments and model-based interpretation. Journal of contaminant hydrology, 110(3), 130–142.</li> </ul>
636 637 638 639 640 641 642 643 644 645 646 647 648 649 650 651	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009). Enhancement of dilution and transverse reactive mixing in porous media: Experiments and model-based interpretation. Journal of contaminant hydrology, 110(3), 130–142.</li> <li>Saffman, P. (1959). A theory of dispersion in a porous medium. Journal of Fluid</li> </ul>
636 637 638 639 640 641 642 643 644 645 646 645 646 647 648 650 651 652	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009). Enhancement of dilution and transverse reactive mixing in porous media: Experiments and model-based interpretation. Journal of contaminant hydrology, 110(3), 130–142.</li> <li>Saffman, P. (1959). A theory of dispersion in a porous medium. Journal of Fluid Mechanics, 6(03), 321–349.</li> </ul>
636 637 638 640 641 642 643 644 645 646 647 648 649 650 651 652 653	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837-855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009). Enhancement of dilution and transverse reactive mixing in porous media: Experiments and model-based interpretation. Journal of contaminant hydrology, 110(3), 130-142.</li> <li>Saffman, P. (1959). A theory of dispersion in a porous medium. Journal of Fluid Mechanics, 6(03), 321-349.</li> <li>Scheven, U. (2013). Pore-scale mixing and transverse dispersivity of randomly</li> </ul>
636 637 638 639 640 641 642 643 644 645 644 645 646 647 648 650 651 652 653 653	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837-855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009). Enhancement of dilution and transverse reactive mixing in porous media: Experiments and model-based interpretation. Journal of contaminant hydrology, 110(3), 130-142.</li> <li>Saffman, P. (1959). A theory of dispersion in a porous medium. Journal of Fluid Mechanics, 6(03), 321-349.</li> <li>Scheven, U. (2013). Pore-scale mixing and transverse dispersivity of randomly packed monodisperse spheres. Phys. Rev. Lett., 110(21), 214504.</li> </ul>
636 637 638 649 641 642 643 644 645 644 645 646 647 648 650 651 655	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128 (2), 837-855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009). Enhancement of dilution and transverse reactive mixing in porous media: Experiments and model-based interpretation. Journal of contaminant hydrology, 110(3), 130-142.</li> <li>Saffman, P. (1959). A theory of dispersion in a porous medium. Journal of Fluid Mechanics, 6(03), 321-349.</li> <li>Scheven, U. (2013). Pore-scale mixing and transverse dispersivity of randomly packed monodisperse spheres. Phys. Rev. Lett., 110(21), 214504.</li> <li>Sherman, T., Engdahl, N. B., Porta, G., &amp; Bolster, D. (2021). A review of spa-</li> </ul>
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636 637 638 640 641 642 643 644 645 646 647 648 649 650 651 652 653 655 655 655 655	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837-855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009). Enhancement of dilution and transverse reactive mixing in porous media: Experiments and model-based interpretation. Journal of contaminant hydrology, 110(3), 130-142.</li> <li>Saffman, P. (1959). A theory of dispersion in a porous medium. Journal of Fluid Mechanics, 6(03), 321-349.</li> <li>Scheven, U. (2013). Pore-scale mixing and transverse dispersivity of randomly packed monodisperse spheres. Phys. Rev. Lett., 110(21), 214504.</li> <li>Sherman, T., Engdahl, N. B., Porta, G., &amp; Bolster, D. (2021). A review of spatial markov models for predicting pre-asymptotic and anomalous transport in porous and fractured media. Journal of Contaminant Hydrology, 236, 103734. Retrieved from https://www.sciencedirect.com/science/article/pii/co102020202020202020202020202020202020202</li></ul>
636 637 638 639 640 641 642 643 644 645 646 645 646 651 651 652 653 654 655 655 655 655 655	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837-855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009). Enhancement of dilution and transverse reactive mixing in porous media: Experiments and model-based interpretation. Journal of contaminant hydrology, 110(3), 130-142.</li> <li>Saffman, P. (1959). A theory of dispersion in a porous medium. Journal of Fluid Mechanics, 6(03), 321-349.</li> <li>Scheven, U. (2013). Pore-scale mixing and transverse dispersivity of randomly packed monodisperse spheres. Phys. Rev. Lett., 110(21), 214504.</li> <li>Sherman, T., Engdahl, N. B., Porta, G., &amp; Bolster, D. (2021). A review of spatial markov models for predicting pre-asymptotic and anomalous transport in porous and fractured media. Journal of Contaminant Hydrology, 236, 103734. Retrieved from https://www.sciencedirect.com/science/article/pii/S016977220303235 doi: https://doi.org/10.1016/j.jconhyd.2020.103734</li> </ul>
636 637 638 639 640 641 642 643 644 645 644 645 646 649 650 651 655 655 655 655 655 655 655 655 655	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837-855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009). Enhancement of dilution and transverse reactive mixing in porous media: Experiments and model-based interpretation. Journal of contaminant hydrology, 110(3), 130-142.</li> <li>Saffman, P. (1959). A theory of dispersion in a porous medium. Journal of Fluid Mechanics, 6(03), 321-349.</li> <li>Scheven, U. (2013). Pore-scale mixing and transverse dispersivity of randomly packed monodisperse spheres. Phys. Rev. Lett., 110(21), 214504.</li> <li>Sherman, T., Engdahl, N. B., Porta, G., &amp; Bolster, D. (2021). A review of spatial markov models for predicting pre-asymptotic and anomalous transport in porous and fractured media. Journal of Contaminant Hydrology, 236, 103734. Retrieved from https://www.sciencedirect.com/science/article/pii/S016977220303235 doi: https://doi.org/10.1016/j.jconhyd.2020.103734</li> <li>Souzy, M., Lhuissier, H., Méheust, Y., Borgne, T. L., &amp; Metzger, B. (2020, March).</li> </ul>
636 637 638 639 640 641 642 643 644 645 646 647 648 649 650 651 655 655 655 655 655 655 655 655 655	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837-855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advaatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009). Enhancement of dilution and transverse reactive mixing in porous media: Experiments and model-based interpretation. Journal of contaminant hydrology, 110(3), 130-142.</li> <li>Saffman, P. (1959). A theory of dispersion in a porous medium. Journal of Fluid Mechanics, 6(03), 321-349.</li> <li>Scheven, U. (2013). Pore-scale mixing and transverse dispersivity of randomly packed monodisperse spheres. Phys. Rev. Lett., 110(21), 214504.</li> <li>Sherman, T., Engdahl, N. B., Porta, G., &amp; Bolster, D. (2021). A review of spatial markov models for predicting pre-asymptotic and anomalous transport in porous and fractured media. Journal of Contaminant Hydrology, 236, 103734. Retrieved from https://www.sciencedirect.com/science/article/pii/S016977220303235 doi: https://doi.org/10.1016/j.jconhyd.2020.103734</li> <li>Souzy, M., Lhuissier, H., Méheust, Y., Borgne, T. L., &amp; Metzger, B. (2020, March). Velocity distributions, dispersion and stretching in three-dimensional porous media. Inverse of Ehrid Mechanics of 201.</li> </ul>

10.1017/jfm.2020.113 doi: 10.1017/jfm.2020.113 663 Valocchi, A. J., Bolster, D., & Werth, C. J. (2018, December). Mixing-limited re-664 Transport in Porous Media, 130(1), 157-182. actions in porous media. Re-665 trieved from https://doi.org/10.1007/s11242-018-1204-1 doi: 10.1007/ 666 s11242-018-1204-1 667 Vanderborght, J. (2001). Concentration variance and spatial covariance in second-668 order stationary heterogeneous conductivity fields. Water resources research, 669 37(7), 1893-1912.670

- Weller, H. G., Tabor, G., Jasak, H., & Fureby, C. (1998). A tensorial approach to
   computational continuum mechanics using object-oriented techniques. *Comput. Phys.*, 12(6), 620-631. doi: 10.1063/1.168744
- Whitaker, S. (1967, 05). Diffusion and dispersion in porous media. AIChE Journal,
   13, 420 427. doi: 10.1002/aic.690130308

# Temporal evolution of solute dispersion in three-dimensional porous rocks

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# Key Points:

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8	• Pore-scale simulations of temporal evolution of solute dispersion in three-dimensional
9	porous rocks
10	• Systematic study of effective and ensemble dispersion coefficients as measures for
11	solute spreading and mixing
12	• Time evolution of dispersion coefficients is determined by medium structure, pore-

• Time evolution of dispersion coefficients is determined by medium structure, porescale flow heterogeneity and diffusion

