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Draft. Please, Do Not Quote.

March 15, 2022

Abstract

This paper analyzes whether repealing net neutrality (NN) improves or decreases the capacity of a regulator to make internet service providers (ISPs) extend broadband coverage through universal service obligations (USOs). We model a two-sided market where a monopolistic ISP links content providers (CPs) to end users with a broadband network of a given bandwidth. A regulator determines whether to submit the ISP to NN or to allow it to supply paid priority (P) services to CPs. She can also impose a broadband USO to the ISP, i.e. she can mandate the broadband market coverage. We show that the greater is the network bandwidth, the more likely the repeal of net neutrality increases ISP profits and social welfare. Regulation can still be necessary, however, as there are bandwidth ranges for which the ISP would benefit from a repeal of NN while such a repeal is detrimental to society.

Keywords: Internet, Net Neutrality, Universal Service Obligations, Prioritization, Regulation

JEL: D21, K23, L12, L51, L96

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1 Introduction

Most countries impose two types of regulation on Internet Service Providers (ISPs): Net Neutrality (NN) and Universal Service Obligations (USOs). The first prohibits ISPs from "speeding up, slowing down or blocking Internet traffic based on its source, ownership or destination" (Kramer et al. [21]). It aims to promote investment, innovation and competition among content providers (CPs) and more generally, to ensure free speech (Katz [20]). The latter forces ISPs to cover a given percentage of the territory with a minimum broadband standard, set in terms of download and upload speeds. Its goal is to avoid a "digital divide" among citizens of different regions.

Because of the growth of data intensive content on internet, ISPs argue that it is nowadays counterproductive to treat in the same way CPs that require high speed of transmission and that do not tolerate delays, like streaming, from those that are far less demanding on those counts, like emails. Peitz and Schuett [24] show that when content have different sensitivities to delay, letting the ISP organize a paid prioritization service could improve welfare. As a result, there are debates within regulatory agencies and among academics on the ongoing relevance of NN, while the Government of the US has already repealed the NN rules in 2018. In contrast, there is a clear tendency to strengthen USOs almost everywhere in the world, including in the US.

A striking feature of the debates and economic analysis on Internet regulation is that they treat NN and USOs policies independently. On the one hand, the growing literature on NN studies its impact on social welfare, content innovation and network investment (Calzada and Tselekounis [8]). The literature gives two interpretations to NN: in the first, the ISPs cannot charge CPs, i.e. NN is interpreted as a zero price rule on the CP side; in the other, ISPs cannot offer quality differentiated access to CPs, implying that they cannot prioritize some CPs' traffic, and consequently they cannot ask for a payment to prioritized content. Although economic models differ with respect of the assumed market structure (monopoly, oligopoly, vertically integrated firms, etc.) for ISPs as well as for CPs, they are similar in their objectives to compare market equilibria, where each agent maximizes profit or utility, with and without NN. In other words, the only regulation considered is NN and the benchmark case is free market. This ignores the fact that, in reality, there generally exist USOs and subsidies for network extension in remote areas. Subsidizing mechanisms potentially change both the market size and the market configuration.

On the other hand, the early literature on USOs takes their existence as given and evaluate compensations schemes for the ISPs in their capacities to cover the cost burden imposed on the universal service provider (USP) without altering firms' competitive behavior in markets (Gautier and Wauthy [18], for instance). Models also analyze the impact on social welfare and on entry incentives of a uniform price constraint, which is often part of USOs and which prohibits the ISPs to offer tariffs that differ among markets, i.e. to price discriminate (Valletti et al [31], for instance).

The literature on USOs is mute on the NN both because discussions to repeal the latter are rather recent and that, under NN, one can analyze Internet USOs with the same methodological approach than the one applied to other industries already studied, such as electricity, natural gas or traditional telecommunications. Indeed, since CPs do not pay specific charges to ISPs, their uploads are virtually free and, consequently, investment incentives in networks for ISPs come only from the end users willingness to pay for the service, as is the case for the other industries. In other words, with the zero price rule, the ISPs look like one sided firms selling access to internet to consumers. However, this vision is misleading and the behavior of the CP side of the market must be taken into account, especially when the NN rules are repealed.

The separate treatment of NN and USOs is clearly unsatisfactory as both regulations impact directly on Internet service pricing and on the incentives to invest in broadband networks. To evaluate fully the global impact of the repealing of NN on the industry performance in terms of pricing and investment, one has to consider whether it increases or decreases the regulator's capacity to extend USOs. This amounts to the question of whether a regulator who wishes to extend the network above the firms' profit maximizing coverage is able or not to capture to that end a greater slice of the industry rent following the repealing of NN.

In this paper, we analyze both NN and USO in a single model. We consider a two-sided market where a monopolistic ISP can install a broadband network of a given bandwidth at different locations of a country. A regulator determines whether to submit the ISP to NN or to allow it to supply paid priority (P) services to CPs with the objective to maximize market coverage or, provided it has enough instruments, to maximize welfare. Prioritization gives the opportunity to the ISP to obtain revenue from the prioritized content providers. However, it tilts consumption towards these prioritized CPs and, as a result, can affect adversely consumers if they have strong preferences for the unprioritized service. The overall impact depends on the network bandwitdth.

The greater is the bandwidth, i.e. the greater is the network data transfer rate, the lesser is the impedement of priority on unprioritized content and the more likely it is that prioritization will increase total consumption, ISP profit and/or social welfare. However, because the detrimental effect of prioritization on unprioritized content impacts on welfare but not on the ISP profit, there is a bandwidth range over which the repeal of NN increases the ISP profit while it decreases welfare. A regulation mandating net neutrality is then called for.

The intuitive property that the P regime favors the prioritized content is a basic feature of the NN literature. Across models, however, this property is ensured by using different mechanisms to implement prioritization. On the one hand, in most models, prioritization consists of distinguishing content types by different waiting times in a standard M/M/1 queue system.[1] For tractability, the consumers' utility function is separable in each content consumption and in average download time. The marginal utility of content consumption is constant and consumers do not face any constraint, so that they absorb any increase in content that is supplied. We characterize these models as (content) supply driven. In such models, download times are endogenous and an increase in bandwidth capacity tends to decrease the waiting times gap between contents under priority "because the marginal reduction in waiting time for the fast lane from capacity expansion decreases as the capacity level becomes high".[2] An implicit assumption behind this result is that bandwidth does not impact directly on aggregate content demand.

On the other hand, Economides and Hermalin [16] model priority as a division of the bandwidth into subbandwidths with different capacities. This defines the time necessary to download each content. The utility function is quasi-linear between a numeraire and internet content consumption and marginal utility of each content depends on download time. In contrast with supply-driven models, aggregate content demand depends directly on bandwidth capacity, so we characterize this model as demand driven. A consequence of quasi-linearity and the fact that the price charged for consuming content is independent of the content provider, they obtain the following result: "[G]iven two alternative divisions of the total bandwidth, one is welfare superior to the other if and only if it results in more content being carried in equilibrium than the other." (p. 609). An increase in bandwidth then immediately brings an increase of welfare as it necessarily allows an increased

[1]See, for instance, Reggiani and Valletti [27], Choi and Kim, [12] and Choi et al. [11].
[2]Choi and Kim [13].

traffic. In other words, in Economides and Hermalin [16], the impact of an increase in bandwidth can be assimilated to an income effect in the standard demand theory, while this impact can be assimilated to a substitution in supply driven models.

In order to take into account both these substitution and income effects simultaneously, we introduce a consumer time budget constraint where latence, the delay cost of downloading in terms of time, plays the role of price, and available bandwidth, the role of income. As a result, end users utility is only indirectly affected by prioritization, as is the case in Reggiani and Valletti [27]. However, this indirect impact goes through the demand side of the market rather than through the supply side, so that an increase in bandwidth brings an "income effect", i.e. the idea the consumer can use more data in general at a given download speed. We also assume a Cobb-Douglas utility function so that marginal utility of a particular content is not independent of the consumption of the another content, as in the quasi-linear case. In contrast to Economides and Hermalin [16], this allows for cases where social welfare decreases even though more content is consumed in total and, for our purpose, the possibility that the expansion of network coverage comes with a decrease of the utility of existing consumers, as it is often the case with USO.

With respect to the USO literature, we use the standard framework in which a regulator determines the extent of a total market that the ISP network must cover, acknowledging the fact that some of the sub-markets are not profitable because consumers, in spite of having the same preferences over the network services, are heterogenous with respect to their connection costs to the network. Generally in this large literature,[3] extension of service beyond the profit maximizing coverage must be financed through the industry profit. A recurrent theme is to evaluate the welfare impact of a uniform pricing constraint, which is a ban on third-degree price discrimination. To our knowledge, only one-sided markets have been analyzed. In our model, we rather consider a pricing constraint in a two-sided market, net neutrality, which is a ban on third-degree price discrimination on one side of the market, the content providers.

Our modelling strategy, by relating market coverages possibilities to bandwidth, provides results of practical relevance for the deployment of broadband networks. For instance, to adapt to changing consumption patterns that requires ever more bandwidth, the FCC broadband definition has evolved from 200/200 Kbps dowload/upload speeds, to 4/1 Mbps in 2010 and then to

[3]Early contributions are Anton et al. [1] and Valletti et al. [31].

25Mbps/3Mbps in 2015,[4] and this is probably called for a revision soon.[5]. Broadband definitions also vary across countries.[6] At the same time, most countries share the "Biden Administration's commitment to deploying affordable, high-speed broadband across the country to help bridge America's digital divide and remedy persistent digital inequities" (Bennett et al [4]). We show that the regulatory framework that is the most efficient to reach the common goal of a universal broadband coverage depends crucially on the network bandwidth that is envisionned. This fact could be overlooked as long as internet traffic was fairly homogenous in terms of bandwidth requirements, so that the NN debate could be made independently of the establishment of USO. But its importance should increase as the consumption patterns vary in time and across countries.

In the next section, we present the model of the two-sided internet market that we analyze and we specify the way net neutrality and prioritization are defined and implemented. We also derive end-user demands of CP contents under both net neutrality and prioritization. In section 3, we perform the comparative statics between net neutrality and prioritization for a given market coverage and we present the benchmark cases of welfare and profit-maximizing coverages. Section 4 provides the core results on the choice between net neutrality and prioritization as well as on the determination of market coverage in function of bandwidth. For ease of presentation, these results are obtained under symplifying assumptions and we show that they are robust to their relaxation in Section 6. We discuss extensions of the model in Section 5. Although these extensions cover issues of great practical relevance for implementation of universal service obligations with or without net neutrality, they do not modify qualitatively our core results. We conclude with the next steps we envision with this research.

[4]See BroadbandNow [7].

[5]A number of signals go in this direction. For instance, as soon as in 2016, FCC Commisionner Jessica Rosenworcel [28] claims: "I am proud I was the first to call for a new broadband standard of 100 Megabits. I think anything short of that shortchanges our children, our digital economy, and our future". On its website, Verizon [32]: "If you love to stream HD videos, download large files and enjoy multiplayer gaming, you may want to consider speed plans of 100 Mbps and above". In 2021, a bipartisan group of four senators wrote an open letter urging to "update federal broadband program speed requirements to reflect current and anticipated 21st century uses" (Bennet et al [4]).

[6]For instance, the EU defines a 30 Mbps download speed as fast broadband and a 100 Mbps as ultrafast broadband (Bourreau [6]). Coverage targets are given in both terms. Canada sets broadband coverage targets in terms of 50/10 Mbps download/upload speeds.

2 Basic Model

We consider a two-sided market where a monopolistic ISP connects consumers to content providers in a country composed of a continuum of locations $n \in [0, \infty[$ that are ranked in increasing order of network deployment cost. A regulator oversees the ISP with the aim of maximizing social welfare given the regulatory tools it has in hands. We consider "regulatory frameworks" that differ by the use of either one or both of two different regulatory tools: (i) the enforcement of a "traffic management practice", which is a choice between net neutrality (N) and prioritization (P) and/or (ii) the imposition of universal service obligations, which is the choice of the ISP market coverage.

In this section, we describe a basic model that is sufficient to highlight the main trade-offs involved in the regulator's choices. Some simplifying assumptions are made in favor of readibility and tractability. We show in Section 6 that qualitative results follow through the relaxation of the more technical of these assumptions. In section 5, we extend the model to consider an ISP participation constraint and the concommitent instauration of a USO fund to deal with this contraint. This enhances practical relevance of the model without modifying fundamental results.

2.1 Content Providers (CPs)

There are two types of content providers denoted by $j = 0, 1$. CPs value traffic on their websites or applications and they have an ad-sponsored business model. Each CP's total revenue is equal to the click probability times the revenue per click and we denote by a this expected benefit per unit of traffic. Operating costs are normalized to zero. Denoting by X_j the total traffic per location (in MB) of CP j, a CP's profit is $\Pi_j = anX_j$. We let $X \equiv X_0 + X_1$ be the total traffic per location.

Although consumers distinguish content from each CP through their preferences, in this basic model, CPs are homogenous in terms of technology. However, in order to consider the impact of prioritization on content diversity, we introduce CP heterogeneity in section 6.3.

2.2 Internet Service Provider (ISP)

The ISP operates a network of bandwidth μ to link CPs to consumers in n locations. The maximum traffic that the ISP network can handle depends on bandwidth and the traffic management practiced in case of congestion. We use the standard M/M/1 queue system to model congestion, as it "is well known to be a very good approximation for the arrival process in real systems".[7] However, instead of using the M/M/1 model to determine, for a given bandwidth, the average content delivery delay in function of traffic, we determine traffic in function of delay. Waiting times, or its reciprocal, transmission speed, is then the quality of service advertized by the ISP.

Two traffic management practices are possible: net neutrality and prioritization.