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#### 14 Abstract

We study the temporal evolution of solute dispersion in three-dimensional porous rocks 15 of different heterogeneity and pore structure. To this end, we perform direct numerical 16 simulations of pore-scale flow and transport in a sand-like medium, which exhibits mild 17 heterogeneity, and a Berea sandstone, which is characterized by strong heterogeneity as 18 measured by the variance of the logarithm of the flow velocity. Solute dispersion is quan-19 tified by effective and ensemble dispersion coefficients. The former is a measure for the 20 typical width of the plume, the latter for the deformation, that is, the spread of the mix-21 ing front. Both dispersion coefficients evolve from the molecular diffusion coefficients to-22 ward a common finite asymptotic value. Their evolution is governed by the interplay be-23 tween diffusion, pore-scale velocity fluctuations and the medium structure, which deter-24 mine the characteristic diffusion and advection time scales. Dispersion in the sand-like 25 medium evolves on the transverse diffusion time across a characteristic streamtube di-26 ameter, which is the mechanism by which pore-scale flow variability is sampled by the 27 solute. Dispersion in the Berea sandstone in contrast is governed by both the diffusion 28 time across a typical streamtube, and the diffusion time along a pore conduit. These in-29 sights shed light on the evolution of mixing fronts in porous rocks, with implications for 30 the understanding and modeling of transport phenomena of microbes and reactive so-31 lutes in porous media. 32

#### **1 Introduction**

The transport of solutes in porous media is driven by the phenomenon of disper-34 sion, which results from the interplay between advective spreading and diffusion. The 35 former is triggered by the spatial variability of the fluid speed which is controlled by the 36 geometry of the connected pore network (Datta et al., 2013; Alim et al., 2017; Valocchi 37 et al., 2018; Puyguiraud et al., 2021) while the later is ubiquitously controlled by the con-38 centration gradients. The heterogeneity of the porous medium that triggers the flow speed 39 distribution is therefore a primary parameter that controls dispersion from pre-asymptotic 40 to Fickian regime (Dentz et al., 2004; Sherman et al., 2021). Transport in porous me-41 dia is considered in many fields of academic and industrial applications from materials 42 science, engineering and medicine to groundwater hydrology, environmental technolo-43 gies and petroleum engineering, and at many scales from microfluidic applications to ground-44 water management. Beside being necessary for understanding and predicting the spread-45 ing of chemicals such as pollutants or bionutrients, modeling dispersion is required also 46 to understand and predict solute-solute and solute-minerals reactions that can produce 47 new solute species and trigger mineral dissolution and precipitation features, for instance. 48

Dispersion in porous media has been extensively studied from the pore to the re-49 gional scale for decades (Saffman, 1959; Whitaker, 1967; Gelhar & Axness, 1983; Dagan, 50 1990; Dentz et al., 2023). Here we focus on hydrodynamic dispersion due velocity fluc-51 tuations caused by the heterogeneity of the pore space. A main challenge concerns how 52 continuum-scale solute transport can be modeled by macroscopic parameters, such as 53 the dispersion coefficient, that can be inferred experimentally, by using direct pore scale 54 simulations or upscaling methods such as volume averaging or stochastic modeling (Brenner, 55 1980; Ahmadi et al., 1998; Koch & Brady, 1985; Scheven, 2013; Bijeljic & Blunt, 2006; 56 Le Borgne et al., 2011; Souzy et al., 2020; Puyguiraud et al., 2021). Similar challenges 57 are encountered for reactive transport that is controlled by the time resolved distribu-58 tion of the solutes and their mixing. If the reaction thermodynamics and kinetics are known, 59 then the goal is to be able to model the local reaction rate from knowing dispersion prop-60 erties (Battiato et al., 2009; Battiato & Tartakovsky, 2011). However, it is well known 61 that the advection-dispersion equation parameterized by constant asymptotic dispersion 62 coefficients are not suited to evaluate the effective reaction rates, because it assumes full 63 mixing whereas incomplete mixing is the rule during the pre-asymptotic (non-Fickian) 64 dispersion regimes (Rolle et al., 2009; Le Borgne et al., 2010; Dentz et al., 2011; Le Borgne 65

et al., 2011; Puyguiraud et al., 2021). Nevertheless, diffusion and transverse mixing tend
to homogenize concentration and full mixing can be expected in the asymptotic regime,
as long as the characteristic length of heterogeneity is finite. Clearly, the convergence
rate toward asymptotic dispersion and full mixing depend on the medium heterogeneity, but characterizing the relationship is still challenging and requires investigating both
mixing and spreading mechanisms at all scales.

Solute dispersion and its pre-asymptotic behavior have been analyzed in terms of 72 breakthrough curves, the time evolution of the spatial variance of concentration or par-73 74 ticle distributions, or directly from particle velocities, using experiments and direct numerical pore scale simulations (Hulin & Plona, 1989; Khrapitchev & Callaghan, 2003; 75 Bijeljic et al., 2004; Gouze et al., 2021; Puyguiraud et al., 2021; Gouze et al., 2023). These 76 studies, accounting for the heterogeneity as a whole, show that the pore structure shapes 77 the evolution of dispersion during the pre-asymptotic regime and then determine the asymp-78 totic value. Hulin and Plona (1989) and Khrapitchev and Callaghan (2003) study the 79 reversibility of pore-scale dispersion upon flow reversal, which addresses the issue of un-80 der which conditions hydrodynamic dispersion describes solute mixing or advective so-81 lute spreading. As mentioned above, the fundamental mechanisms of hydrodynamic dis-82 persion are pore-scale velocity fluctuations and diffusion. The former mechanism is re-83 versible in the Stokes regime, which holds for typical applications in groundwater resources. 84 Irreversibility, or actual solute mixing is induced by the interaction of spatial velocity 85 fluctuations and molecular diffusion (Dentz et al., 2023). Consider for example a solute 86 that evolves from an extended areal source. At early times, the solute front deforms due 87 to velocity variability within the source distribution, which leads to a complex concen-88 tration distribution, which nevertheless is partially reversible. Hydrodynamic dispersion 89 coefficients that are defined in terms of the spatial variance of the global solute distri-90 bution, measure at pre-asymptotic this advective spreading rather than actual solute mix-91 ing. 92

This issue was recognized by Kitanidis (1988) in the context of solute dispersion 93 in heterogeneous porous formations, and Bouchaud and Georges (1990) in the context of random walks in quenched disordered media. These authors propose to define disper-95 sion coefficients from the second-centered moments of the solute or particle distributions 96 that evolve from a point-like initial condition. In the absence of local scale dispersion 97 or molecular diffusion, these dispersion coefficients are exactly zero. In the following, we 98 refer to this concept as *effective dispersion*. The dispersion concept based on the spa-99 tial variance of the solute concentration evolving from an extended areal or line source, 100 is called *ensemble dispersion* in the following. As outlined above, at preasymptotic times 101 ensemble dispersion measures advective solute spreading rather than mixing. In fact, it 102 measures the center of mass fluctuations of the partial plume that evolves from the point 103 injections that constitute the spatially extended initial distribution (Bouchaud & Georges, 104 1990). Several authors studied these dispersion concept in the context of mixing and dis-105 persion in porous media on the continuum scale characterized by spatially variable hy-106 draulic conductivity (Attinger et al., 1999; Dentz et al., 2000; Fiori, 2001; Fiori & Da-107 gan, 2000; Vanderborght, 2001; Dentz & de Barros, 2015; De Barros et al., 2015; de Bar-108 ros & Dentz, 2016). Dentz et al. (2000) analyzed the time evolution of the effective and 109 ensemble dispersion coefficients. They showed that the time resolved ensemble disper-110 sion coefficient is usually larger than the effective dispersion until the effective disper-111 sion growth rate increases due transverse local dispersion and diffusion and eventually 112 converges with the ensemble dispersion coefficient. This increase of the effective disper-113 sion value denotes the convergence of average local mixing toward macroscopic mixing 114 that accounts for heterogeneity as a whole. Because it is a quantitative way to discrim-115 inate mixing from spreading, the notion of effective dispersion has been discussed and 116 used by several authors for the modeling of experimental and numerical reactive trans-117 port data (Cirpka, 2002; Jose et al., 2004; Perez et al., 2019, 2020; Puyguiraud et al., 2020). 118 As discussed above, most works that analyze effective and ensemble dispersion to quan-119

tify the impact of spatial heterogeneity on solute mixing and spreading consider continuum scale fluctuations of the hydraulic conductivity. To the best of our knowledge, the
concept of effective dispersion has not been studied for transport in three-dimensional
porous media despite its potential to explain the overestimation of pore-scale mixing and
reaction by constant asymptotic hydrodynamic dispersion coefficients (Kapoor et al., 1998;
Gramling et al., 2002; Perez et al., 2019).

In the present communication we investigate in detail the temporal evolution of mix-126 ing and spreading mechanisms occurring in porous media, in order to evaluate the dif-127 ferent regimes in relation with the porous media structure. To this end, we perform three-128 dimensional direct numerical simulations of pore-scale flow and solute transport in a sand-129 pack medium and in a Berea sandstone of distinctly different heterogeneity levels, that 130 can be measured, for instance, by the variance the logarithm of the flow velocity distri-131 bution. Solute dispersion is quantified by the temporal evolution of the effective and of 132 the ensemble dispersion coefficients. This paper is organized as follows: the methodol-133 ogy used to calculate flow and transport and measure dispersion are presented in Sec-134 tion 2. In Section 3, we present the analyze of the dispersion behavior in the sand pack 135 and Berea samples and discuss how these information can help us depicting the differ-136 ent dispersion stages in relation with the porous media structure. Section 4 presents the 137 conclusions of the study. 138

# 139 2 Methodology

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#### 2.1 Pore-scale flow and transport

Flow in three-dimensional porous media, described as dual solid-void structures, is described by the Stokes equation together with the continuity equation (Leal, 2007),

$$\nabla^2 \mathbf{u}(\mathbf{x}) = -\frac{1}{\mu} \nabla p(\mathbf{x}), \qquad \nabla \cdot \mathbf{u}(\mathbf{x}) = 0, \qquad (1)$$

where  $\mu$  is the dynamic viscosity,  $\mathbf{u}(\mathbf{x})$  is the Eulerian velocity and  $p(\mathbf{x})$  is the fluid pressure at position  $\mathbf{x} = (x_1, x_2, x_3)$ . Here, flow is driven by the macroscopic pressure gradient, which is aligned with the *x*-axis of the coordinated system. Zero-flux boundary conditions are set at the solid-void interface and at the lateral domain boundaries.

Transport of solutes is described by the advection-diffusion equation (ADE) for the solute concentration  $c(\mathbf{x}, t)$ 

$$\frac{\partial c(\mathbf{x},t)}{\partial t} + \nabla \cdot [\mathbf{u}(\mathbf{x}) - D\nabla] \ c(\mathbf{x},t) = 0, \tag{2}$$

where  $c(\mathbf{x}, t)$  is the solute concentration at position  $\mathbf{x}$  and time t, and D is the molecular diffusion coefficient. The advection-diffusion equation (2) is equivalent to the Langevin equation (Risken, 1996)

$$\frac{\mathrm{d}\mathbf{x}(t)}{\mathrm{d}t} = \mathbf{u}[\mathbf{x}(t)] + \sqrt{2D}\boldsymbol{\xi}(t), \tag{3}$$

where  $\boldsymbol{\xi}(t)$  is a Gaussian white noise with mean  $\langle \xi_i \rangle = 0$  and covariance  $\langle \xi_j(t)\xi_k(t) \rangle = \delta_{jk}\delta(t-t'); \ \delta_{jk}$  is the Kronecker delta.

The average pore length  $\ell_0$ , the mean streamwise flow velocity  $\langle v \rangle = \langle |v(\mathbf{x})| \rangle$  and the diffusion coefficient D set the advection time  $\tau_v = \ell_0 / \langle v \rangle$  and the characteristic diffusion time  $\tau_D = \ell_0^2 / D$ . The two time scales are compared by the Péclet number  $Pe = \tau_D / \tau_v = \langle v \rangle \ell_0 / D$ .

## <sup>164</sup> 2.2 Mixing versus spreading

In this section, we discuss plume mixing versus spreading in terms of effective and
 ensemble dispersion coefficients. Then, we pose an approximate model based on the con cept of effective dispersion to upscale pore-scale mixing to the continuum scale.

We analyze the mixing and dispersion of a solute by considering the concentration distribution  $c(\mathbf{x}, t)$  for the normalized plane source

$$c(\mathbf{x}, t = 0) = \rho(\mathbf{x}) = \phi^{-1}\delta(x_1)\frac{\mathbb{I}(\mathbf{x} \in \Omega_f)}{wh},$$
(4)

where  $\Omega_f$  denotes the fluid domain and  $\mathbb{I}(\cdot)$  is the indicator function, which is one if its argument is true and zero else. w and h denote the width and height of the medium and  $\phi$  is porosity. The injection plane is large enough such that

$$\int_{\Omega} d\mathbf{x} \rho(\mathbf{x}) = \phi, \tag{5}$$

where  $\Omega$  denotes the bulk domain, that is, the union of fluid domain and solid domain. The solute distribution can be decomposed into partial plumes  $g(\mathbf{x}, t | \mathbf{x}')$  that satisfy Eq. (2)

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$$g(\mathbf{x}, t = 0 | \mathbf{x}') = \delta(\mathbf{x} - \mathbf{x}') \mathbb{I}(\mathbf{x}' \in \Omega_f).$$
(6)

181 Then, we can write the concentration distribution  $c(\mathbf{x}, t)$  as

$$c(\mathbf{x},t) = \int_{\Omega} d\mathbf{x}' \rho(\mathbf{x}') g(\mathbf{x},t|\mathbf{x}').$$
(7)

Note that  $g(\mathbf{x}, t|y', z')$  is the Green function of the transport problem. In the following, we define a surrogate model for the Green function using the concept of effective dispersion.

# 2.2.1 Effective and ensemble dispersion coefficients

In order to define effective and ensemble dispersion coefficients, we consider the moments of the Green function  $g(\mathbf{x}, t | \mathbf{x}')$  and the concentration distribution  $c(\mathbf{x}, t)$ . The first and second moments of  $g(\mathbf{x}, t | \mathbf{x}')$  are defined by

$$m_i(t; \mathbf{x}') = \int d\mathbf{x} x_i g(\mathbf{x}, t | \mathbf{x}'), \tag{8}$$

$$m_{ij}(t; \mathbf{x}') = \int d\mathbf{x} x_i x_j g(\mathbf{x}, t | \mathbf{x}').$$
(9)

The first moments  $m_i(t; \mathbf{x}')$  determine the center of mass position of  $g(\mathbf{x}, t | \mathbf{x}')$ . The second centered moments

$$\kappa_{ij}(t;\mathbf{x}') = m_{ij}^{(2)}(t;\mathbf{x}') - m_i^{(1)}(t;\mathbf{x}')m_j^{(1)}(t;\mathbf{x}')$$
(10)

are measures for the spatial extension of the Green function. The average of  $\kappa_{ij}(t; \mathbf{x}')$ over all Green functions defines the effective second centered moment

$$\kappa_{ij}^{\text{eff}}(t) = \int d\mathbf{x}' \rho(\mathbf{x}') \kappa_{ij}(t; \mathbf{x}').$$
(11)

It is a measure for the average width of the Green function. The temporal rate of growth of  $\kappa_{ij}^{\text{eff}}(t)$  is given by the effective dispersion coefficients

$$D_{ij}^{\text{eff}}(t) = \frac{1}{2} \frac{d}{dt} \kappa_{ij}^{e}(t), \qquad (12)$$

The effective dispersion coefficient measures the rate of growth of the spatial variance of a concentration distribution that evolves from a point-like initial condition.

In full analogy, we define the first and second moments of  $c(\mathbf{x}, t)$  as

$$m_i(t) = \int d\mathbf{x} x_i c(\mathbf{x}, t) = \int d\mathbf{x}' \rho(\mathbf{x}') m_i(t; \mathbf{x}'), \qquad (13)$$

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 $m_{ij}(t) = \int d\mathbf{x} x_i x_j c(\mathbf{x}, t) = \int d\mathbf{x}' \rho(\mathbf{x}') m_{ij}(t; \mathbf{x}').$ (14)

As per the second equality signs, the moments are determined by taking ensemble averages over the moments of the set of Green functions and as such are named the ensemble moments in the following. The second centered ensemble moments are defined by

$$\kappa_{ij}^{\text{ens}}(t) = m_{ij}(t) - m_i(t)m_j(t).$$
 (15)

They are measures for the spatial extension of the concentration distribution, or equivalently for the ensemble of Green functions. The temporal rate of growth of the second centered ensemble moments is measured by the ensemble dispersion coefficients

$$D_{ij}^{\rm ens}(t) = \frac{1}{2} \frac{d}{dt} \kappa_{ij}^{\rm ens}(t).$$
(16)

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228 229 The difference between the ensemble and effective variances,

$$\delta\kappa_{ij}^{m}(t) = \int d\mathbf{x}'\rho(\mathbf{x}') \left[m_{i}^{(1)}(t;\mathbf{x}') - m_{i}^{(1)}(t)\right] \left[m_{j}^{(1)}(t;\mathbf{x}') - m_{j}^{(1)}(t)\right],\tag{17}$$

quantifies the variance of the center of mass fluctuations of the Green functions that constitute the solute plume. Along the same lines, the difference between the ensemble and
effective dispersion coefficients measures the dispersion of the center of mass positions
of the Green functions that constitute the solute plume

$$\delta D_{ij}^m(t) = \frac{1}{2} \frac{d}{dt} \delta \kappa_{ij}^m(t).$$
(18)

In the following, we study the effective and ensemble dispersion coefficients as well as the center of mass fluctuations in streamwise direction, that is, for i = j = 1.