Net Neutrality (N) Under net neutrality, traffic is managed under a best effort service so that the ISP announces an average transmission speed in MB/s. This average speed turns out to be the reciprocal of the average waiting time $\bar{\omega}$, which is given by the M/M/1 queue model:

$$\bar{\omega} = \frac{1}{\mu - \lambda X}$$

where λ is the frequency (in s^{-1}) of data transmission to the network, which we assume identical across contents. Data arriving at a speed exceeding λX would involve an infinite waiting time and is therefore considered as not being served by the network. The ISP announces the average speed of transmission $\frac{1}{\bar{\omega}}$ as its quality of service under NN. For bandwidth μ and a normalized quality of service $\bar{\omega}$ to 1, the network is then able to support total data transmission $X = \frac{\mu-1}{\lambda}$. We also normalize λ to 1, so that $\mu - 1$ represents the data transmission capacity of the system.[8] The network has thus a capacity constraint given by:

$$X_0 + X_1 = \mu - 1 \tag{1}$$

Prioritization (P) With a given bandwidth μ, the ISP can alternatively route traffic with a prioritization system under which half the traffic $\mu - 1$ observed under neutrality is given precedence in case of congestion. In other words, there is an equal endowment of capacity that is a priori allocated to the priority traffic and the non-priority class.[9]

[7]Choi and Kim [12], p. 452. The M/M/1 queue system is also used in Choi et al. [11], Reggiani and Valletti [27], Choi and Kim [13], Bourreau et al. [5], and Kramer and Weiworra [22].

[8]Depending on the context, variable μ can then be referred either to bandwidth (in MB/s) or to data transmission capacity (in MB).

[9]This initial endowment is arbitrary and made for readability. In section 6.1, we show that qualitative results are unchanged if we use any share ρ that does not exceed the consumption share observed under neutrality of the content to be prioritirized with the repeal of neutrality.

Instead of posting an average speed $\frac{1}{\bar{\omega}} = 1$, the ISP announces priority speed $\frac{1}{\omega_0} > 1$ and "regular" speed $\frac{1}{\omega_1} < 1$ that result from the M/M/1 queue:[10]

$$\begin{aligned} \omega_0 &= \frac{1}{\mu - \frac{1}{2}(\mu - 1)} = \frac{1}{\frac{1}{2}(\mu + 1)} \\ \omega_1 &= \mu\omega_0 \end{aligned} \tag{2}$$

These speeds must still meet the overall average delay $\bar{\omega} = 1$: as a result, consumptions X_0 and X_1 under prioritization must be such that waiting times ω_0 and ω_1 weighted by the consumption shares in transmission equal 1:[11]

$$\omega_0 \cdot \frac{X_0}{\mu - 1} + \omega_1 \cdot \frac{X_1}{\mu - 1} = \bar{\omega} = 1 \tag{3}$$

Using (2) and multiplying both sides by $\frac{\mu-1}{\omega_0}$, this can be written as:

$$X_0 + \mu X_1 = \frac{1}{2}\left(\mu^2 - 1\right) \tag{4}$$

Note that, by construction, consumption vector $(X_0, X_1) = (\frac{1}{2}(\mu - 1), \frac{1}{2}(\mu - 1))$ is feasible under both traffic management practices.

Cost and Revenue

The cost of establishing a network that covers markets $[0, n]$ with a bandwidth μ is:

$$C(n, \mu) = \frac{1}{2}kn\mu^2 + \frac{1}{2}c\mu n^2 \tag{5}$$

Note that because a bandwidth μ has a maximum data transmission capacity of $\mu - 1$, this function is defined on $\mathbb{R}_+ \times [1, \infty)$. $C(n, 1)$ can thus be interpreted as a fixed cost of serving n markets, as the network must install bandwidth $\mu = 1$ before being able to transmit any data in a finite time.

To cover these costs, the ISP charges a fixed fee to end users. The ISP can then extract completely the consumers' surplus. A fixed fee is also charged to providers of prioritized content under P traffic management, so that the ISP can appropriate the incremental profit brought to beneficiaries of prioritization. This assumes that prioritized CPs have no bargaining power. We relax this assumption in Section 6.2.

[10]In view of the fact that CPs are homogenous in this basic model, the choice of content to be prioritized is arbitrary at this stage. We provide a rationale for the choice of content to be prioritized in Section 6.3.

[11]An equivalent interpretation is to say that X_0 and X_1 must meet capacity constraint $\omega_0 X_0 + \omega_1 X_1 = \mu - 1$.

2.3 Consumers

There is a mass 1 of identical consumers in each location and we use the Cobb-Douglas function to represent their preferences:

$$U\left(X_0, X_1\right) = X_0^{\alpha} X_1^{\beta} \tag{6}$$

where $\alpha + \beta < 1$.

Total consumption is constrained by the data transmission capacity $\mu - 1$ and by the delay of transmission for each content ω_0^i and ω_1^i that prevails under traffic management $i = N, P$:

$$\omega_0^i X_0 + \omega_1^i X_1 = \mu - 1 \tag{7}$$

This can be interpreted as a standard budget constraint where delays play the role of prices and effective capacity, the role of income. Dividing both sides of (7) by ω_0^i, i.e. by considering the prioritized content as the numeraire, one obtains constraints (1) and (4) for the net neutrality and priority management techniques, respectively. Compared to (1), constraint (4) displays a higher "relative price" for unprioritized content. Prioritization thus gives incentives to decrease the share of unprioritized content to prioritized content in total consumption. In fact, the budget constraint under prioritization is obtained by pivoting the net neutrality contraint around $(X_0, X_1) = \left(\frac{1}{2}(\mu - 1), \frac{1}{2}(\mu - 1)\right)$, starting with a slope of $\frac{\omega_1^N}{\omega_0^N} = 1$ to attain slope$\frac{\omega_1^P}{\omega_0^P} = \mu$. Under priority, an increase in capacity μ thus brings both income and substitution effects, while it only brings an income effect under neutrality.

Letting $\left(X_0^i(\mu), X_1^i(\mu)\right)$, $i = N, P$ be the optimal solution under i, we obtain:

$$\begin{array}{ll} X_0^N(\mu) = \frac{\alpha}{\alpha+\beta}(\mu - 1) & X_1^N(\mu) = \frac{\beta}{\alpha+\beta}(\mu - 1) \\ X_0^P(\mu) = \frac{1}{2}\frac{\alpha}{\alpha+\beta}\left(\mu^2 - 1\right) & X_1^P(\mu) = \frac{1}{2}\frac{\beta}{\alpha+\beta}\frac{\mu^2-1}{\mu} \end{array} \tag{8}$$

From these demand functions, we can interpret a change from neutrality to priority as a simultaneous increase of "income" from $\mu - 1$ to $\mu^2 - 1$ and of the unprioritized "content" price from 1 to μ. We accordingly decompose the impact of a change in μ in an income effect and a substitution effect.

Note that if $\alpha \geq \beta$, consumption of prioritized content is greater than $\frac{1}{2}(\mu - 1)$ under neutrality. As prioritization makes the budget constaint pivot around $\frac{1}{2}(\mu - 1)$ while it reduces the relative price of the prioritized content, $\left(\frac{1}{2}X^N(\mu), \frac{1}{2}X^N(\mu)\right)$ is feasible under prioritization. Consequently, by a revealed preference argument, consumers prefer the prioritization regime. Prioritization involves a trade-off between a slower unprioritized content and greater capacity only when consumers initially give more weight on the unprioritized content. For this reason, hereafter we assume that $\alpha < \beta$.

We define the indirect utility function in regime i as $V^i(\mu) \equiv U^i\left(X_0^i(\mu), X_1^i(\mu)\right)$. From (6) and (8), we obtain:

$$V^N(\mu) = v(\mu-1)^{\alpha+\beta} \quad (9)$$

$$V^P(\mu) = \left(\frac{1}{2}\right)^{\alpha+\beta} v\left(\mu^2-1\right)^{\alpha+\beta}\mu^{-\beta} \quad (10)$$

where $v \equiv \left(\frac{\alpha}{\alpha+\beta}\right)^{\alpha}\left(\frac{\beta}{\alpha+\beta}\right)^{\beta}$.

2.4 Market Functioning and Regulation

The ISP sells broadband connection to covered users at a fixed charge p. Users will agree to subscribe if their net utility $V^i(\mu) - p$ is larger than their outside option, that we normalize to zero. Hence, the ISP can extract all the surplus from the consumers and $p^i = V^i(\mu)$.

Under net neutrality, the ISP does not have financial relations with the CP. The per location revenue is thus $R^N(\mu) \equiv V^N(\mu)$.

Under prioritization, the ISP gives an advantage to the prioritized content. Consequently, the traffic of the prioritized content increases by $X_0^P - X_0^N$ per location. We assume that content providers have no bargaining power for the implementation conditions of P management, so that the ISP is able to extract $a\left(X_0^P - X_0^N\right)$ per location from priotization.[12] Under P management, the ISP per location revenue is then:

$$R^P(\mu) \equiv V^P(\mu) + a\left(X_0^P(\mu) - X_0^N(\mu)\right)$$

A benevolent regulator monitors this two-sided market. Depending on the regulatory framework considered, it can determine the traffic management practice N or P, or the market coverage n, or both. The per location social benefit functions that she enters in her welfare function are given by:

$$B^i(\mu) \equiv V^i(\mu) + aX^i(\mu), \; i = N, P$$

[12] We relax this assumption in Section 6.2.

If n markets are covered, the ISP profit is given by $\Pi^i(n,\mu) \equiv nR^i(\mu) - C(n,\mu)$, while social welfare is given by $W^i(n,\mu) = nB^i(\mu) - C(n,\mu)$.

Our analysis consists in comparing the performance of three regulatory frameworks. Under traffic management regulation (TMR), the regulator determines whether the ISP operates under net neutrality or prioritization, while the ISP chooses market coverage. Conversely, under universal service obligations (USO), the regulator imposes the market coverage and the ISP chooses the traffic management practice. Finally, under full regulation (FR), the regulator imposes both the traffic management practice and market coverage. We gauge the performance of these regulatory frameworks in terms of coverage and social welfare against the benchmark cases of a unregulated market (UM), where the ISP chooses both the traffic management regime and market coverage in order to maximize profit, and the first-best outcome (FB), where the welfare-maximizing regime and coverage are considered notwithstanding any market or institutional constraint.[13]

3 Preliminary Results

In this section, we develop the fundamental results of the model that will lie behind the analyzis of the ISP's and regulator's choices. We first make a comparison of the traffic management practices in terms of ISP revenue and social benefit and then deduce optimal coverages and management technique for benchmark cases of unregulated markets and first-best.

3.1 Net Neutrality vs Priority: Comparative Statics

As a first step, we compare market outcomes obtained under neutrality and priority for given μ and n. Because costs are independent of the traffic management regime, we can abstract from them, so that outcome comparisons are made in terms of per location traffic, ISP revenue, and social benefit. Hereafter, for any function $F^i(\mu)$, $i = N, P$, we let $\Delta F \equiv F^P(\mu) - F^N(\mu)$.

For traffic, note that even though the change from neutrality to priority brings a positive income effect for both types of contents, the increased delay on the unprioritized content can make

[13] In our basic model, full regulation is assimilated to the first-best benchmark. A distinction is introduced in section 2.2 with the conjunction of an ISP participation constraint and the impossibility for the regulator to freely make monetary transfers to the ISP.

consumers reduce total consumption. The next Lemma presents the threshold bandwidth for which prioritization increases total content consumption and utility.[14]

Lemma 1 *If* $\alpha < \beta$,

(a) $\Delta X \geq 0$ *if and only if* $\mu \gtreqless \mu_X \equiv \frac{\beta}{\alpha}$

(b) There exists a $\mu_V > \mu_X > 1$ *such that* $V^P(\mu) \geq V^N(\mu)$ *if and only if* $\mu \geq \mu_V$.

In contrast to Economides-Hermalin [16], where consumers' utility increases if and only if total consumption is increased, if $\mu \in (\mu_X, \mu_V)$, utility under priority is less than utility under neutrality even though total consumption is higher under priority. The difference comes from the fact that transmission speeds in [16] are decision variables, so that they are set independently of capacity, instead of being determined by the M/M/1 queue, which introduces an interdependence of transmission speeds with capacity under priority.[15] Combined with the assumption that utility is additively separable in contents, content demands are independent in Economides and Hermalin [16], so that an increase of bandwitdh "is similar to more total income in a conventional consumer-choice model".[16] This income effect is also present in our model under both neutrality and priority,[17] but the interdependence of transmission speeds and capacity under priority adds a substitution effect. As a result, the change from neutrality to priority can involve both an increase in total consumption and a decrease in utility if the substitution effect, absent in Economides and Hermalin [16], is such that the utility loss associated to the decrease of the unprioritized content consumption is not compensated by the increase in total consumption.

The next Proposition shows that prioritization gains a comparative advantage over neutrality as bandwidth is increased. However, the exact threshold for which prioritization dominates neutrality depends on the function considered.

[14]Again, because prioritization dominates neutrality in all respects whenever $\alpha \geq \beta$, we focus on the case where $\alpha \leq \beta$. The result nevertheless holds for $\alpha \geq \beta$, as we then obtain a threshold $\mu_X < 1$, so that $\mu > \mu_X$, $\forall \mu > 1$, meaning that total consumption is increased under priority whatever is μ. We would also obtain $\mu_V = 1$, meaning that $V^P > V^N$, $\forall \mu > 1$.

[15]See equation (2).

[16]Economides and Hermalin [16], p. 609.

[17]The Cobb-Douglas utility function can be considered as additively separable by taking its log form. Moreover, in both models, transmission speeds are by definition the same for all contents under neutrality.