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# 2.3 Numerical simulations

In the following, we describe the studied porous media, the numerical solution of the pore-scale flow problem and of the transport problem using random walk particle tracking.

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# 2.3.1 Porous media and fluid flow

We study two three-dimensional porous media of different complexity, (i) a Berea 237 sandstone sample and (ii) a sand pack sample illustrated in Figure 1 The Berea sample 238 displays a complex pore structure with a porosity of  $\phi = 0.18$ , see also (Puyguiraud et 239 al., 2021). This type of porous rock is considered to be a pertinent large-scale homoge-240 neous proxy of high permeability sedimentary reservoirs (Churcher et al., 1991). The sand 241 pack sample has a high porosity of  $\phi = 0.37$  with a more regular structure of the pore 242 space. The sand-pack image (Sand Pack LV60C) was obtained from the Imperial Col-243 lege image repository (Imperial College Consortium on Pore-scale Imaging and Modelling, 244 2014). It is a compact packing of irregular quartz grains of variable size that is a proxy 245 of sub-surface aquifers (Di Palma et al., 2019). The difference between the two porous 246 medium samples can be illustrated by the distribution of flow speeds (Alhashmi et al., 247 2016) shown in Figure 1. The flow heterogeneity is measured by the variance  $\sigma_f^2$  of the 248



**Figure 1.** Eulerian velocity pdfs for the sand pack (blue circles) and the Berea sandstone (red squares). Inlay: The three-dimensional pore geometry of (left) the sand pack sample (5mm<sup>3</sup>) and of (right) the Berea sandstone (1mm<sup>3</sup>). The grey and blue colors represent the pore space and the solid phase, respectively.

natural logarithm  $f = \ln v$  of the flow speed v. For the Berea sandstone sample, we obtain  $\sigma_f^2 = 10$ , for the sand pack sample  $\sigma_f^2 = 2$ , that is, the Berea sample is significantly more heterogeneous. The characteristic pore length scale is  $\ell_0 = 1.5 \times 10^{-6}$  m both for the Berea and sand pack samples.

<sup>253</sup> Both pore geometries are based on X-Ray microtomography images. The geome-<sup>254</sup> tries are meshed using regular hexahedron cells (voxels). This type of mesh has two ma-<sup>255</sup> jor advantages. Firstly, it perfectly fits the voxels of the X-Ray tomography images, and <sup>256</sup> secondly, it allows for a simple and computationally efficient velocity interpolation scheme, <sup>257</sup> which is required for the transport simulation based on random walk particle tracking (Mostaghimi <sup>258</sup> et al., 2012). Each of the images is decomposed in 900<sup>3</sup> voxels of length  $l_m = 1.060 \cdot$ <sup>259</sup>  $10^{-6}$ m for the Berea sandstone and  $l_m = 5.001 \cdot 10^{-6}$ m for the sand pack.

Fluid flow in the pore space is solved numerically using the SIMPLE algorithm implemented in OpenFOAM (Weller et al., 1998). Pressure boundary conditions are set at the inlet (x=0) and outlet  $(x = 900l_m)$  of the domains. No-slip boundary conditions are prescribed at the void-solid interface and at the lateral boundaries of the domain. Once the solver has converged, the flow velocities are extracted at the centers of the interfaces of the mesh (that is, at the six faces of each of the regular hexahedra that form the mesh) in the normal direction to the face.

The ratio between the mean flow speed  $\langle v \rangle$  and the mean flow velocity  $\langle u \rangle$  in streamwise direction defines the advective tortuosity  $\chi = \langle v \rangle / \langle u \rangle$ . For the Berea sample, we find  $\chi = 1.64$ , and for the sand pack  $\chi = 1.32$ . Since for Stokes flow, the flow velocities scale with the pressure gradient, the flow field is determined for a unit gradient and then scaled for the Péclet scenario under consideration. For example, for Pe = 200, the mean flow speeds are  $\langle v \rangle = 2.67 \times 10^{-3}$  m/s. The mean streamwise velocities can be obtained from the respective tortuosity values.

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# 2.3.2 Random walk particle tracking

Solute transport is modeled using random walk particle tracking (Noetinger et al.,
2016). The numerical simulation is based on the discretized version of the Langevin equation (3),

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$$\mathbf{x}(t + \Delta t) = \mathbf{x}(t) + \mathbf{u}[\mathbf{x}(t)]\Delta t + \sqrt{2D\Delta t}\boldsymbol{\zeta}(t), \tag{19}$$

where  $\boldsymbol{\zeta} = (\zeta_1, \zeta_2, \zeta_3)$ . The  $\zeta_i$  are independent random variables that are uniformly dis-280 tributed in  $\left[-\sqrt{3},\sqrt{3}\right]$ . The central limit theorem ensures that the sum of these uniform 281 random variables is Gaussian distributed with zero mean and unit variance. The par-282 ticle velocities  $\mathbf{u}[\mathbf{x}(t)]$  are interpolated from the velocities at the voxel faces using the 283 algorithm of Mostaghimi et al. (2012), which implements a quadratic interpolation in 284 the void voxels that are in contact with the solid and thus guarantees an accurate rep-285 resentation of the flow field in the vicinity of the solid-void interface. The time step is 286 variable and chosen such that the particle displacement at a given step is shorter than 287 or equal to the side length of a voxel. The time step varies from  $\Delta t = 10^{-8}$  s at early 288 times to get an accurate resolution of the moments to  $\Delta t = 10^{-3}$ s at late times to en-289 sure faster simulations. The diffusion coefficient is set to  $D = 10^{-9} \text{ m}^2/\text{s}$ . 290

To investigate the effective and ensemble dispersion coefficients,  $1.5 \times 10^7$  particles are uniformly placed at a plane perpendicular to the mean flow direction, see Figure 2 for the Berea sandstone. A similar setup is used for the sand-pack. We consider this scenario for Pe = 200 and Pe = 2000.

# <sup>295</sup> **3** Dispersion behavior

In this section, we analyze the dispersion behavior in the sand pack and Berea samples. Figure 2 displays three snapshots of the concentration distribution for the Berea



Figure 2. Snapshots of the conservative simulation for the Berea sandstone for Pe = 2000 at three different times  $t = 0.15\tau_v$ ,  $t = 0.8\tau_v$  and  $t = 5\tau_v$ . The density of particles represents the concentration.

sandstone at Pe = 2000. The concentration distribution is heterogeneous and characterized by fast solute transport along preferential flow paths and retention in slow flowing regions. In the following, we discuss the evolution of the mean displacement, and the longitudinal effective and ensemble dispersion coefficients defined in Section 2.2 for the sand pack and the Berea sandstone samples. In the following figures, time is non-dimensionalized by the advection time  $\tau_v$ .

# 3.1 Center of mass

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Figure 3 shows the evolution of the streamwise center of mass position  $m_1(t)$  of the 305 global solute distribution  $c(\mathbf{x},t)$  in the top panels. The bottom panels show the rate of 306 change  $\delta D_{11}^m(t)$  of the variance of the center of mass positions  $m_1(t|\mathbf{x}')$  of partial plumes 307  $g(\mathbf{x},t|\mathbf{x}')$  defined by (18). The center of mass of the global plume moves with the mean 308 flow velocity  $\langle u \rangle$ , while the center of mass velocities of the partial plumes evolve from 309 the velocities at the respective injection points toward the mean flow velocity. At short 310 times  $t \ll \tau_v$ , that is, travel distances shorter than the average pore size, the center of 311 mass velocities are approximately constant, which implies  $m_1(t; \mathbf{x}') = u_1(\mathbf{x}')t$  and there-312 fore 313

$$\delta D_{11}^m(t) = \sigma_0^2 t, \tag{20}$$

where  $\sigma_0^2$  denotes the initial velocity variability. The initial particle velocities persist until the plume starts sampling the flow field by transverse diffusion across streamlines, and by advection along the streamlines. This ballistic early time regime is observed for both the sand pack and Berea samples.