Since $\Delta X_0 > 0$, the ISP has always an additional income from the CPs but this might be insufficient to compensate the lower revenue from the consumers. The ISP prefers prioritization when $\Delta R = \Delta V + a\Delta X_0 > 0$. This threshold from which priority increases the ISP revenue is less than the one that increases welfare. Indeed, since $\Delta X_1 < 0$ and $\Delta B = \Delta R + a\Delta X_1 < \Delta R$, it takes a greater bandwidth to make priority welfare improving than to make it ISP revenue improving.

Considering that ΔB is equivalently equal to $\Delta V + \Delta X$, that $\Delta V(\mu_X) < 0$ and $\Delta X(\mu_V) > 0$, the minimum bandwidth necessary to obtain a social benefit increase is lower than the one necessary for obtaining an indirect utility increase but greater than the one necessary to obtain a traffic increase.

Proposition 1 *There exist a μ_R and a $\mu_B > \mu_X$ such that $\mu_R < \mu_B < \mu_V$ and*

1. $0 \geq \Delta R \geq \Delta B$, $\forall \mu \leq \mu_R$
2. $\Delta R > 0 \geq \Delta B$ *for* $\mu_R < \mu \leq \mu_B$
3. $\Delta R > \Delta B > 0$, $\forall \mu > \mu_B$

Proposition 1 implies that, for a given μ and n, if the ISP prefers net neutrality, then the regulator also prefers net neutrality. If the regulator prefers prioritization, then the ISP also prefers prioritization. More importantly, there exist cases where the regulator prefers net neutrality while the ISP prefers the priority regime. This is illustrated in Figure 1.

[Figure 1: Social welfare, ISP profit and consumer surplus differences]

The main message from the comparative statics is thus that the change from net neutrality to prioritization is the more valuable the greater is the bandwidth. The regulator and the ISP however differ on the exact threshold for which they consider prioritization preferable to net neutrality.

3.2 Benchmark Coverages

In order to evaluate the performance of regulatory frameworks in the next section, we use two benchmarks: the first-best welfare maximizing benchmark and the unregulated market benchmark where the ISP maximizes its profit. In this section, we consider that bandwidth is exogenous and we determine the benchmark coverages for a given bandwidth. This corresponds to a case where bandwidth is primarily determined by the current state of technology. Historically, the interaction of technological improvement (from copper networks to fiber, for instance) and content data transmission requirements (from emails to streaming, for instance) has resulted in a ever increasing minimum standard for bandwidth to be considered as part of a high-speed broadband service. By relating regulatory regimes to bandwidth levels, the model will suggest how the choice of these regimes should evolve in time.[18]

Unregulated Market Benchmark (UM) With no coverage regulation, the ISP maximizes its profit in either neutrality or prioritization:

$$\max_n \Pi^i (n, \mu) = nR^i(\mu) - C(n, \mu)$$

The first order condition is:

$$R^i(\mu) - C'_n (n, \mu) = 0 \tag{11}$$

Denoting the interior solution of 11 by n^i_I, we obtain:

$$n^i_I (\mu) = N\left(R^i(\mu), \mu\right)$$

where $N(x, \mu) = \frac{1}{c}\left(\frac{x}{\mu} - \frac{1}{2}k\mu\right)$ an increasing function of x. This interior solution is valid if it guarantees a positive profit to the ISP i.e. if $\Pi^i\left(n^i_I(\mu), \mu\right) \geq 0$. For the moment, we leave aside the funding issues which require additional assumption on the cost function. This problem will be analyzed in greater details in Section 5.1.

Since network deployment costs are independent of the traffic management practice, the practice that leads to the greater ISP coverage is the one that conveys the greater revenue. We then obtain the following proposition from Proposition 1.[19]

Proposition 2 *As* $\mu \lesseqqgtr \mu_R$,

$$\begin{aligned} n^P_I (\mu) &\lesseqqgtr n^N_I (\mu) \\ \Pi^P\left(n^P_I (\mu), \mu\right) &\lesseqqgtr \Pi^N\left(n^N_I (\mu), \mu\right) \end{aligned}$$

[18]Baranes (2014) analyzes the impact on the choice of regulatory regime when an old and a new technology cohabit (say a copper and fiber). By fixing μ, we consider here a case where one technology has come to dominate.

[19]Hereafter, it is implicitly assumed that $\mu \geq \underline{\mu}^N_I$

Intuitively, since network deployment costs are independent of revenue, the technique that leads to the greater optimal coverage is the one that conveys the greater revenue. From Proposition 1, this corresponds to net neutrality if $\mu \leq \mu_R$ and to priority if $\mu > \mu_R$. In turn, as coverage and revenue become correlated, the regime that has the greater profit-maximizing coverage also leads to the greater profit. The profit-maximizing coverage is thus $n_I(\mu) \equiv \max\left(n_I^N(\mu), n_I^P(\mu)\right)$ and the profit-maximizing regime is $\arg\max_i n_I^i(\mu)$.

First-Best Benchmark (FB) Under regime $i = N, P$, the first-best coverage is the solution to the following problem:

$$\max_n W^i(n,\mu) = nB^i(\mu) - C(n,\mu)$$

From first-order condition,

$$B^i(\mu) = C'_n(n,\mu)$$

Denoting the interior solution by n_*^i, we obtain :

$$n_*^i(\mu) = N\left(B^i(\mu), \mu\right) \tag{12}$$

As $B^N(\mu) - R^N(\mu) = aX^N(\mu) > 0$ and $B^P(\mu) - R^P(\mu) = a\left(X_1^P(\mu) + X_0^N(\mu)\right) > 0$, social benefit is greater than ISP revenue under both management practices. As a result, the unregulated coverage is lower than the first-best coverage for a given μ. This fact and Proposition 1 then lead straightforwardly to the following Proposition.

Proposition 3 *(a) For $i = N, P$ and $\forall \mu$, $n_I^i(\mu) < n_*^i(\mu)$*

(b) As $\mu \lesseqqgtr \mu_B$,

$$\begin{aligned} n_*^P(\mu) &\lesseqqgtr n_*^N(\mu) \\ W^P\left(n_*^P(\mu), \mu\right) &\lesseqqgtr W^N\left(n_*^N(\mu), \mu\right) \end{aligned}$$

The first-best coverage is thus $n_*(\mu) \equiv \max\left(n_*^N(\mu), n_*^P(\mu)\right)$ and the first-best regime is $\arg\max_i n_*^i(\mu)$. Of course, the result that first-best coverage is greater than the profit-maximizing coverage is the basic feature of any model on universal service. But in our model, the regulator can go counter not only to the ISP preferred coverage but also to its preferred traffic management practice. Interactions between coverages and regulatory frameworks are analyzed in the next section.

4 Choices of Traffic Management Practice and Market Coverage

In practice, the net neutrality debate and traffic management regulation have by and large been pursued independently of coverage considerations in general and on the presence or absence of universal service obligations in particular. We now analyze the interactions between the choice of the traffic management practice N or P and the choice of market coverage under different regulatory frameworks: the traffic management regulation (TMR) where the regulator chooses N or P but the coverage is chosen by the ISP, the universal service obligations (USO) where the regulator chooses the market coverage but not the traffic management practice and the full regulation (FR) where the regulator chooses the coverage and the traffic management. Our objectives are to identify the optimal regulatory framework and the cost of incomplete regulations.

4.1 Traffic Management Regulation (TMR)

Under TMR, the regulator can choose the traffic management practice N or P but cannot impose universal service obligations, so that market coverage is chosen by the ISP. Then, the fact that net neutrality has a comparative advantage for low bandwidth remains. However, since coverage is chosen by the ISP, for whom the comparative advantage of net neutrality vanishes at a lower level of bandwidth than for the regulator, the threshold capacity that makes the regulator prefer priority over net neutrality is lower than μ_B. Moreover, this threshold is also greater than μ_R since welfare is still greater under neutrality than under prioritization at $\mu = \mu_R$.

Proposition 4 *There exists a* $\tilde{\mu}_2 \in (\mu_R, \mu_B)$ *such that* $W^P\left(n_I^P(\tilde{\mu}_2), \tilde{\mu}_2\right) \lesseqqgtr W^N\left(n_I^N(\tilde{\mu}_2), \tilde{\mu}_2\right)$ *as* $\mu \lesseqqgtr \tilde{\mu}_2$.

Let n_T *represents the coverage choice of the IS. Then*

(a) If $\mu \leq \mu_R$*, the regulator chooses* N *and* $n_T = n_I^N = n_I$

(b) If $\mu \in (\mu_R, \tilde{\mu}_2]$*, the regulator chooses* N *and* $n_T = n_I^N < n_I$

(c) If $\mu > \tilde{\mu}_2$*, the regulator chooses* P *and* $n_T = n_I^P = n_I$

Cases (a) and (c) are those where the choice of the regulator is aligned to the preferences of the ISP, so that the presence of the regulator turns out to be irrelevant: welfare is the same than under unregulated market since the ISP sets the profit maximizing coverage. The regulator makes a difference in case (b) where it imposes neutrality while the ISP would have chosen prioritization. Note, however, that this makes the ISP choose a coverage that is lower than the one it would have freely chosen. Rather surprisingly, the regulator reduces coverage and works to the detriment of unserved markets, to provide a higher utility in served markets.

4.2 Universal Service Obligations (USO)

Under USO, the regulator can choose market coverage n_U, i.e. can impose universal service obligations, but does not have the power to determine the traffic management practice. We assume that ISP participation is not an issue in the sense that the ISP does not make a negative profit when the regulator imposes n_*^i, whatever is μ and $i = N, P$.[20] Then, independently of the coverage imposed by the regulator in the first stage, the ISP chooses P if and only if $\mu > \mu_R$, since priority leads to a higher revenue whatever is the coverage. As a result, for $\mu < \mu_R$ or $\mu > \mu_B$, the regime choice of the ISP corresponds to the one that the regulator would have chosen and this allows the regulator to impose the first-best coverage. For $\mu \in (\mu_R, \mu_B)$, however, the choice is $n_U = n_*^P < n_*^I$ and the regulator is unable to attain first-best even though n_U is the welfare-maximizing coverage given the traffic management practice chosen by the ISP.

Proposition 5 *Let n_U be the welfare-maximizing coverage under USO regulation. Then*

(a) If $\mu \leq \mu_R$, the ISP chooses N and $n_U = n_*^N = n_*$

(b) If $\mu \in (\mu_R, \mu_B]$, the ISP chooses P and $n_U = n_*^P < n_*$

(c) If $\mu > \mu_B$, the ISP chooses P and $n_U = n_*^P = n_*$

4.3 Full Regulation (FR)

Under FR, the regulator can impose both the traffic management practice and universal service obligations. Under the assumption that the ISP makes a non-negative profit at welfare-maximizing coverage, the regulator can attain the first-best if it imposes both the regulatory regime and universal service obligations. Hereafter, we thus assimilate full regulation to the FB benchmark.[21]

[20] We relax this assumption in Section 5.1.

[21] The distinction between full regulation and FB is made in section 5.1.

4.4 Comparisons: traffic management and market coverage

In this section, we compare the coverage and the chosen traffic management practice under the four possible regulatory regimes (FR, TMR, USO, UM). The comparison is summarized in Proposition 6 and illustrated in two figures.

Proposition 6 *(a) If $\mu < \tilde{\mu}_1 < \mu_R$, then $n_U = n_*^N > n_T = n_I^N > n_*^P > n_I^P$ and net neutrality is chosen under the four regulatory frameworks.*

(b) If $\tilde{\mu}_1 \leq \mu < \mu_R$, then $n_U = n_*^N > n_*^P > n_T = n_I^N > n_I^P$ and net neutrality is chosen under the four regulatory frameworks.

(c) If $\mu_R \leq \mu < \tilde{\mu}_2$; then $n_*^N > n_U = n_*^P > n_I^P \geq n_T = n_I^N$ net neutrality is chosen under FB and TMR, while prioritization is chosen under USO and UM.

(d) If $\tilde{\mu}_2 \leq \mu < \mu_B$, then $n_*^N > n_U = n_*^P > n_T = n_I^P \geq n_I^N$ net neutrality is chosen under FB, while prioritization is chosen under TMR, USO and UM.

(e) If $\mu \geq \mu_B$, then $n_U = n_*^P \geq n_*^N > n_T = n_I^P > n_I^N$ and Prioritization is chosen under four criteria.

Figure 3 shows the choice of the traffic management practice N or P in the four regulatory frameworks FR, TMR, USO and UM, respectively. For example $[N, N, P, P]$ means that prioritization is chosen in regulatory frameworks FR and TMR, while net neutrality is chosen under USO and UM.

[Figure 3: Net Neutrality or Prioritization : regulatory frameworks [FB,TMR,USO,UM]]

We see that, whatever is the regulatory framework, net neutrality has a comparative advantage for low bandwidths and priority, for high bandwidths. Moving from $\mu = 1$ to the right, there exists for each regulatory framework a bandwidth threshold from which priority becomes superior. When the traffic management practice is chosen by the regulator (in FR, and TMR), net neutrality prevails for a greater bandwidth range than when it is chosen by the ISP, since neutrality allows the regulator to avoid the loss of fringe revenue that prioritization brings while this loss has no impact on the ISP. The switch to prioritization comes at a lower bandwidth under TMR than under FR because it is the ISP that chooses coverage based on the revenue function rather than on the social benefit function. In contrast, when traffic management is chosen by the ISP (in USO and UM), the switch to prioritization appears at a common bandwidth level because the coverage decision is already made at the ISP decision stage, so that the ISP chooses in a situation of a fixed cost and a per location revenue that is independent of coverage.

Figure 4 now illustrates the choice of coverage under each regulatory framework in function of bandwidth.

[Figure 4 : Proposition 6. n_T: blue thick line, n_U: red thick line]

We see that coverage is always higher when it is chosen by the regulator (FR, USO) than by the ISP (TMR, UM). Discontinuity in the coverage paths appear for USO and TMR frameworks as decisions of traffic management practice and coverage are made by different agents: with USO, the switch to prioritization is made when profit are the same at μ_R while welfare-maximizing coverages chosen by the regulator would be the same at $\mu_B > \mu_R$; with TMR, the switch to prioritization is made at $\tilde{\mu}_2$ while profit-maximizing coverages chosen by the ISP would be the same at $\mu_R < \tilde{\mu}_2$. Again, this reflects the fact that, in cases where the regulator and the ISP favor different regimes, the regulator prefers neutrality while the ISP prefers prioritization.

4.5 Comparisons: the cost of incomplete regulation

From Propositions 3 and 5, it is clear that the first best can be achieved with USO only, when there is no disagreement between the regulator and the ISP on the preferred traffic management regime, that is either for low ($\mu \leq \mu_R$) bandwidth where they both prefer N or for high bandwidth ($\mu \geq \mu_B$) where they both prefer P. For those bandwidth values, regulating traffic management is useless and imposing USO is sufficient for having the first best. For the remaining intermediate values of μ, only full regulation can achieve the first best. For this parameter range, we discuss the cost of incomplete regulation i.e. the relative merits of TMR versus USO. The comparison is done in the following proposition.

Proposition 7 *(i) If $\mu \leq \mu_R$ or $\mu \geq \mu_B$, USO lead to first-best. (ii) It exists $\tilde{\mu}_u \in (\mu_R, \tilde{\mu}_2)$ such that TMR leads to a higher welfare than USO if $\mu \in [\mu_R, \tilde{\mu}_u]$ and USO lead to a higher welfare than TMR for $\mu \geq \tilde{\mu}_u$.*

For $\mu \geq \tilde{\mu}_2$, TMR is useless as it replicates the unregulated market situation. Therefore for those parameters USO dominates TMR. For $\mu \in (\mu_R, \tilde{\mu}_2)$, USO allow the regulator to bring the welfare maximizing coverage given the P management practice chosen by the ISP. However, this management practice is not itself the welfare-maximizing one, so that the result is short of the first-best. Similarly, TMR allows to change the traffic management practice to N but at the cost of reducing market coverage. The optimal single-instrument policy of Proposition 7trades-off these two dimensions.[22]

5 Extensions

5.1 USO Funding

In our main analysis, we assumed that the ISP participation was not an issue when the regulator imposed $n_*(\mu)$. This is equivalent to assume there is no cost of public funds if the regulator has to subsidize the ISP for providing the USO coverage. Although this is in line with seminal papers on universal services, such as Anton et al. [1], Valletti et al. [31], the question of the choice of the funding mechanism and its impact on the ISP behavior has quickly become central in the literature.[23]

In this section, we take into account the possibility that the optimal USO coverage brings a deficit to the ISP so that there exists a ISP participation constraint that can be tight. We consider first the case where no compensation mechanism exists. This can be considered as one polar benchmark case, while our main model focused on another polar case of total compensation with no transaction cost. In conformity to the USO literature, we then focus on "self-funded" mechanisms where the ISP losses are "funded through cross subsidies or through taxes levied on consumers or firms involved in the market".[24] Note, however, that we analyze the case where funds

[22]Note that even in the range $(\mu_R, \tilde{\mu}_u)$, where welfare is higher under TMR than under USO, USO nevertheless bring a coverage n_*^P that is higher than the coverage n_I^N that is brought about by the regulator's choice of net neutrality under TMR.

[23]Seminal contributions on the subjects are Chone et al. [14], [15].

[24]Chone et al [15], p. 1249. Note that models using direct subsidies can be considered as a special case of a model with a US fund where an exogenous shadow cost of public funds replaces the endogenous value of the Lagrange multiplier associated to the ISP participation constraint.

are levied from the CP side of the market, which is absent in the USO literature, rather than from consumers or from ISP competitors to the USO provider in an oligopilistic market.[25]

As these three scenarios suggest, assumptions on the capacity of the regulator to raise funds from CPs can easily abound. This is because tax avoidance from CPs is facilitated by the relative difficulty for governments to monitor CP activities for fiscal purposes. This difficulty is enhanced by the absence of geographical frontiers for data transmission.[26] However, the key point of this section is that, although the market coverage under USO is quantitatively modified by the ISP participation constraints and the presence of monetary transfers to the ISP, qualitative results are maintained: one the one hand, taking into account the participation constraint can reduce USO coverage compared to FB coverage; on the other hand, the creation of a US fund can counteract this effect through the increase of ISP revenue and, as we show that this increase is relatively greater for neutrality than for priority, the bandwidth threshold for which the ISP prefers priority under USO is then greater than μ_R.

Benchmark: No US Fund We assume first that the regulator is unable to make any transfer to the ISP. In such a case, we must check whether its participation is ensured, i.e. whether there exists a range of bandwidth levels for which

$$\Pi^i\left(n_*^i(\mu),\mu\right) = n_*^i(\mu)\,R^i(\mu) - C\left(n_*^i(\mu),\mu\right) \geq 0 \tag{13}$$

We will denote by M_*^i the set of bandwidths satisfying (13). Conditions for existence of such a set are given in the following Lemma.

Lemma 2 *There exists a value a_π^i such that*

(a) if $a > a_\pi^i$, then $M_*^i = \varnothing$

(b) if $a \leq a_\pi^i$, then there exist $\underline{\mu}_*^i < \bar{\mu}_*^i$ such that $\Pi^i\left(n_*^i\left(\underline{\mu}_*^i\right),\underline{\mu}_*^i\right) = \Pi^i\left(n_*^i\left(\bar{\mu}_*^i\right),\bar{\mu}_*^i\right) = 0$ and $M_*^i = [\underline{\mu}_*^i,\bar{\mu}_*^i]$

[25] In section 5.2, we adapt the analysis our analysis for the possibility of foreign ownership of the unprioritized CPs, on the one hand, and the prioritized CP, on the other hand.

[26] Accordingly, the OECD [23] observes, on the hand, that "because the digital economy is increasingly becoming the economy itself, it would not be feasible to ring-fence the digital economy from the rest of the economy for tax purposes.", but on the other hand, that "certain business models and key features of the digital economy exacerbate base erosion and profit shifting risks."

A seemingly counterintuitive result is that the ad price a must not be too high for the first-best to be implementable by the ISP. The reason is that the first-best coverage increases as CPs become more profitable and, as coverage costs are convex, the first-best coverage can prove to be too costly for the ISP. For sufficiently low levels of ad prices, the ISP can implement the FB coverages only for intermediate levels of bandwidth. Indeed, on the one hand, there is a fixed cost effect when covering the FB level: because $C\left(n_*^i(1), 1\right) > 0$, a minimum bandwidth must be attained before FB coverage becomes profitable. On the other hand, there is a profitability effect: as ISP profits are lower than the social welfare and as this gap is increasing with the bandwidth level, for highest bandwidth levels, the ISP revenues cannot overcome the cost of implementing FB coverages.

For bandwidth levels not in M_*^i, i.e. for μ such that $\Pi^i\left(n_*^i(\mu), \mu\right) < 0$, the choice of coverage under USO regulation is given by the ISP participation constraint. Coverage is thus the value n_π^i such that

$$\Pi^i\left(n_\pi^i(\mu), \mu\right) = n_\pi^i(\mu) R^i(\mu) - C\left(n_\pi^i(\mu), \mu\right) = 0$$

This implies $n_\pi^i(\mu) = 2n_I^i(\mu)$. The ISP will again choose the regime N if and only if $\mu \leq \mu_R$. Of course UM, TMR and FB are not impacted by the participation constraint and results of Proposition 6 still hold. We can then wrap up the results in the following Proposition.

Proposition 8 *If $a > \max\{a_\pi^P, a_\pi^N\}$ then $M_*^i = \varnothing$ for $i = N, P$ and*

(a) If $\mu < \mu_R$, then $n_U = n_\pi^N > n_\pi^P > n_T = n_I^N > n_I^P$ and net neutrality is chosen under the four regulatory frameworks

(b) If $\mu_R \leq \mu < \tilde{\mu}_2$; then $n_U = n_\pi^P \geq n_\pi^N > n_I^P \geq n_T = n_I^N$ and prioritization is chosen under UM and USO, while net neutrality is chosen under TMR and FB

(c) If $\tilde{\mu}_2 \leq \mu < \mu_B$, then $n_U = n_\pi^P > n_\pi^N > n_T = n_I^P > n_I^N$ and prioritization is chosen under UM, TMR and USO, while net neutrality is chosen under FB

(d) If $\mu \geq \mu_B$, then $n_U = n_\pi^P > n_\pi^N > n_T = n_I^P > n_I^N$ and Prioritization is chosen under four criteria.

If $a < \max\{a_\pi^P, a_\pi^N\}$, depending on the value of μ, M_*^N or M_*^P is not empty and for some intermediate levels of bandwidth levels, the regulator will be able to raise the USO coverage from $n_\pi^i(\mu)$ to $n_*^i(\mu)$.

US fund Assume now that the regulator is able to establish a US fund by seizing shares t_0 and t_1 of the prioritized and unprioritirized CPs rents, respectively.[27] We let the shares be different for the two CPs as tax avoidance possibilities can in general differ across CPs.[28]

Collecting money on the CP side to finance infrastructure extension is an argument often used to justify the repeal of net neutrality and for allowing paid prioritization. A US fund has the same purpose but it does not distort the consumers' demand for content.

The first impact of the fund is to enlarge the set M_*^i for which the ISP is able to supply the FB coverage. But it can also modify the ISP behavior. Hereafter, we consider μ in M_*^i given the existence of the fund and check whether the fund modifies results under USO.

As the total tax proceeds are transferred to the ISP, the ISP revenues become:

$$\begin{aligned} R_t^N(\mu) &= V^N(\mu) + t_0 a X_0^N(\mu) + t_1 a X_f^N(\mu) > R^N(\mu) \\ R_t^P(\mu) &= V^P(\mu) + a(1-t_0)\Delta X_0 + t_0 a X_0^P(\mu) + t_1 a X_f^P > R^P(\mu) \end{aligned}$$

where, under prioritization, the ISP is able to directly charge the additionnal large CP profit, net of taxation. We thus have:

$$\Delta R_t = \Delta V + a\Delta X_0 + t_1 a \Delta X_f < \Delta R$$

Let n_t^i be the profit maximizing coverage under CP taxation. From the fact that $N(x,\mu)$ is increasing in x, we immediately see that $n_t^i > n_I^i$. Moreover, since $\Delta R_t < \Delta R$, we have that the threshold bandwidth μ_t that is such that $\Delta R_t = 0$ is greater than μ_R. Prioritization becomes relatively less attractive than neutrality under USO funding because its introduction reduces funds from the fringe content without improving funding from the large content since the incremental transfer was already ensured without US fund. As a result, under USO regulation, prioritization

[27] Alternatively, t_0 and t_1 can be considered as unit taxes on CPs. We let the shares differ for prioritized and unprioritized to make the analysis compatible with the extension on content diversity in section 6.3.

[28] For instance, it can be difficult to recover taxes from small CPs as their activities are difficult to monitor for the government – think for instance of bloggers or influencers. On their part, large CPs, even when they are domestic, can for instance practice tax shifting across countries. Fuchs [17] gives empirical evidence that some big digital companies intensively employ intangibles registered in low tax jurisdictions (as Ireland) and can operate in the market without necessarily being physically present. We come back below to the additional problem of foreign ownership. In section 6.3, the prioritized content will be supplied by a large CP, while the unprioritized content will come from a fringe of small CPs.

is chosen for a lower range of bandwidths by the ISP than under UM. The range of disagreement between the regulator and the ISO on the preferred regime is reduced. There is also a discrepency between the ISP choice of regime under UM and USO that did not existed without the US fund.

As usual, V and B are not modified by monetary transfers. Note that if the regulator is able to seize the totality of the CPs rent, i.e. if $t_0 = t_1 = 1$, then $R_t^i = B^i$, and the ISP espouses the regulator's preferences and simply maximizes welfare. As long as $t_1 < 1$, however, $\mu_t < \mu_B$.

In a nutshell, abstracting from the possibility of establishing a US fund does not modify the main results of section 4, except for the fact that neutrality becomes favored by the ISP for a larger broadband range.

5.2 Foreign CPs

Although ISPs are generally regulated at the national level, content providers operate on a global market, as was quickly coined by the name *World Wide Web.* Apart from funding problems described in section 5.1, this raises the additional problem that some of the network value added is not considered in the welfare calculations of the regulator. In this section, we adapt the model for foreign ownership. While US funding impacted on the ISP revenue without modifying the social benefit, foreign ownership impacts on the social benefit without impacting on the revenue.

Foreign Unprioritized CP Assume that the unprioritized CP is a foreign firm. The regulator thus excludes its profit in its calculation of welfare, so that the social benefit becomes:

$$B_0^i(\mu) = B^i(\mu) - aX_1^i(\mu) = V^i(\mu) + aX_0^i(\mu) \tag{14}$$

The FB then becomes $n_0^i = N\left(B_0^i(\mu), \mu\right)$ where $N(x, \mu)$ has been defined in (12). Since $B^i(\mu) > B_0^i(\mu) > R^i(\mu)$, it is clear that $n_*^i > n_0^i > n_I^i$. This is not surprising as welfare is lower under foreign ownership than under domestic ownership.

Note, however, that $\Delta B_0 = \Delta R$, so that disagreements over the choice of the regulatory regime disappear. The fact that prioritization reduces the unprioritized CP profit, which the regulator took into account under domestic ownership, is ignored under foreign ownership. As a result, regulation TMR becomes equivalent to UM and USO becomes equivalent to FB, the first-best coverage now being n_0^i. Letting $\hat{\mu} \in (\tilde{\mu}_1, \mu_R)$ be such that $B_0^P(\mu) \gtreqqless R^N(\mu)$, we obtain the following Proposition.