3.1.1

# 3.1.1 Sand pack sample

The evolution of  $\delta D_{11}^m(t)$  for the sand pack sample is characterized by two regimes. 321 The early time ballistic regime, and a sharp decay after a maximum that is assumed on 322 the advective time scale  $\tau_v$ . This is at first counter-intuitive because transverse diffusion 323 is the only mechanisms that makes the partial plume sample the flow heterogeneity such 324 that the differences between the center of mass positions of different partial plumes de-325 crease. Thus, one would expect that the relevant time scale is set by the characteristic 326 pore length and diffusion, that is, by the diffusion time  $\tau_D$ . Sampling occurs indeed by 327 diffusion in transverse direction. However, the distance  $\ell_c$  to sample a new velocity de-328 pends on the flow rate because streamtubes in low velocity regions are wider than in high 329 velocity regions. Since the flow rate is constant in a streamtube,  $Q_c = \ell_c^2 \langle v \rangle$ , with  $Q_c$ 330



Figure 3. Temporal evolution of the center of mass position of the (black solid line) global plume, and (orange dashed lines) selected partial plumes for the sand-pack with (top left) Pe = 200 and (top right) Pe = 2000, and the Berea sample with (bottom left) Pe = 200 and (bottom right) Pe = 2000. The dashed vertical lines denote (black) the advection time scale  $\tau_v$ , (yellow and orange) the respective diffusion time scales  $\tau_D$ .

a characteristic flow rate, the decorrelation length becomes  $\ell_c = \sqrt{Q_c/\langle v \rangle}$ . Thus, the time scale at which particles decorrelate is

$$\tau_c = \frac{\ell_c^2}{D} = \frac{Q_c}{D\ell_0} \tau_v. \tag{21}$$

From Figure 3, we observe that  $\tau_c \approx \tau_v$ , which means that the characteristic flow rate is  $Q_c \approx D\ell_0$ .

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# 3.1.2 Berea sandstone sample

For the Berea sample, we observe three different regimes for  $\delta D_{11}^m(t)$ . The early time 338 regime is ballistic as discussed above. The start of the second regime is marked by the 339 advective time scale  $\tau_v$  as observed for the sand pack. Here, however,  $\delta D_{11}^m(t)$  does not 340 assume a maximum on the advective time scale and then decays, but keeps increasing 341 until the diffusion time  $\tau_D$ , where it reaches maximum and then shows a rapid decay. 342 The behavior in the second time regime is characterized by the transverse velocity sam-343 pling of particles that are initialized at moderate to high flow velocities on the one hand 344 and the persistence of particles in low velocity conducts on the other hand, which gives 345 rise to the observed sub-linear increase of  $\delta D_{11}^m(t)$ . These low velocities are eliminated 346 on the time scale  $\tau_D$ , which sets the maximum transition time along a conduct. In other 347 words, transition times of particles that move a low velocities along a conduct are cut-348 off at the diffusion time scale (Puyguiraud et al., 2021). 349

In summary, the evolution of the center of mass fluctuations is marked by the ad-350 vection time scale for the sand pack sample, and by the advection and diffusion time scales 351 for the Berea sample. The fact that the intermediate regime is not present for the sand 352 pack sample can be explained by the spatial medium structures of the two samples shown 353 in Figure 1. The structure of the Berea sample can be seen as a connected network of 354 conducts, while the sand pack is more a connected network of pore bodies. These dif-355 ferences are also reflected in the evolutions of the effective and ensemble dispersion co-356 efficients discussed in the next section. 357

#### 3.2 Ensemble and effective dispersion

Figures 4 and 5 show the evolution of the effective and ensemble dispersion coef-359 ficients for the sand pack and Berea samples. One observes a marked difference between 360 the ensemble and effective dispersion coefficients at short and intermediate times. At early 361 times  $t < \tau_0 = D/\langle v \rangle^2 = P e^{-1} \tau_v$ , diffusion dominates over advection, and both the 362 ensemble and effective dispersion coefficients are equal to the molecular diffusion coef-363 ficient D. For  $\tau_0 < t < \tau_v$ , advection starts dominating over diffusion. As outlined in 364 the previous section, particles are transported at their initial velocities that persist over 365 the characteristic length scale  $\ell_0$ . Thus, the ensemble dispersion coefficients evolve bal-366 367 listically in this regime

$$D_{11}^{\rm ens}(t) = \sigma_0^2 t, \tag{22}$$

where  $\sigma_0^2$  is the initial velocity variance. It behaves in the same way as  $\Delta D_{11}^m(t)$ , see Eq. (20).

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This effect of the center of mass fluctuations between partial plumes is removed by 372 the definition of the effective dispersion coefficients as the average dispersion coefficient 373 of the partial plumes. For  $\tau_0 < t < \tau_v$ , a partial plume is translated by its initial ve-374 locity. As its size increases by diffusion, the plume gets sheared by the transverse veloc-375 ity contrast. Therefore, the effective dispersion coefficients  $D_{11}^{\text{eff}}(t)$  first remain at the value 376 of the molecular diffusion coefficient and then increase steeply due to shear dispersion. 377 Figures 4b and 5b show that the increase of the effective dispersion coefficients occurs 378 for high Pe at earlier non-dimensional times than for low Pe. This indicates that the 379 shear rate does not scale linearly with  $\langle u \rangle$ . In fact, a typical shear rate can be written 380 381 as

$$\gamma = \frac{\langle v \rangle}{\ell_{\gamma}},\tag{23}$$

where  $\ell_{\gamma}$  is the scale of transverse velocity contrast. The latter is proportional to the typical streamtube size. That is, as  $\ell_{\gamma}^2 \langle v \rangle = \text{constant}$ , we have  $\ell_{\gamma} \sim \langle v \rangle^{-1/2}$ . The characteristic shear length scale decreases with increasing flow rate, and thus the shear rate scales as  $\gamma \sim \langle u \rangle^{3/2}$ . Thus, the characteristic shear time scale  $\tau_{\gamma} = \gamma^{-1} \propto \tau_v / \langle v \rangle^{1/2}$ . This dependence explains the differences in the time behaviors of the effective dispersion coefficients for different *Pe*.

The early time ballistic and shear dispersion behaviors for  $t < \tau_v$  are observed for both the sand pack and Berea samples. For  $t > \tau_v$  the dispersion behaviors are different.

# 3.2.1 Sand pack sample

Figures 4a–d show the evolution of the ensemble and effective dispersion coefficients for the sand pack sample. For times  $t > \tau_v$ , that is for mean travel distances larger than the average pore size, particles start sampling different flow velocities along their trajectories, and the ballistic behavior for the ensemble dispersion coefficients breaks down, see Figure 4a.



Figure 4. Dispersion coefficients of the sand pack. Top panels: (Black solid lines) Ensemble and (blue solid lines) effective dispersion coefficients for (a) Pe = 200 and (b) Pe = 2000. Bottom panels: (c) Ensemble dispersion coefficients for (red solid line) Pe = 2000 and (orange solid line) Pe = 200 for the sand pack, and (d) corresponding effective dispersion coefficients. The vertical dashed lines denote the decorrelation time scale  $\tau_c = \tau_v$ . The horizontal dash-dotted lines denote the asymptotic short time and long time values.

For purely advective transport, the ensemble dispersion coefficients continue growing non-linearly with time, which can be traced back to the broad distribution of transition time across pores (Puyguiraud et al., 2019). At finite Pe, the ensemble dispersion coefficients first follow the purely advective behavior and eventually cross over toward their asymptotic value on the time scale. The effective dispersion coefficients shown in Figure 4 cross over toward their asymptotic values, also on the time scale  $\tau_v$ . As shown in Figures 4c and d, they converge with  $D_{11}^{ens}(t)$ .