Proposition 9 *(a) If* $\mu < \hat{\mu}_1 < \mu_R$*, then* $n_0^N > n_I^N > n_0^P > n_I^P$ *and net neutrality is chosen*

(b) If $\hat{\mu}_1 \leq \mu < \mu_R$*, then* $n_0^N > n_0^P > n_I^N > n_I^P$ *and net neutrality is chosen*

(c) If $\mu \geq \mu_R$*, then* $n_0^P \geq n_0^N > n_I^P > n_I^N$ *and prioritization is chosen.*

Foreign Prioritized CP We now turn to the case where the prioritized CP is the foreign firm.[29] Then the social benefit becomes

$$\begin{aligned} B_1^N(\mu) &= V^N(\mu) + aX_1^N(\mu) = B^N(\mu) - aX_0^N(\mu) < B^N(\mu) \\ B_1^P(\mu) &= V^P(\mu) + a\Delta X_0(\mu) + aX_1^P(\mu) = B^P(\mu) - aX_0^N(\mu) < B^P(\mu) \end{aligned}$$

so that $\Delta B_1(\mu) = \Delta B(\mu)$. Although social benefit is lower under foreign ownership than under domestic ownership, prioritization brings the same change in the social benefit for both cases since the increased rent to the prioritized CP is seized whatever is the ownership structure. As the ISP revenue is not impacted by the prioritized CP ownership, bandwidth thresholds μ_R and μ_B stay the same, i.e. preferences of both the regulator and the ISP over the regulatory regimes are maintained.

However, since $B_1^i(\mu) < B^i(\mu)$, the first-best coverage n_f^i under foreign ownership of the priotirized CP is lower than n_*^i. The threshold $\tilde{\mu}_f$ for which $B_1^P(\mu)$ becomes greater than R^N is then greater than $\tilde{\mu}_1$. We also define $\hat{\mu}_f \in (\mu_R, \mu_B)$ as the bandwidth level such that $W_f^P(n_I^P, \hat{\mu}_f) = W_f^N(n_I^N, \hat{\mu}_f)$

Proposition 10 *If the prioritized CP is a foreign firm, results in Proposition 6 applies by replacing accordingly* $\tilde{\mu}_1$ *by* $\tilde{\mu}_f > \tilde{\mu}_1$*,* n_*^i *by* $n_f^i < n_*^i$ *and* $\tilde{\mu}_2$ *by* $\hat{\mu}_f$*.*

6 Robustness Checks

6.1 Priority Capacity Endowment

For ease of presentation in section 2.2, we established a prioritization system under which half the traffic observed under neutrality is given precedence in case of congestion. In this section, we show that our results are qualitatively similar for any system that gives priority to a share ρ that is greater than $\frac{\alpha}{\alpha+\beta}$, i.e. greater than the share of X_0^N in total traffic under net neutrality.

[29] This is a likely case for any country except the United States.

To understand this fact, assume that a share $\rho \in [\frac{\alpha}{\alpha+\beta}, 1)$ of traffic under net neutrality is given precedence under priority. Then the waiting times become:

$$\omega_0 = \frac{1}{\mu - \rho(\mu - 1)} = \frac{1}{(1-\rho)\mu + \rho} \tag{15}$$

$$\omega_1 = \mu\omega_0 \tag{16}$$

The bounds on ρ are set so that the priority service is able to accommodate X_0^N, so that a switch to priority service can be done without modifying consumption of the prioritized content. If ρ is less than $\frac{\alpha}{\alpha+\beta}$, it is impossible to prioritize content $X_0^N(\mu) = \frac{\alpha}{\alpha+\beta}(\mu - 1)$ at the quoted (minimum) speed $\frac{1}{\omega_0}$. Having $\rho > 1$ would result in a quoted speed $\frac{1}{\omega_0}$ greater than the net neutrality speed $\frac{1}{\bar{\omega}}$, which is in contradiction with the concept of prioritization.

Substituting waiting times (15) and (16) in (3) and taking X_0 as the numeraire gives the following prioritization "budget constraint":

$$X_0 + \mu X_1 = (\mu - 1) \cdot ((1-\rho)\mu + \rho) \tag{17}$$

It is easily seen that constraint (4) is a particular case of (17) with $\rho = \frac{1}{2}$. Accordingly, (17) is geometrically the result of pivoting the net neutrality budget constraint around $(X_0, X_1) = (\rho(\mu - 1), (1-\rho)(\mu - 1))$, while constraint (4) follows the same procedure for $\rho = \frac{1}{2}$.

Note that the only difference in the two constraints lies in the RHS, so ρ impacts on the "income effect" of prioritization, but not on the substitution effect. Since the RHS is decreasing in ρ, the greater is ρ, the lower is the "income effect" of prioritization, so that V^P, R^P and B^P become decreasing functions with respect to ρ. Thus, qualitative results of our analysis are not modified, but quantitatively, the comparative advantage of prioritization becomes the greater the lower is ρ: whatever is the regulatory framework considered in Section 4, the range of bandwidths for which priority is the preferred regime is the larger the lower is ρ. Compared to our earlier analysis, taking $\rho > \frac{1}{2}$ would improve (quantitatively) the comparative advantage of neutrality, while taking $\rho < \frac{1}{2}$ gives a greater comparative advantage to prioritization.

At the lower bound $\rho = \frac{\alpha}{\alpha+\beta}$, prioritization implies a budget pivot around $\left(X_0^N(\mu), X_1^N(\mu)\right)$, so that the optimal choice of contents under net neutrality is feasible under priority whatever is

μ. In that case, prioritization has always a comparative advantage.[30] In the limit case $\rho = 1$, the budget pivot around corner point $(\mu - 1, 0)$ and makes any optimal choice under prioritization feasible under neutrality, so that neutrality has always the comparative advantage.

Apart from the ease of presentation, the choice of $\rho = \frac{1}{2}$ can be justified by an equity argument towards content providers. Note that $\left(\rho X_0^N(\mu), (1-\rho) X_1^N(\mu)\right)$ is the unique point that is feasible under both traffic management practices. So, $\rho = \frac{1}{2}$ is the unique value of ρ that ensures that in the case each content provider generates the same traffic under one management practice, this equal share of network is also feasible under the other regime. As both content providers generate the same revenue by unit of traffic, that this revenue is dependent on traffic but not on the transmission time, so that they are unaffected by changes of transmission times if this is not accompanied by a change in traffic, both practices treat CPs equally when they are equally responsible for network usage and contribute equally to social benefit.

Of course, one can debate over such an interpretation of equity. The main point is that the analysis is not modified by any feasible choice of ρ.

6.2 ISP Bargaining Power

So far, we assumed that the ISP was able to fully capture the rent of the prioritized content provider. Even though this polar case of market power is somewhat in line with the assumption of a monopolistic ISP, it can be harder to justify in cases the prioritized content providers also have bargaining power. In this section, we show that our results in fact hold in all respects provided that the ISP has a "sufficiently large" bargaining power. We also show how these results are modified when this is not the case, i.e., when the bargaining power rather lies on the side of the content providers, as would be the case, for instance, if the prioritized content providers are part of the GAFAM group.

Let η be an index of the ISP relative bargaining power *vis-à-vis* the prioritized CP, so that the revenue that the ISP can extract from the gain provided to the CP is now $R^P(\mu, \eta) = V + \eta a \Delta X_0$. Then, $\Delta R = \Delta V + \eta a \Delta X_0$. The lower is η, the more heavily the relative profitabiliy of prioritization

[30] Note, however, that this is no longer the case if we consider the impact of prioritization on content diversity as done in section 6.3. Then, the loss of diversity impacts negatively on utility and fringe content profit, so that the superiority of prioritization is not ensured.

rests on the change of consumers' utility. Since the change of social benefit is independent of the ISP bargaining power, this means that the change of revenue brought by prioritization can now be lower than the change of the social benefit. As a result, the threshold bandwidth for which the ISP begins to prefer prioritization over net neutrality can be greater than the corresponding threshold for social benefit.

Lemma 3 *There exists a* $\hat{\eta}$ *such that* $\mu_R(\eta) \lesseqqgtr \mu_B$ *if and only if* $\eta \gtreqqless \hat{\eta}$.

For $\eta > \hat{\eta}$, the only impact of having $\eta < 1$ is that the range of bandwidths for which preferences of the ISP and the regulator over prioritization diverge is increased. This is because the ISP, by getting a lower part of industry revenue, takes into account a lower part of the social benefit in its evaluation of prioritization. Otherwise, results obtained in sections 2 to 4 are maintained once we substitute $\mu_R(\eta)$ to μ_R in lemmas and propositions.

Hereafter, we thus focus on the case $\eta \leq \hat{\eta}$. Then, the additional revenue obtained from the implementation of prioritization becomes too low for making the ISP evaluate more highly prioritization than the regulator, so that we now have $\Delta R \leq \Delta B$. Accordingly, we obtain the following proposition, where prioritization still eventually becomes the preferred choice as bandwidth is increased, but where it is now the ISP that is less prone to adopt prioritization.

Proposition 11 *1. If* $\eta \leq \hat{\eta}$

2. $0 \geq \Delta B \geq \Delta R$, $\forall \mu \leq \mu_B(\eta)$

3. $\Delta B \geq 0 \geq \Delta R$ *for* $\mu_B \leq \mu \leq \mu_R(\eta)$

4. $\Delta B \geq \Delta R > 0$, $\forall \mu > \mu_R(\eta)$

The following lemma restates results from Lemma **??** and Proposition 4 by taking into account the reversed order of μ_R and μ_B.

Lemma 4 *If* $\eta < \hat{\eta}$*,there exist bandwidth thresholds* $(\tilde{\mu}_1, \tilde{\mu}_2)$ *such that* $\tilde{\mu}_1 < \mu_B < \tilde{\mu}_2 < \mu_R(\eta)$ *and:*

1. $B^P(\tilde{\mu}_1) = R^N(\tilde{\mu}_1)$

2. $W^N\left(n_I^N(\tilde{\mu}_2), \tilde{\mu}_2\right) = W^P\left(n_I^P(\tilde{\mu}_2), \tilde{\mu}_2\right)$

Results for the case $\eta \leq \hat{\eta}$ can be summed up in the following proposition, which echoes Proposition 6 for the case $\eta > \hat{\eta}$.

Proposition 12 *If $\eta < \hat{\eta}$ and if*

(a) *If $\mu < \tilde{\mu}_1 < \mu_B$, $n_U = n_*^N > n_T = n_I^N > n_*^P > n_I^P$ and net neutrality is chosen under the four regulatory frameworks*

(b) *If $\tilde{\mu}_1 \leq \mu < \mu_B : n_U = n_*^N > n_*^P \geq n_T = n_I^N > n_I^P$ and net neutrality is chosen under the four regulatory frameworks*

(c) *If $\mu_B \leq \mu < \tilde{\mu}_2 : n_*^P > n_U = n_*^N \geq n_T = n_I^N > n_I^P$ and prioritization is chosen under FB, while net neutrality is chosen under UM, TMR and USO*

(d) *If $\tilde{\mu}_2 \leq \mu < \mu_R : n_*^P > n_U = n_*^N \geq n_I^N > n_T = n_I^P$ and prioritization is chosen under FB and TMR, while neutrality is chosen under UM and USO*

(e) *If $\mu \geq \mu_R : n_U = n_*^P > n_*^P \geq n_T = n_I^P > n_I^N$ and prioritization is chosen under the four regulatory frameworks*

[Figure 5: Criteria with ISP low bargaining power]

The results in Proposition 6 were established with the highest bargaining power for the ISP. This can be viewed as a favorable upper bound in terms of private coverages in the analysis. Of course, since the bargaining power concerns only monetary transfers between the ISP and CPs, welfare is not impacted by it. But if prioritization is meant to help investment in networks, as is often advanced by its proponents, and if the choice of the traffic management practice is left to the ISP, as is the case for UM and USO regulations, the bargaining power of the ISP *vis-à-vis* CPs becomes an important factor to take into account. The related question of the taxing power of the regulator is treated in the next section 5.1.

Note that relaxing the assumption that the ISP has full market power does not modify the fact that the comparative advantage of net neutrality is the more likely the lower is the bandwidth.

6.3 Content Diversity

A primary benefit that is attributed to net neutrality is to promote network access for content providers, thus ensuring competition, innovation and diversity at this end of the market. Reggiani and Valletti [27] confirm this conjecture. In this section, we show that these considerations are easily integrated in our model.

Assume that the anonymous CPs we considered up to now are in fact belonging to two classes that are different in nature. The CP denoted 0 is a large CP (say Google or Facebook), while the CP formerly denoted 1 becomes a fringe of m small CPs (each denoted by j). CPs in the fringe face a fixed entry cost F so that their individual profits are given by $\Pi_j = anX_j - jF$. There is free entry in fringe content supply, so that Π_m will be nil at equilibrium and the number of fringe content types m is endogenous. We let $X_f \equiv \sum_{i=1}^{m} X_j$ be aggregate fringe traffic and $X = X_0 + X_f$ be the total traffic on the network.

Consumers now value diversity of the fringe contents. More precisely, the utility function has the following separable form in the large and fringe contents:

$$U\left(X_0, Z_f\left(X_1, ..., X_m\right)\right) = X_0^{\alpha} \cdot Z_f$$

where Z_f is a CES index of the overall fringe consumption that takes into account both substituability among contents and the number of varieties:[31]

$$Z_f \equiv \left(\sum_{j=1}^{m} X_j^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\beta\sigma}{\sigma-1}} \tag{18}$$

In this expression, $\sigma > 1$ is the elasticity of substitution between any two fringe contents and $\beta < \frac{\sigma-1}{\sigma}$ is the degree of homogeneity of the CES function. The bound on β is set in order to ensure strict concavity of the index.

As the same transmission time applies to any fringe content X_j and the weight given on X_j in Z_f is the same for all j, total fringe consumption will be evenly distributed among fringe CPs, i.e. $X_j = \frac{X_f}{m}$. Substituting this value in (18) gives $Z_f = X_f^{\beta} m^{\frac{\beta}{\sigma-1}}$ and the consumer problem can be written as:

$$\max_{X_0, X_f} \quad X_0^{\alpha} X_f^{\beta} m^{\delta}$$
$$\text{s.t.} \quad \omega_0^i X_0 + \omega_f^i X_f = \mu - 1$$

where $\delta \equiv \frac{\beta}{\sigma-1}$ [ajouter condition sur σ pour assurer concavité plus tard par rapport à n] and where ω_0^i and ω_f^i are transmission times of large and fringe CPs, respectively, under regulatory

[31] This representation of preferences in a model of monopolistic competition is borrowed from Belleflamme and Peitz [3], p. 88.

regime i. Note that in this formulation, consumers value diversity per se. Since m is a constant in this problem, optimal solutions are still given by (8). Note the case of no fringe diversity that we considered up to now is a particular case where $m = 1$.

Equilibrium number of fringe CPs is obtained from the zero-profit condition. Recalling that $X_m = \frac{X_f}{m}$, we obtain:

$$m^i(n,\mu) = \left(\frac{anX_f^i(\mu)}{F}\right)^{\frac{1}{2}},\ i = N, P \tag{19}$$

The fringe aggregate profit is the

$$\Pi_f^i(n,\mu) = anX_f^i - \frac{(m^i)^2}{2}nF = \frac{1}{2}anX_f^i(\mu)$$

Whatever is the traffic management practice, both an increase in capacity and coverage favor diversity and there is complementarity between capacity and coverage with respect to diversity. However, since demand of fringe content is lower under prioritization than under neutrality, a shift from neutrality to priority lowers diversity of content. The difference, however, tends to be attenuated when market coverage is increased because, as the fixed cost of fringe CPs is independent of coverage, the exit of firms because of lower demand becomes less severe as coverage is increased.

Proposition 13 *In both regimes N and P,* $m'_\mu > 0$, $m'_n > 0$, $m''_{\mu n} > 0$. *Moreover, letting* $\Delta m = m^P(n,\mu) - m^N(n,\mu)$, *we obtain* $\Delta m < 0$, $\frac{\partial(\Delta m)}{\partial n} < 0$ *and* $\frac{\partial^2(\Delta m)}{\partial n^2} > 0$

Per location utility function $V^i = U(X_0(\mu), X_f(\mu), m(n,\mu))$ then depends on market coverage:

$$V^N(n,\mu) = n^{-\frac{1}{2}\delta} v(\mu - 1)^{\alpha+\beta+\frac{1}{2}\delta} \tag{20}$$

$$V^P(n,\mu) = \left(\frac{1}{2}\right)^{\alpha+\beta} n^{-\frac{1}{2}\delta} v\left(\mu^2 - 1\right)^{\alpha+\beta+\frac{1}{2}\delta} \mu^{-\left(\beta+\frac{1}{2}\delta\right)} \tag{21}$$

where $v \equiv \left(\frac{a}{f}\right)^{\frac{1}{2}\delta}\left(\frac{\alpha}{\alpha+\beta}\right)^{\alpha}\left(\frac{\beta}{\alpha+\beta}\right)^{\gamma\beta+\frac{1}{2}\delta}$. Note that per location indirect utility is convex with respect to n, but aggregate utility nV^i, which enter the objective functions of the regulator and the ISP, are concave in n. The functional forms of (20) and (21) with respect to μ are identical to those of (9) and (10). Results for this version of the model are thus qualitatively similar to those obtained with those in sections 3 and 4. However, as fringe content becomes more valuable with diversity while neutrality is the regime that is the most favorable to diversity, the ranges of intervals for which prioritization meets criteria in proposition 6 are simply reduced.

In summary, taking into account diversity introduces a positive network externality from unprioritized content demand: the greater revenue that induces a greater fringe content demand helps support more variety, which is retroactively valued as such. As both coverage and capacity induces more fringe content consumption, they both contribute to this positive network externality. These impacts are, however, less important for priority. As a result, both the ISP and the regulatory agency recur less often to prioritization when diversity of content is taken into account.

Note that our results on diversity are in line with those of Reggiani and Valletti [27] in particular and with the general argument in favor of net neutrality that was first stated by Wu [34].

7 Conclusion

We have integrated the analysis of net neutrality and universal service obligations in a single model. We have shown that the comparative advantage of prioritization over net neutrality for extending broadband market coverage is positively correlated with bandwidth or, in other words, with the data transmission capacity of the network. The reason is that the greater is the network capacity, the greater is the relative gain of prioritization in terms of total traffic carried against the utility loss associated to unprioritized content displacement. This result is independent of various assumptions on the ISP bargaining power *vis-à-vis* the content providers, on the capacity of the regulator to establish a universal service fund as well as on the foreign or domestic ownership of content providers. It seems to fit well the history of internet, as questions about the relevance of net neutrality in the economic literature came up at a time a "virtuous" circle was emerging between broadband technological progress and increasing consumption of time-sensitive content.

In this preliminary version of the paper, we consider bandwidth as a parameter, assuming its level is driven by exogenous technological progress. An immediate extension of the model is to endogenize the choice of bandwidth. A first step would be to study the profit-maximizing and welfare-maximizing bandwidths for a given market coverage. This would allow us to determine a "bandwidth universal service". Preliminary work suggests that both profit-maximizing and welfare-maximizing bandwidths are decreasing with market coverage, because a higher coverage induces a higher fixed cost for the deployment of the broadband network. As net neutrality has a comparative advantage with low bandwidth, and provided that CPs revenue are not too large, this implies that net neutrality would be adopted for high coverage, or equivalently, for large countries with important fixed deployment costs. This suggests that the eventual repeal of net neutrality will come later in large countries with low population density, such as Canada. The second step would be to determine simultaneously the choices of coverage and bandwidth.

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Appendix

Proof of Lemma 1

(a) From (8), $\Delta X = X^P(\mu) - X^N(\mu)$ and

$$\Delta X = \frac{(\mu-1)^2}{2(\alpha+\beta)\mu}(\alpha\mu-\beta) \lesseqgtr 0 \text{ as } \mu \lesseqgtr \mu_X \equiv \frac{\beta}{\alpha}$$

(b) From (9) and (10),

$$\frac{V^P}{V^N} = \left(\frac{1}{2}\right)^{\alpha+\beta}(\mu+1)^{\alpha+\beta}\mu^{-\beta}$$

so that

$$V^P \lesseqgtr V^N \text{ as } F(\mu) \equiv \frac{\ln\left(\frac{\mu+1}{2}\right)}{\ln\mu - \ln\left(\frac{\mu+1}{2}\right)} \lesseqgtr \mu_X \tag{22}$$

As $F(\mu)$ is an strictly increasing function of μ such that $\lim_{\mu\to1} F(\mu) = 1$ and $\lim_{\mu\to\infty} F(\mu) = \infty$, then it exists $\mu_V : F(\mu_V) = \mu_X > 1$. Note that

$$1 \le F(\mu) \le \mu \text{ for } \mu \ge 1 \tag{23}$$

so $\mu_V > F(\mu_V) = \mu_X$. Moreover $\mu - F(\mu)$ is an strictly increasing concave function of μ.

Proof of Proposition 1. Note that for all $\mu > 0$, $\Delta B(\mu) = \Delta V(\mu) + a\Delta X_0(\mu) + a\Delta X_f(\mu) < \Delta V(\mu) + a\Delta X_0(\mu) = \Delta R(\mu)$ since $\Delta X_f(\mu) < 0$. For the large-biased case ($\alpha \ge 0.5$), since $\Delta V(\mu) \ge 0$ and $\Delta X \ge (\mu)\, 0$, $\forall\mu \ge 1$, we have $\mu_R = \mu_B = 1$. Consider now the fringe biased case ($\alpha \le 0.5$). Since $\Delta V(\mu) \lesseqgtr 0$ as $\mu \lesseqgtr \mu_V$, $a\Delta X_0(1) = 0, a\lim_{\mu\to\infty}\Delta X_0(\mu) \to \infty$ and $\frac{\partial(\Delta X_0(\mu))}{\partial\mu} > 0$, $\forall\mu$, there exists a $\mu_R < \mu_V$ such that $\Delta R(\mu) = \Delta V(\mu) + a\Delta X_0(\mu) \lesseqgtr 0$ as $\mu \lesseqgtr \mu_R$.

Similarly, since $\Delta R \lesseqgtr 0$ as $\mu \lesseqgtr \mu_R$, $a(\Delta X(1)) = 0$, $\lim_{\mu\to\infty} a(\Delta X(\mu)) \to \infty$, and $\Delta B < 0 = \Delta R$ at $\mu = \mu_R$, there exists a $\mu_B > \mu_R$ such that $\Delta B \lesseqgtr 0$ as $\mu \lesseqgtr \mu_B$. As $\mu_X < \mu_V$, $\Delta B(\mu_X) = \Delta V(\mu_X) < 0$, so that $\mu_B > \mu_X$, and $\Delta B(\mu_V) = \Delta X(\mu_V) > 0$, so that $\mu_B < \mu_V$.

Proof of Proposition 2. Assume that $\mu < \mu_R$. From Lemma 1, $\Delta R < 0$ so that $n_I^P < n_I^N$. $\Delta R < 0$ also implies that $\Pi^P(n_I^P, \mu) < \Pi^N(n_I^P, \mu)$; since profits $\Pi^i(n, \mu)$ are strictly concave in n, n_I^N is a unique solution and $\Pi^N(n_I^P, \mu) < \Pi^N(n_I^N, \mu)$. We thus have $\Pi^P(n_I^P, \mu) < \Pi^N(n_I^N, \mu)$.

Proof of Proposition 3. As $B^i(\mu) > R^i(\mu)$ for all μ, by definitions (12) and (11) of coverages, we have the first result.. Now, consider the case where $\mu < \mu_B$. From Lemma 1, $\Delta B < 0$, so that $n_*^P < n_*^N$. Moreover, $\Delta B < 0$ also implies that $W^P\left(n_*^P, \mu\right) < W^N\left(n_*^P, \mu\right)$; since welfare $W^i\left(n, \mu\right)$ is strictly concave in n, n_*^N is a unique solution and $W^N\left(n_*^P, \mu\right) < W^N\left(n_*^N, \mu\right)$. We thus have $W^P\left(n_*^P, \mu\right) < W^N\left(n_*^N, \mu\right)$. The proof is similar for $\mu \geq \mu_B$.

Proof of Proposition 4. Note that for all μ, coverage solutions write $N\left(G, \mu\right) = \frac{G}{c\mu} - \frac{1}{2c}k\mu$ which is an increasing function in G. So we have

$$\begin{aligned} & W^P\left(n_I^P\left(\mu\right), \mu\right) - W^N\left(n_I^N\left(\mu\right), \mu\right) \\ = \ & N\left(R^P\left(\mu\right), \mu\right) B^P\left(\mu\right) - C\left(n_I^P\left(\mu\right), \mu\right) - N\left(R^N\left(\mu\right), \mu\right) B^N\left(\mu\right) + C\left(n_I^N\left(\mu\right), \mu\right) \end{aligned}$$

At $\mu \leq \mu_R$,we have:

$$\begin{aligned} & W^P\left(n_I^P\left(\mu_R\right), \mu_R\right) - W^N\left(n_I^N\left(\mu_R\right), \mu_R\right) \\ < \ & \left[N\left(R^P\left(\mu_R\right), \mu_R\right) - N\left(R^N\left(\mu_R\right), \mu_R\right)\right] B^N\left(\mu_R\right) - C\left(n_I^P\left(\mu_R\right), \mu_R\right) + C\left(n_I^N\left(\mu_R\right), \mu_R\right) = 0 \end{aligned}$$

where the inequality comes from the fact that $B^P\left(\mu_R\right) < B^N\left(\mu_R\right)$ and the equality, from the fact that $R^P\left(\mu_R\right) = R^N\left(\mu_R\right)$ and $n_I^N\left(\mu_R\right) = n_I^P\left(\mu_R\right)$. Similarly, at $\mu = \mu_B$, we have:

$$\begin{aligned} & W^P\left(n_I^P\left(\mu_B\right), \mu_B\right) - W^N\left(n_I^N\left(\mu_B\right), \mu_B\right) \\ > \ & N\left(R^N\left(\mu_B\right), \mu_B\right)\left(B^P\left(\mu_B\right) - B^N\left(\mu_B\right)\right) + C\left(n_I^N\left(\mu_B\right), \mu_B\right) > 0 \end{aligned}$$

where the inequality comes from the fact that $R^P\left(\mu_B\right) > R^N\left(\mu_B\right)$ and the equality, from the fact that $B^P\left(\mu_B\right) = B^N\left(\mu_B\right)$. By continuity, there exists a $\tilde{\mu}_2 \in \left(\mu_R, \mu_B\right)$ such that $W^P\left(n_I^N\left(\tilde{\mu}_2\right), \tilde{\mu}_2\right) = W^N\left(n_I^P\left(\tilde{\mu}_2\right), \tilde{\mu}_2\right)$. Moreover from Propositions 1 and 2, for $\mu < \mu_R < \mu_B$, $n_*^N\left(\mu\right) > n_I^N\left(\mu\right) > n_I^P\left(\mu\right)$ and $W^P\left(n, \mu\right) - W^N\left(n, \mu\right) < 0$, we have

$$W^P\left(n_I^P\left(\mu\right), \mu\right) < W^N\left(n_I^P\left(\mu\right), \mu\right) < W^N\left(n_I^N\left(\mu\right), \mu\right) < W^N\left(n_*^N\left(\mu\right), \mu\right)$$

So this proves that $W^P\left(n_I^P\left(\mu\right), \mu\right) - W^N\left(n_I^N\left(\mu\right), \mu\right) < 0$ for $\mu < \mu_R$. Identically, for $\mu > \mu_B > \mu_R$, $n_I^P\left(\mu\right) > n_I^N\left(\mu\right)$ and $W^P\left(n, \mu\right) - W^N\left(n, \mu\right) > 0$ so

$$W^N\left(n_I^N\left(\mu\right), \mu\right) < W^P\left(n_I^N\left(\mu\right), \mu\right) < W^P\left(n_I^P\left(\mu\right), \mu\right) < W^P\left(n_*^P\left(\mu\right), \mu\right)$$

This proves that $\tilde{\mu}_2$ is unique and $W^P\left(n_I^P\left(\mu\right), \mu\right) \lesseqqgtr W^N\left(n_I^N\left(\mu\right), \mu\right)$ as $\mu \lesseqqgtr \tilde{\mu}_2$.