As mentioned in Section 3.1, these behaviors are at first sight counter-intuitive be-406 cause we expect the deviation from the purely advective behavior observed for  $D_{11}^{\text{ens}}(t)$ and the convergence of  $D_{11}^{\text{eff}}(t)$  toward  $D_{11}^{\text{ens}}(t)$  to be governed by diffusion. For ensem-408 ble dispersion, diffusion is the mechanism that decorrelates subsequent (low) velocities 409 in time and thus leads to the separation of  $D_{11}^{ens}(t)$  from the (anomalous) purely advec-410 tive behavior. Similarly, the mechanism by which the effective dispersion coefficients con-411 verge toward the ensemble dispersion coefficients is due to decorrelation of the particles 412 that start from the same point, which is due to diffusion in transverse direction. Thus 413 one would expect that the dispersion coefficients evolve on the diffusion time scale  $\tau_D$ . 414

As discussed in Section 3.1.1, the decorrelation mechanism is indeed transverse diffusion across a length scale that is related to a typical streamtube width. Thus, the decorrelation time  $\tau_c$  is given by Eq. (21), which is proportional to  $\tau_v$ . This observation explains the temporal evolution of the ensemble and effective dispersion coefficients for  $t < \tau_v$ .

# 3.2.2 Berea sandstone sample

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Figures 5a-d show the evolution of the ensemble and effective dispersion coefficients 421 for the Berea sandstone sample. As seen in Figure 5a, the initial ballistic behavior for 422 the ensemble dispersion coefficients breaks down on the time scale  $\tau_v$  when particles start 423 sampling different flow velocities along their trajectories. For purely advective transport, we observe anomalous dispersion characterized by a super-linear growth of the ensem-425 ble dispersion coefficients, which can be traced back to broad distributions of advective 426 particle transition times (Puyguiraud et al., 2019). Unlike for the sand pack, here the 427 cross-over toward the constant asymptotic long time values occurs on the diffusion time 428 scale  $\tau_D$ . As discussed in Section 3.1.2, here the temporal decorrelation of low velocities 429 is due to diffusion along pore channels with the characteristic time scale  $\tau_D$  (Puyguiraud 430 et al., 2021). Similary, the convergence of the effective dispersion coefficient shown in Fig-431 ure 5b occurs on the time scale  $\tau_D$ . 432

The cross-over of the effective to the ensemble dispersion coefficients shown in Fig-433 ures 5c and d occurs on the decorrelation time scale  $\tau_c$ , see Eq. (21). This time scale is 434 set by transverse diffusion across streamtubes, which is the mechanisms by which par-435 ticles that originate at the same initial position start decorrelating and sampling differ-436 ent flow velocities. The independent sampling of flow velocities along trajectories between 437 different particles is the ensemble mechanism of dispersion as measured by the ensem-438 ble dispersion coefficients, and therefore effective and ensemble dispersion converge on 439 the scale  $\tau_c$ . 440

# 441 4 Conclusions

We investigate solute dispersion in three-dimensional porous rocks using detailed numerical simulations of pore-scale flow and transport. We consider a sand-like medium, and a Berea sandstone sample. The two media have quite distinct pore structure, which manifests in distinct pore-scale flow variability. The latter is quantified by the distribution of Eulerian flow speeds. The degree of flow heterogeneity is measured by the variance of the logarithm of the flow speed, which is significantly higher for the Berea sam-



**Figure 5.** Dispersion coefficients for the Berea sandstone sample. Top panels: (a) Ensemble dispersion coefficients for (red solid line) Pe = 2000 and (orange solid line) Pe = 200, and (b) corresponding effective dispersion coefficients. The vertical dashed lines denote the corresponding diffusion time scale  $\tau_D = \tau_v Pe$ . Bottom panels: (Black solid lines) Ensemble and (blue solid lines) effective dispersion coefficients for (a) Pe = 200 and (b) Pe = 2000. The vertical black dashed lines denote the decorrelation time scale  $\tau_c = \tau_v$ , the blue dashed lines the respective diffusion time scales. The horizontal dash-dotted lines denote the asymptotic short time and long time values.

ple than for the sand pack sample. Solute dispersion is quantified by effective and en-448 semble dispersion coefficients. The former is defined in terms of the spatial average of 449 the second-centered moments of the partial plumes (Green functions) that constitute the 450 global solute distribution. Ensemble dispersion coefficients are defined in terms of the 451 second centered moments of the global solute plume. Thus, the effective dispersion co-452 efficients can be seen as a measure for the typical width of a mixing front, while the en-453 semble dispersion coefficients are a measure for its deformation due to the flow variabil-454 ity within the initial plume. The mechanisms that cause hydrodynamic dispersion are 455 pore-scale flow variability and molecular diffusion, and govern the evolution of both the 456 effective and ensemble dispersion coefficients. They eventually converge toward the same 457 asymptotic value, which quantifies the impact of spatial heterogeneity on large-scale mix-458 ing. 459

The early time behavior of the ensemble coefficient is ballistic as a result of the spa-460 tial persistence of flow velocities in the initial plume. The effective coefficients on the other 461 hand are significantly smaller than their ensemble counterparts. Their early time evo-462 lution is dominated by shear dispersion, which results from the velocity gradients within the partial plumes, whose lateral extent initially increases by diffusion. The two disper-464 sion coefficients start converging when the lateral extent of the partial plumes is large 465 enough for the efficient sampling of the flow heterogeneity, and it is here, where disper-466 sion in the sand pack and Berea sandstone behave differently. For the sand pack, the evo-467 lution of effective dispersion is marked by the characteristic diffusion time across a stream-468 tube, which sets the time for both convergence to ensemble dispersion and its asymp-469 totic behavior. For the Berea sandstone, this time scale marks the time for convergence 470 of effective and ensemble dispersion, which, however, still evolve non-linearly with time 471 until they assume their asymptotic long time value on the time scale for diffusion over 472 a typical pore length. These behaviors can be traced back to the network-like medium 473 structure in case of the Berea sample, and the strong connectivity of pores in the sand 474 pack. Thus, the evolution of solute dispersion reflects the medium structure, which de-475 termines the microscopic mass transfer mechanisms. While the behavior of ensemble dis-476 persion can be captured by travel-time based approaches like the continuous time ran-477 dom walk in terms of flow variability and medium structure, it is still elusive how to quan-478 tify effective dispersion in these terms. 479

We argue that it is first important to realize that solute dispersion evolves in time, 480 and on time scales that are relevant for the understanding of transport phenomena of 481 reactive solutes and microbes, for example. Second, it is important to realize that there 482 is a conceptual and quantitative difference between solute spreading, as quantified by 483 ensemble dispersion, and solute mixing, which is represented here by effective dispersion 484 because it measures the typical rate of growth of the width of a partial plume that evolves 485 from a point-like injection. The temporal evolution of effective dispersion from molec-486 ular diffusion to asymptotic hydrodynamic dispersion sheds light on the evolution of mix-487 ing fronts in porous media, and may explain phenomena of incomplete mixing observed 488 for fast chemical reactions in porous media. 489

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# 496 References

<sup>497</sup> Ahmadi, A., Quintard, M., & Whitaker, S. (1998, September). Transport in chem-