Proof of Proposition 5. At the second stage, the ISP chooses the regime independently of coverage, so that the regime is N if $\mu \leq \mu_R$ and P if $\mu > \mu_R$. If $\mu \leq \mu_R$ or $\mu > \mu_B$, the regime chosen by the ISP is also the regime preferred by the regulator, so that the regulator can impose the welfare-maximizing coverage. If $\mu \in (\mu_R, \mu_B]$, the ISP chooses P while N is the welfare-maximizing regime, so that the regulator chooses $n_*^P < n_*$.

Proof of Proposition 6

(a) if $\mu < \tilde{\mu}_1$, then $R^N(\mu) > B^P(\mu)$ so that $n_I^N > n_*^P$, which implies that $n_*^N > n_I^N > n_*^P > n_I^P$. As $n_I^N > n_I^P$, this also implies that N meets criterion 2. Moreover, since $\mu < \mu_R$, we have $R^N(\mu) > R^P(\mu)$, so that

$$W^N\left(n_I^N, \mu\right) > W^P\left(n_I^P, \mu\right)$$

and N meets criterion 1. Since $\mu < \mu_B$, we have $B^N(\mu) > B^P(\mu)$, so that

$$W^N\left(n_*^N, \mu\right) > W^P\left(n_*^P, \mu\right)$$

so that N meets criterion 3.

(b) if $\tilde{\mu}_1 \leq \mu < \mu_R$ then $B^P(\mu) \geq R^N(\mu)$ so that $n_*^P \geq n_I^N$. Since $\mu < \mu_B$, $n_*^N > n_*^P$ and since $\mu < \mu_R$, $n_I^N > n_I^P$ We thus have that

$$n_*^N > n_*^P \geq n_I^N > n_I^P$$

and that N meets criterion 2. Criteria 1 and 3 are checked in the same way as in (a).

(c) if $\mu_R \leq \mu < \tilde{\mu}_2$ then $W^N\left(n_I^N, \mu\right) > W^P\left(n_I^P, \mu\right)$, so that N meets criterion 1. Since $\mu \geq \mu_R$, $n_I^P \geq n_I^N$ and $\Pi^P\left(n_I^P, \mu\right) \geq \Pi^N\left(n_I^N, \mu\right)$. Since $\mu < \mu_B$, $n_*^N > n_*^P$ and $W^N\left(n_*^N, \mu\right) > W^P\left(n_*^P, \mu\right)$. We thus have that

$$n_*^N > n_*^P > n_I^P \geq n_I^N$$

and that P meets criterion 2, while N meets criterion 3.

(d) $\tilde{\mu}_2 \leq \mu < \mu_B$, then $W^P\left(n_I^N, \mu\right) > W^N\left(n_I^P, \mu\right)$, so that P meets criterion 1. Since $\mu_R > \mu > \mu_B$, coverages are set as in (c). Criteria 2 and 3 are checked as in (c).

(e) if $\mu \geq \mu_B$ then $n_*^P \geq n_*^N$ and $W\left(n_*^P, \mu\right) \geq W\left(n_*^N, \mu\right)$ and P meets criterion 2 and 3. Since $\mu > \tilde{\mu}_2$, $n_*^P > n_I^N$ and since $\mu > \mu_R$, $n_I^P > n_I^N$ and

$$n_*^P \geq n_*^N > n_I^P > n_I^N$$

Criterion 1 is checked as in (d).

Proof of Proposition 7. First part is directly derived from Proposition 5 as the regulator can impose the welfare-maximizing coverage. For $\mu \in (\tilde{\mu}_2, \mu_B)$, we have $n_U = n_*^P > n_T = n_I^P$ so this yields $W^P\left(n_*^P(\mu), \mu\right) > W^P\left(n_I^N(\mu), \mu\right)$, which proves the second part. Last, if $\mu \in (\mu_R, \tilde{\mu}_2]$, from Proposition 6, $n_*^N > n_*^P > n_I^P \geq n_I^N$ and if $\mu = \tilde{\mu}_2$: $W^P\left(n_*^P(\tilde{\mu}_2), \tilde{\mu}_2\right) > W^P\left(n_I^P(\tilde{\mu}_2), \tilde{\mu}_2\right) = W^N\left(n_I^P(\tilde{\mu}_2), \tilde{\mu}_2\right) > W^N\left(n_I^N(\tilde{\mu}_2), \tilde{\mu}_2\right)$. Now we have

$$\begin{aligned} & W^P\left(n_*^P(\mu), \mu\right) - W^N\left(n_I^N(\mu), \mu\right) \\ = \; & \Pi^P\left(n_*^P(\mu), \mu\right) - \Pi^N\left(n_I^N(\mu), \mu\right) + a\left(X_1^P(\mu) - X_1^N(\mu)\right) \\ < \; & \Pi^P\left(n_I^P(\mu), \mu\right) - \Pi^N\left(n_I^N(\mu), \mu\right) + a\left(X_1^P(\mu) - X_1^N(\mu)\right) \end{aligned}$$

as $\Pi^P\left(n_I^P(\mu), \mu\right) > \Pi^P(n, \mu)$ for all n. So if $\mu = \mu_R$

$$W^P\left(n_*^P(\mu_R), \mu_R\right) - W^N\left(n_I^N(\mu_R), \mu_R\right) < a\left(X_1^P(\mu_R) - X_1^N(\mu_R)\right) < 0$$

since $X_1^P(\mu) - X_1^N(\mu) = -\frac{\beta}{2}\frac{(\mu-1)^2}{\mu(\alpha+\beta)} < 0$. By continuity, there exists a $\tilde{\mu}_u \in (\mu_R, \tilde{\mu}_2)$ such that $W^P\left(n_*^P(\tilde{\mu}_u), \tilde{\mu}_u\right) = W^N\left(n_I^N(\tilde{\mu}_u), \tilde{\mu}_u\right)$ and $W^P\left(n_*^P(\mu), \mu\right) \lesseqqgtr W^N\left(n_I^N(\mu), \mu\right)$ as $\mu \lesseqqgtr \tilde{\mu}_u$.

Proof of Lemma 3. From Proposition 1, $\mu_R(1) < \mu_B$. Since $\Delta R = \Delta V$ at $\eta = 0$, we have $\mu_R(0) = \mu_V$, which is greater than μ_B from Lemma 1 and Proposition 1. By continuity there exists a $\hat{\eta} \in (0,1)$ such that $\mu_R(\eta) \lesseqqgtr \mu_B$ as $\eta \gtreqqless \hat{\eta}$.

Proof of Proposition 8. Proof of Proposition 6 applies with the restriction that $\tilde{\mu}_1 = \mu_R$.

Proof of Proposition 9. Proof of Proposition 6 applies by replacing $\tilde{\mu}_1$ by $\hat{\mu}_1$ and imposing the restriction that $\mu_B = \tilde{\mu}_2 = \mu_R$.

Proof of Proposition 10. First, as we have $B_1^i(\mu) < B^i(\mu)$ for $i = N, P$, maximum USO coverages are $n_f^i = N\left(B_1^i(\mu), \mu\right) < n_*^i = N\left(B^i(\mu), \mu\right)$. Second, as $B_1^i(\mu) - R^i(\mu) = aX_1^i(\mu) > 0$ then $n_f^i > n_I^i$ for all $\mu \geq 1$. Third, as $\Delta B_1 = \Delta B$, for $\mu \leq \mu_B$, $n_f^N \geq n_f^P$ and conversely. Fourth, as $B_1^P(\mu) - R^N(\mu) < B^P(\mu) - R^N(\mu)$ and $B^P(\tilde{\mu}_f) > R^N(\tilde{\mu}_f)$, similar arguments as in the proof of Lemma **??** applies, so that there exists a $\tilde{\mu}_f \in (\tilde{\mu}_1, \mu_R)$ such that $B_1^P(\tilde{\mu}_f) = R^N(\tilde{\mu}_f)$. Then, for

$\mu \leq \tilde{\mu}_f, n_f^P \leq n_I^N$. Furthermore, as $W_1^P\left(n_I^P,\mu\right)-W_1^N\left(n_I^N,\mu\right) = N\left(R^P\left(\mu\right),\mu\right)B_1^P(\mu)-C\left(n_I^P,\mu\right)-N\left(R^N(\mu),\mu\right)B_1^N(\mu)+C\left(n_I^N,\mu\right)$ and we follow same steps as in the proof of Proposition **??**, above, to show that, by continuity, there exists a $\hat{\mu}_f \in (\mu_R,\mu_B)$ such that it exists : $W_1^P\left(n_I^P,\hat{\mu}_f\right) = W_1^N\left(n_I^N,\hat{\mu}_f\right)$. Finally, others arguments are unchanged from Proposition 6 and are omitted in this proof. Then Proposition 6 applies for n_f^i and n_I^i by substituting n_f^i to n_*^i, $\tilde{\mu}_f$ to $\tilde{\mu}_1$ and $\hat{\mu}_f$ to $\tilde{\mu}_2$.

Proof of Proposition 11. We assume $\alpha < 0.5$. Note that

$$\Delta B - \Delta R \lesseqqgtr 0 \text{ as } \eta \gtreqqless \hat{\eta}$$

If $\eta > \hat{\eta}$, the proof of Lemma 1 applies once we substitute $\mu_R(\eta)$ to μ_R in it.

Consider now the case where $0 < \eta \leq \hat{\eta}$. Since $\Delta V \lesseqqgtr 0$ as $\mu \lesseqqgtr \mu_V$, $a\eta\Delta X_0(1) = 0$, $a\lim_{\mu\to\infty}\eta\Delta X_0 \to \infty$ and $\frac{\partial(\Delta X_0)}{\partial\mu} > 0$, $\forall\mu$, there exists a $\mu_R(\eta) < \mu_V$ such that $\Delta R = \Delta V + a\Delta X_0 \lesseqqgtr 0$ as $\mu \lesseqqgtr \mu_R$.

Similarly, since $\Delta R \lesseqqgtr 0$ as $\mu \lesseqqgtr \mu_R(\eta) > 0$, $a(\Delta X(1)) = 0$, $\lim_{\mu\to\infty} a(\Delta X(\mu)) \to \infty$, [$a(\Delta X(\mu))$ is convex: vérifier si cela a été montré] and $\Delta B \geq 0 = \Delta R$ at $\mu = \mu_R(\eta)$, there exists a $\mu_B \leq \mu_R(\eta)$ such that $\Delta B \lesseqqgtr 0$ as $\mu \lesseqqgtr \mu_B$.

Proof of Lemma 4

1. Consider a $\mu < \mu_B < \mu_R(\eta)$. As $B^P(\mu)$ and $R(\mu)$ are independent of η.

2. We have:

$$\begin{aligned} & W^P\left(n_I^P(\mu),\mu\right) - W^N\left(n_I^N(\mu),\mu\right) \\ = \; & N\left(R^P(\mu),\mu\right)B^P(\mu) - C\left(n_I^P(\mu),\mu\right) - N\left(R^N(\mu),\mu\right)B^N(\mu) + C\left(n_I^N(\mu),\mu\right) \end{aligned}$$

   At $\mu = \mu_B$, we have:

$$\begin{aligned} & W^P\left(n_I^P(\mu_B),\mu_B\right) - W^N\left(n_I^N(\mu_B),\mu_B\right) \\ < \; & N\left(R^P(\mu_B),\mu_B\right)\left(B^P(\mu_B) - B^N(\mu_B)\right) - C\left(n_I^P(\mu_B),\mu_B\right) < 0 \end{aligned}$$

   where the first equality comes from the fact that $R^P(\mu_B) < R^N(\mu_B)$ and the second, from the fact that $B^P(\mu_B) = B^N(\mu_B)$.

   At $\mu = \mu_R(\eta)$,we have:

$$\begin{aligned} & W^P\left(n_I^P(\mu_R),\mu_R\right) - W^N\left(n_I^N(\mu_R),\mu_R\right) \\ > \; & N\left(R^P(\mu_R),\mu_R\right) - N\left(R^N(\mu_R),\mu_R\right)B^N(\mu_R) + C\left(n_I^N(\mu_R),\mu_R\right) - C\left(n_I^P(\mu_R),\mu_R\right) = 0 \end{aligned}$$

where the first equality comes from the fact that $B^P(\mu_R) > B^N(\mu_R)$ and the second, from the fact that $R^P(\mu_R) = R^N(\mu_R)$ and $n_I^N(\mu_R) = n_I^P(\mu_R)$.

Proof of Proposition 12. If $\eta > \hat{\eta}$, the proof of Proposition 6 applies once we substitute $\mu_R(\eta)$ to μ_R in it. Consider now the case of $\eta < \hat{\eta}$.

(a) The case $\mu < \tilde{\mu}_1(\eta)$ is identical to case (a) of Proposition 6 because μ is less than all bandwidth threshold.

(b) The case $\tilde{\mu}_1(\eta) \leq \mu < \mu_B < \mu_R(\eta)$ is identical to case (b) of Proposition 6 as μ is lower than both μ_B and μ_R.