498	ically and mechanically heterogeneous porous media. Advances in Water Re-
499	sources, $22(1)$ , 59–86. doi: $10.1016/s0309-1708(97)00032-8$
500	Alhashmi, Z., Blunt, M., & Bijeljic, B. (2016). The impact of pore structure hetero-
501	geneity, transport, and reaction conditions on fluid-fluid reaction rate studied
502	on images of pore space. Transport in Porous Media, 115(2), 215–237.
503	Alim, K., Parsa, S., Weitz, D. A., & Brenner, M. P. (2017, Oct). Local pore size cor-
504	relations determine flow distributions in porous media. <i>Phys. Rev. Lett.</i> , 119,
505	144501. Retrieved from https://link.aps.org/doi/10.1103/PhysRevLett
506	$\frac{119.144501}{1000} \text{ doi: } 10.1105/PhysRevLett.119.144501}$
507	Attinger, S., Dentz, M., Kinzelbach, H., & Kinzelbach, W. (1999). Temporal be-
508	Mach 286 77-104
509	Battiato I & Tartakovsky D M (2011 MAR 1) Applicability regimes for macro-
510	scopic models of reactive transport in porous media [Article] Journal of Con-
512	taminant Hudrologu, 120-21 (SI), 18-26, doi: 10.1016/j.iconhyd.2010.05.005
512	Battiato I. Tartakovsky D. M. Tartakovsky A. M. & Scheibe T. (2009) On
514	breakdown of macroscopic models of mixing-controlled heterogeneous reactions
515	in porous media. Advances in water resources, 32(11), 1664–1673.
516	Bijeljic, B., & Blunt, M. J. (2006). Pore-scale modeling and continuous time random
517	walk analysis of dispersion in porous media. Water Resources Research, $42(1)$ .
518	doi: 10.1029/2005WR004578
519	Bijeljic, B., Muggeridge, A. H., & Blunt, M. J. (2004). Pore-scale modeling of longi-
520	tudinal dispersion. Water Resources Research, $40(11)$ .
521	Bouchaud, JP., & Georges, A. (1990). Anomalous diffusion in disordered me-
522	dia: statistical mechanisms, models and physical applications. <i>Physics reports</i> ,
523	195(4-5), 127-293.
524	Brenner, H. (1980). Dispersion resulting from flow through spatially periodic porous
525	media. Proc. Roy. Soc. A, 297, 81-133.
526	Churcher, P., French, P., Shaw, J., & Schramm, L. (1991). Rock properties of berea
527	sandstone, baker dolomite, and indiana ninestone. In Spe international sympo-
528	Circle $O(\Lambda)$ (2002) Choice of dispersion coefficients in reactive transport calcula
529	tions on smoothed fields I Contam Hudrol 58(3-4) 261-282
530	Dagan G (1990) Transport in heterogeneous porous formations: Spatial moments
532	ergodicity, and effective dispersion. Water Resources Research, 26(6), 1281–
533	1290.
534	Datta, S. S., Chiang, H., Ramakrishnan, T. S., & Weitz, D. A. (2013, Aug).
535	Spatial fluctuations of fluid velocities in flow through a three-dimensional
536	porous medium. Phys. Rev. Lett., 111, 064501. Retrieved from https://
537	link.aps.org/doi/10.1103/PhysRevLett.111.064501 doi: 10.1103/
538	PhysRevLett.111.064501
539	De Barros, F., Fiori, A., Boso, F., & Bellin, A. (2015). A theoretical framework for
540	modeling dilution enhancement of non-reactive solutes in heterogeneous porous
541	media. Journal of contaminant hydrology, 175, 72–83.
542	de Barros, F. P., & Dentz, M. (2016). Pictures of blockscale transport: Effective ver-
543	sus ensemble dispersion and its uncertainty. Advances in Water Resources, 91,
544	11-22.
545	Dentz, M., Cortis, A., Scher, H., & Berkowitz, B. (2004). Time behavior of so-
546	The transport in neterogeneous media: transition from anomalous to normal transport $Adv$ Water Resour $\frac{97(2)}{155-173}$
547	Dentz M & de Barros F (2015) Mixing scale dependent dispersion for transport
540	in heterogeneous flows J Fluid Mech 777 178–105
550	Dentz, M., Hidalgo, J. J., & Lester, D. (2023). Mixing in porous media: Concepts
551	and approaches across scales. Transport in Porous Media, 146, 5–53. doi: 10
552	.1007/s11242-022-01852-x

553	Dentz, M., Kinzelbach, H., Attinger, S., & Kinzelbach, W. (2000). Temporal behav-
554	ior of a solute cloud in a heterogeneous porous medium: 1. point-like injection.
555	Water Resources Research, 36(12), 3591-3604. Retrieved from https://
556	agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2000WR900162 doi: https://doi.org/10.1029/2000WR900162
557	Dontz M. Lo Borgno, T. Englort A. & Bijolije B. (2011) Mixing spreading and
558	reaction in heterogeneous media: A brief review Iournal of contaminant hu-
559	drology 190 1–17
500	Di Palma P R Guvannon N Parmigiani A Huber C Haße F $\&$ Romano
501	$E_{\rm c}$ (2019) Impact of synthetic porous medium geometric properties on solute
502	transport using direct 3d pore-scale simulations Geoffuids 2019
505	Figri A $(2001)$ On the influence of local dispersion in solute transport through
565	formations with evolving scales of heterogeneity Water Resources Research
566	37(2), 235–242.
567	Fiori, A., & Dagan, G. (2000, September). Concentration fluctuations in aquifer
568	transport: a rigorous first-order solution and applications. J. of Cont. Hydrol.,
569	45(1-2), 139-163.
570	Gelhar, L. W., & Axness, C. L. (1983). Three-dimensional stochastic analysis of
571	macrodispersion in aquifers. Water Resour. Res., $19(1)$ , $161-180$ .
572	Gouze, P., Puyguiraud, A., Porcher, T., & Dentz, M. (2021). Modeling longi-
573	tudinal dispersion in variable porosity porous media: Control of velocity
574	distribution and microstructures. Frontiers in Water, 3. Retrieved from
575	https://www.frontiersin.org/articles/10.3389/frwa.2021.766338 doi:
576	10.3389/frwa.2021.766338
577	Gouze, P., Puyguraud, A., Roubinet, D., & Dentz, M. (2023). Pore-scale transport
578	in rocks of different complexity modeled by random walk methods. Transport
579	in Porous Meaia, 14b, $39-158$ . doi: 10.1007/ $s11242-021-01675-2$
580	Gramling, C. M., Harvey, C. F., & Meigs, L. C. (2002, jun). Reactive transport in
581	tion Environmental Science & Technology 26(11) 2508 2514 Detrieved from
582	tion. Environmental Science & Technology, $30(11)$ , $2508-2514$ . Retrieved from http://dx.doi.org/10.1021/oc0157144. doi: 10.1021/oc0157144
583	Hulin I P & Plong T I (1080 August) "Echo" tracer dispersion in percus me
584	dia Physics of Fluids 4: Fluid Dynamics 1(8) 1341-1347 Retrieved 2023-07-
585	13 from https://doi_org/10_1063/1_857309_doi: 10_1063/1_857309
500	Imperial College Consortium on Pore-scale Imaging and Modelling (2014–10)
588	<i>LV60C sandnack</i> (Tech Rep.) Retrieved from https://figshare.com/
589	articles/dataset/LV60C sandpack/1189272 doi: 10.6084/m9.figshare
590	.1189272.v1
591	Jose, S. C., Rahman, M. A., & Cirpka, O. A. (2004). Large-scale sandbox
592	experiment on longitudinal effective dispersion in heterogeneous porous
593	media. Water Resources Research, 40(12). Retrieved from https://
594	agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2004WR003363 doi:
595	https://doi.org/10.1029/2004WR003363
596	Kapoor, V., Jafvert, C. T., & Lyn, D. A. (1998). Experimental study of a bimolecu-
597	lar reaction in Poiseuille flow. Water resources research, 34(8), 1997–2004.
598	Khrapitchev, A. A., & Callaghan, P. T. (2003, September). Reversible and ir-
599	reversible dispersion in a porous medium. Physics of Fluids, 15(9), 2649–
600	2660. Retrieved 2023-03-10, from https://aip.scitation.org/doi/
601	10.1063/1.1596914(Publisher: American Institute of Physics)doi:
602	10.1063/1.1596914
603	Kitanidis, P. K. (1988). Prediction by the method of moments of transport in a het-
604	erogeneous formation. J. Hydrol, $102(1-4)$ , $453-473$ .
605	Koch, D. L., & Brady, J. F. (1985, May). Dispersion in fixed beds. Journal of
606	Fluid Mechanics, 154, 399-427. Retrieved from https://doi.org/10.1017/
607	s0022112085001598 doi: $10.1017/s0022112085001598$

608	Leal, L. G. (2007). Advanced transport phenomena: Fluid mechanics and con-
609	vective transport processes. Cambridge University Press. doi: 10.1017/ CBO9780511800245
611	Le Borgne T Bolster D Dentz M Anna P & Tartakovsky A (2011) Effec-
612	tive pore-scale dispersion upscaling with a correlated continuous time random
613	walk approach. Water Resources Research, $47(12)$ . Retrieved from https://
614	agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2011WR010457 doi:
615	10.1029/2011WR010457
616	Le Borgne T. Dentz, M. Bolster, D. Carrera, J. de Dreuzy, J. B. & Davy, P.
617	(2010, dec). Non-Fickian mixing: Temporal evolution of the scalar dissipation
618	rate in heterogeneous porous media. Advances in Water Resources, 33(12).
619	1468–1475. doi: 10.1016/j.advwatres.2010.08.006
620	Mostaghimi, P., Bijelijc, B., & Blunt, M. J. (2012, 09). Simulation of flow and dis-
621	persion on pore-space images. SPE Journal, 17(04), 1131-1141. doi: 10.2118/
622	135261-PA
623	Noetinger, B., Roubinet, D., Russian, A., Le Borgne, T., Delay, F., Dentz, M.,
624	Gouze, P. (2016). Random walk methods for modeling hydrodynamic trans-
625	port in porous and fractured media from pore to reservoir scale. Transport in
626	Porous Media, 1-41. doi: 10.1007/s11242-016-0693-z
627	Perez, L. J., Hidalgo, J. J., & Dentz, M. (2019). Upscaling of mixing-limited bi-
628	molecular chemical reactions in poiseuille flow. Water Resources Research,
629	55(1), 249-269. Retrieved from https://agupubs.onlinelibrary.wiley
630	.com/doi/abs/10.1029/2018WR022730 doi: https://doi.org/10.1029/
631	2018WR022730
632	Perez, L. J., Hidalgo, J. J., Puyguiraud, A., Jiménez-Martínez, J., & Dentz, M.
633	(2020). Assessment and prediction of pore-scale reactive mixing from ex-
634	perimental conservative transport data. $Water Resources Research, 56(6),$
635	e2019WR026452.
636	Puyguiraud, A., Gouze, P., & Dentz, M. (2019). Upscaling of anomalous pore-scale
636 637	Puyguiraud, A., Gouze, P., & Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.
636 637 638	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. <i>Transport in Porous Media</i>, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the</li> </ul>
636 637 638 639	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126,</li> </ul>
636 637 638 639 640	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> </ul>
636 637 638 639 640 641	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective</li> </ul>
636 637 638 639 640 641 642	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reac-</li> </ul>
636 637 638 639 640 641 642 643	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://</li> </ul>
636 637 638 639 640 641 642 643 644	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi:</li> </ul>
636 637 638 639 640 641 642 643 644 644	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> </ul>
636 637 638 639 640 641 642 643 644 645 646	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837-855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> </ul>
636 637 638 639 640 641 642 643 644 645 646 646	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009).</li> </ul>
636 637 638 640 641 642 643 644 645 646 647 648	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009). Enhancement of dilution and transverse reactive mixing in porous media: Ex-</li> </ul>
636 637 638 639 640 641 642 643 644 645 646 647 648 649	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009). Enhancement of dilution and transverse reactive mixing in porous media: Experiments and model-based interpretation. Journal of contaminant hydrology, 412(2)(2)(2)(2)(2)(2)(2)(2)(2)(2)(2)(2)(2)</li></ul>
636 637 638 639 640 641 642 643 644 645 644 645 646 647 648 649 650	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009). Enhancement of dilution and transverse reactive mixing in porous media: Experiments and model-based interpretation. Journal of contaminant hydrology, 110(3), 130–142.</li> </ul>
636 637 638 639 640 641 642 643 644 645 646 647 648 649 650 651	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009). Enhancement of dilution and transverse reactive mixing in porous media: Experiments and model-based interpretation. Journal of contaminant hydrology, 110(3), 130–142.</li> <li>Saffman, P. (1959). A theory of dispersion in a porous medium. Journal of Fluid</li> </ul>
636 637 638 639 640 641 642 643 644 645 646 645 646 647 648 649 650 651 652	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837–855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009). Enhancement of dilution and transverse reactive mixing in porous media: Experiments and model-based interpretation. Journal of contaminant hydrology, 110(3), 130–142.</li> <li>Saffman, P. (1959). A theory of dispersion in a porous medium. Journal of Fluid Mechanics, 6(03), 321–349.</li> </ul>
636 637 638 640 641 642 643 644 645 646 647 648 649 650 651 652 653	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837-855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009). Enhancement of dilution and transverse reactive mixing in porous media: Experiments and model-based interpretation. Journal of contaminant hydrology, 110(3), 130-142.</li> <li>Saffman, P. (1959). A theory of dispersion in a porous medium. Journal of Fluid Mechanics, 6(03), 321-349.</li> <li>Scheven, U. (2013). Pore-scale mixing and transverse dispersivity of randomly</li> </ul>
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636 637 638 640 641 642 643 644 645 646 647 648 649 650 651 652 653 655 655 655 655	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837-855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advwatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009). Enhancement of dilution and transverse reactive mixing in porous media: Experiments and model-based interpretation. Journal of contaminant hydrology, 110(3), 130-142.</li> <li>Saffman, P. (1959). A theory of dispersion in a porous medium. Journal of Fluid Mechanics, 6(03), 321-349.</li> <li>Scheven, U. (2013). Pore-scale mixing and transverse dispersivity of randomly packed monodisperse spheres. Phys. Rev. Lett., 110(21), 214504.</li> <li>Sherman, T., Engdahl, N. B., Porta, G., &amp; Bolster, D. (2021). A review of spatial markov models for predicting pre-asymptotic and anomalous transport in porous and fractured media. Journal of Contaminant Hydrology, 236, 103734. Retrieved from https://www.sciencedirect.com/science/article/pii/co102020202020202020202020202020202020202</li></ul>
636 637 638 639 640 641 642 643 644 645 646 646 647 648 649 650 651 652 653 654 655 655 655 655 655	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837-855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009). Enhancement of dilution and transverse reactive mixing in porous media: Experiments and model-based interpretation. Journal of contaminant hydrology, 110(3), 130-142.</li> <li>Saffman, P. (1959). A theory of dispersion in a porous medium. Journal of Fluid Mechanics, 6(03), 321-349.</li> <li>Scheven, U. (2013). Pore-scale mixing and transverse dispersivity of randomly packed monodisperse spheres. Phys. Rev. Lett., 110(21), 214504.</li> <li>Sherman, T., Engdahl, N. B., Porta, G., &amp; Bolster, D. (2021). A review of spatial markov models for predicting pre-asymptotic and anomalous transport in porous and fractured media. Journal of Contaminant Hydrology, 236, 103734. Retrieved from https://www.sciencedirect.com/science/article/pii/S016977220303235 doi: https://doi.org/10.1016/j.jconhyd.2020.103734</li> </ul>
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636 637 638 639 640 641 642 643 644 645 646 647 648 649 650 651 655 655 655 655 655 655 655 655 655	<ul> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2019). Upscaling of anomalous pore-scale dispersion. Transport in Porous Media, 128(2), 837-855.</li> <li>Puyguiraud, A., Gouze, P., &amp; Dentz, M. (2021, Apr). Pore-scale mixing and the evolution of hydrodynamic dispersion in porous media. Phys. Rev. Lett., 126, 164501. doi: 10.1103/PhysRevLett.126.164501</li> <li>Puyguiraud, A., Perez, L. J., Hidalgo, J. J., &amp; Dentz, M. (2020). Effective dispersion coefficients for the upscaling of pore-scale mixing and reaction. Advances in Water Resources, 103782. Retrieved from http://www.sciencedirect.com/science/article/pii/S0309170820303006 doi: https://doi.org/10.1016/j.advaatres.2020.103782</li> <li>Risken, H. (1996). The Fokker-Planck equation. Springer Heidelberg New York.</li> <li>Rolle, M., Eberhardt, C., Chiogna, G., Cirpka, O. A., &amp; Grathwohl, P. (2009). Enhancement of dilution and transverse reactive mixing in porous media: Experiments and model-based interpretation. Journal of contaminant hydrology, 110(3), 130-142.</li> <li>Saffman, P. (1959). A theory of dispersion in a porous medium. Journal of Fluid Mechanics, 6(03), 321-349.</li> <li>Scheven, U. (2013). Pore-scale mixing and transverse dispersivity of randomly packed monodisperse spheres. Phys. Rev. Lett., 110(21), 214504.</li> <li>Sherman, T., Engdahl, N. B., Porta, G., &amp; Bolster, D. (2021). A review of spatial markov models for predicting pre-asymptotic and anomalous transport in porous and fractured media. Journal of Contaminant Hydrology, 236, 103734. Retrieved from https://www.sciencedirect.com/science/article/pii/S016977220303235 doi: https://doi.org/10.1016/j.jconhyd.2020.103734</li> <li>Souzy, M., Lhuissier, H., Méheust, Y., Borgne, T. L., &amp; Metzger, B. (2020, March). Velocity distributions, dispersion and stretching in three-dimensional porous media. Inverse of Ehrid Mechanics of 201.</li> </ul>

10.1017/jfm.2020.113 doi: 10.1017/jfm.2020.113 663 Valocchi, A. J., Bolster, D., & Werth, C. J. (2018, December). Mixing-limited re-664 Transport in Porous Media, 130(1), 157-182. actions in porous media. Re-665 trieved from https://doi.org/10.1007/s11242-018-1204-1 doi: 10.1007/ 666 s11242-018-1204-1 667 Vanderborght, J. (2001). Concentration variance and spatial covariance in second-668 order stationary heterogeneous conductivity fields. Water resources research, 669 37(7), 1893-1912.670

- Weller, H. G., Tabor, G., Jasak, H., & Fureby, C. (1998). A tensorial approach to
   computational continuum mechanics using object-oriented techniques. *Comput. Phys.*, 12(6), 620-631. doi: 10.1063/1.168744
- Whitaker, S. (1967, 05). Diffusion and dispersion in porous media. AIChE Journal,
   13, 420 427. doi: 10.1002/aic.690130308