(c) If $\mu_B \leq \mu < \tilde{\mu}_2$, then, because $\mu \geq \mu_B$, $n_*^P > n_*^N$ and $W^P(n_*^P, \mu) \geq W^N(n_*^N, \mu)$ so that P meets criterion 3. But, because $\mu < \tilde{\mu}_2$, $W^N(n_I^N, \mu) > W^P(n_I^P, \mu)$ and N meets criterion 1. Since $\mu < \mu_R(\eta)$, $n_I^N > n_I^P$ and N meets criterion 2. We thus have $n_*^P > n_*^N > n_I^N > n_I^P$.

(d) If $\tilde{\mu}_2 \leq \mu < \mu_R(\eta)$, $W^P(n_I^P, \mu) \geq W^N(n_I^N, \mu)$ and P meets criterion 1. The two other criteria are checked as in (c).

(e) The case $\mu \geq \mu_R(\eta)$, is identical to case (e) of Proposition 6 because because μ is greater than all bandwidth threshold.

Proof of Lemma 2. First let us give a characterization of M_*^i. Putting first-best coverages $n_*^i(\mu) = N(B^i(\mu), \mu)$ in the ISP participation constraint (13), implies that bandwidth levels are such that $R^i(\mu) - \frac{1}{2}B^i(\mu) - \frac{1}{4}k\mu^2 \geq 0$. This also writes $\pi^i(\mu) := R^i(\mu) - aZ^i(\mu) - \frac{1}{8}k\mu^2 \geq 0$, where $Z^N(\mu) = X^N(\mu)$, $Z^P(\mu) = X_0^N(\mu) + X_1^P(\mu)$ and $\Delta Z(\mu) = X_1^P(\mu) - X_1^N(\mu) = -\frac{\beta}{2}\frac{(\mu-1)^2}{\mu(\alpha+\beta)} \leq 0$, decreasing and concave for all $\mu > 1$. Here $Z^N(\mu)$ is increasing and convex and $Z^P(\mu)$ is increasing and concave with μ. Moreover with $\lim_{\mu\to+\infty}\left(R^i(\mu)-aZ^i(\mu)\right)=\lim_{\mu\to+\infty}\left(V^i(\mu)-a\hat{Z}^i(\mu)\right)$ where $\hat{Z}^N(\mu)=X^N(\mu)$ and $\hat{Z}^P(\mu)=X^P(\mu)-2X_0^N(\mu)=\frac{1}{2}(\mu-1)\frac{\alpha\mu^2+(\beta-3\alpha)\mu+\beta}{\mu(\alpha+\beta)}>0.$[32] So

$$\begin{aligned}
\lim_{\mu\to+\infty}\left(V^N(\mu)-aX^N(\mu)\right) &= \lim_{\mu\to+\infty}\left(v(\mu-1)^{\alpha+\beta}-a(\mu-1)\right)=\lim_{\mu\to+\infty}(-a\mu)=-\infty\\
\lim_{\mu\to+\infty}\left(V^P(\mu)-a\hat{Z}^N(\mu)\right) &= \lim_{\mu\to+\infty}\left(\left(\frac{1}{2}\right)^{\alpha+\beta}v\left(\mu^2-1\right)^{\alpha+\beta}\mu^{-\beta}-a\frac{1}{2}(\mu-1)\frac{\alpha\mu^2+(\beta-3\alpha)\mu+\beta}{\mu(\alpha+\beta)}\right)\\
&= \lim_{\mu\to+\infty}\left(v\left(\frac{1}{2}\right)^{\alpha+\beta}\mu^{2\alpha}-a\frac{\alpha\mu^2}{2(\alpha+\beta)}\right)=\lim_{\mu\to+\infty}\left(-a\frac{\alpha\mu^2}{2(\alpha+\beta)}\right)=-\infty
\end{aligned}$$

Define now $m^i=\arg\max_m \pi^i(\mu)$, such as[33] $\left(\pi^i\right)'(\mu)=0\Leftrightarrow r^i(\mu)-\frac{1}{16}k\mu=az^i(\mu)$ where $z^i(\mu)=\left(Z^i\right)'(\mu)>0$. When $a=0$, $m^i=m_0^i>1: r^i(\mu)=\frac{1}{16}k\mu$, as $\lim_{\mu\to1^+}\left(r^i(\mu)-\frac{1}{16}k\mu\right)=+\infty$. Then m^i decreases with a from m_0^i to 1. So $\pi^i\left(m^i\right)=R^i\left(m^i\right)-aZ^i\left(m^i\right)-\frac{1}{8}k\left(m^i\right)^2$ and using the envelop theorem $\frac{\partial\pi^i\left(m^i\right)}{\partial a}=-Z^i\left(m^i\right)<0$, then it exists $a_\pi^i:\pi^i\left(m^i\right)=0$.

- If when $a=0:\pi^i\left(m_0^i\right)=R^i\left(m_0^i\right)-\frac{1}{8}k\left(m_0^i\right)^2<0$ then for all $(a,\mu),\pi^i(\mu)<0$ and M_*^i is empty. This is the case if $k>k_0^i: k_0^i=8R^i\left(m_0^i\right)/\left(m_0^i\right)^2$.
- If when $a=0:\pi^i\left(m_0^i\right)=R^i\left(m_0^i\right)-\frac{1}{8}k\left(m_0^i\right)^2\geq0$, As $\pi^i(1)=-\frac{1}{8}k<0$, for $a\leq a_\pi^i$, it exists $\underline{\mu}^i$ and $\bar{\mu}^i:\pi^i\left(\underline{\mu}^i\right)=\pi^i\left(\bar{\mu}^i\right)=0$ such that $\underline{\mu}^i\leq m^i\leq\bar{\mu}^i$, so that $M_*^i=\left[\underline{\mu}^i,\bar{\mu}^i\right]$ and when $a=a_\pi^i;\underline{\mu}^i=m^i=\bar{\mu}^i$. For $a>a_\pi^i:\pi^i(\mu)<0$ and M_*^i is empty. So if $a\leq a_\pi^i$, $\frac{d\bar{\mu}^i}{da}<0<\frac{d\underline{\mu}^i}{da}$.
- Moreover $\Delta\pi(\mu)=\Delta R(\mu)-a\Delta Z(\mu)$, $\Delta\pi(1)=0$ and $\Delta\pi(\mu_R)=-a\Delta Z(\mu_R)>0$. Then it exists $\mu_\pi:\Delta\pi(\mu_\pi)=0$ such that $\Delta R(\mu_\pi)=a\Delta Z(\mu_\pi)<0$ so $\mu_\pi\leq\mu_R$.

[32] Indeed, the quadratic polynom $\alpha\mu^2+(\beta-3\alpha)\mu+\beta$ has no real roots if $\beta<9\alpha$, and only negative roots if $\beta\geq9\alpha$.

[33] Bien sur ce prend des conditions pour que l'on ait des valeurs uniques mais laisse ça de côté pour l'instant.

En fait j'ai montré pour la dernière extension (sous-section **??**) que

- $r_i:=\left(R^i\right)'(\mu)>0$; $\left(r^i\right)'(\mu)<0$ and so $r_i'-k<0$
- il faut montrer que $\left(R^i\right)''(\mu)-a\left(Z^i\right)''(\mu)-\frac{1}{32}k<0$ ce qui est déjà fait pour $i=N$ mais pas pour $i=P$, A FAIRE
- Cela obligerait à réorganiser les annexes.

Proof of Proposition 13. From (8) and (19), we obtain:

$$
\begin{aligned}
m'_n &= \frac{1}{2}\left(\frac{aX_f^i}{F}\right)^{\frac{1}{2}} n^{-\frac{1}{2}} > 0 \\
m'_\mu &= \frac{1}{2}\left(\frac{an}{F}\right)^{\frac{1}{2}} \left(X_f^i\right)^{-\frac{1}{2}} \left(X_f^i\right)' > 0 \\
m''_{\mu n} &= \frac{1}{4}\left(\frac{an}{F}\right)^{-\frac{1}{2}} \left(X_f^i\right)^{-\frac{1}{2}} \left(X_f^i\right)' > 0
\end{aligned}
$$

Moreover, since

$$\Delta m\left(n,\mu\right) = \left(\frac{an}{F}\right)^{\frac{1}{2}} \left(\sqrt{X_f^P\left(\mu\right)} - \sqrt{X_f^N\left(\mu\right)}\right) < 0$$

we obtain:

$$
\begin{aligned}
\Delta m\left(n,\mu\right) &= \left(\frac{na}{F}\right)^{\frac{1}{2}} \left(\sqrt{X_f^P\left(\mu\right)} - \sqrt{X_f^N\left(\mu\right)}\right) = \left(\frac{na}{F}\right)^{\frac{1}{2}} \left(\sqrt{\frac{1}{2}\frac{\beta}{\alpha+\beta}\frac{\left(\mu^2-1\right)}{\mu}} - \sqrt{\frac{\beta}{\alpha+\beta}\left(\mu-1\right)}\right) < 0 \\
\frac{\partial\left(\Delta m\right)}{\partial n} &= \frac{1}{2}n^{-1}\Delta m < 0 \\
\frac{\partial^2\left(\Delta m\right)}{\partial n^2} &= -\frac{1}{4}n^{-2}\Delta m > 0
\end{aligned}
$$

List of Figures

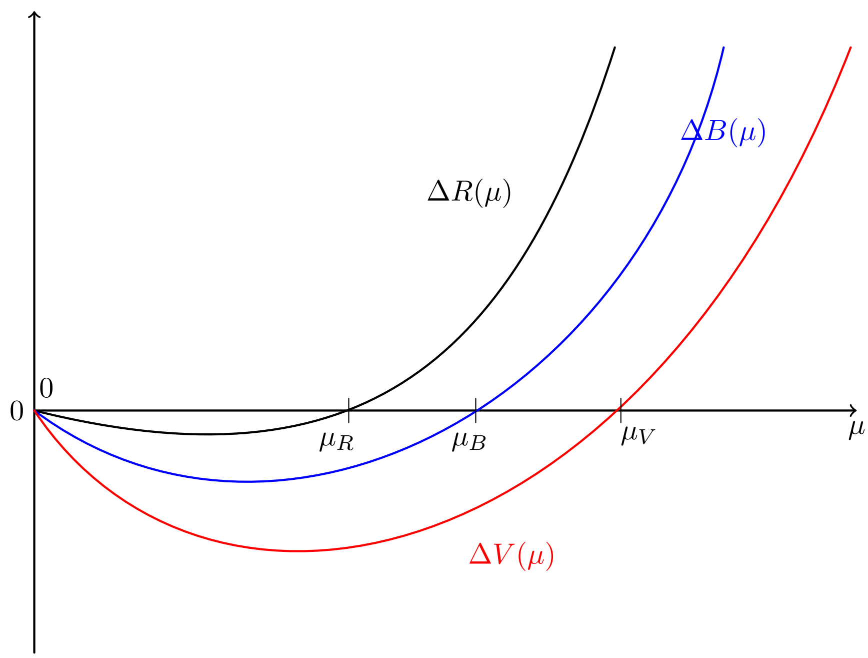

Figure 1: Social welfare, ISP profit and consumer surplus differences

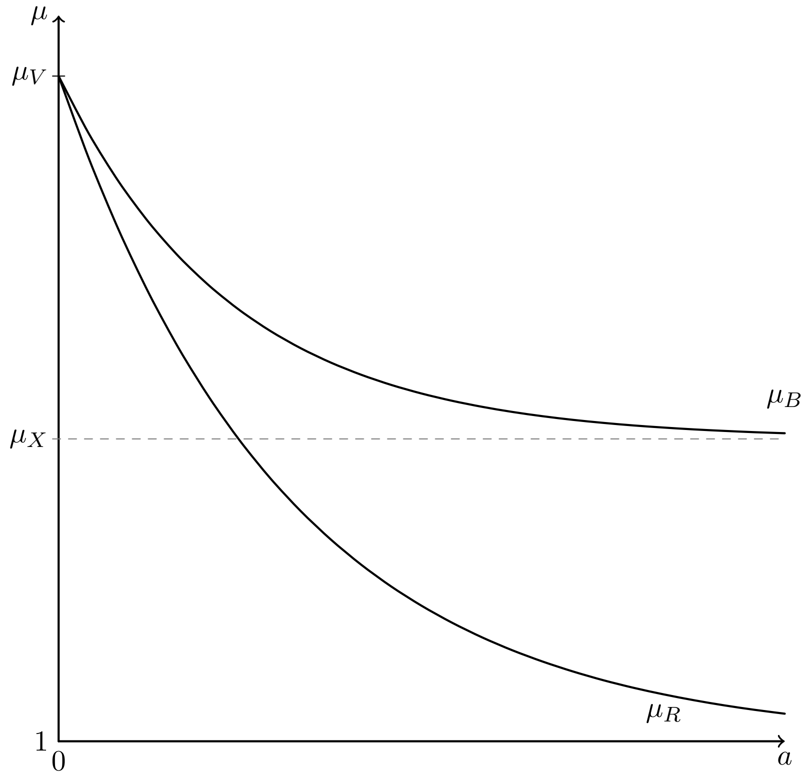

Figure 2: μ_R, μ_B as functions of a

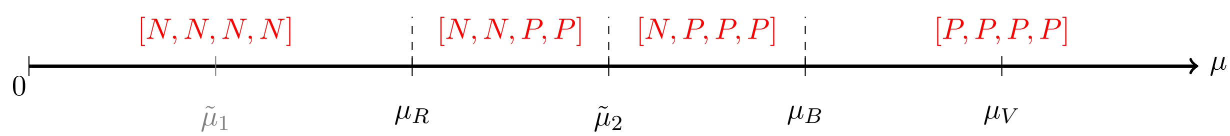

Figure 3: Net Neutrality or Prioritization : criteria [FR, TMR, USO, UM]

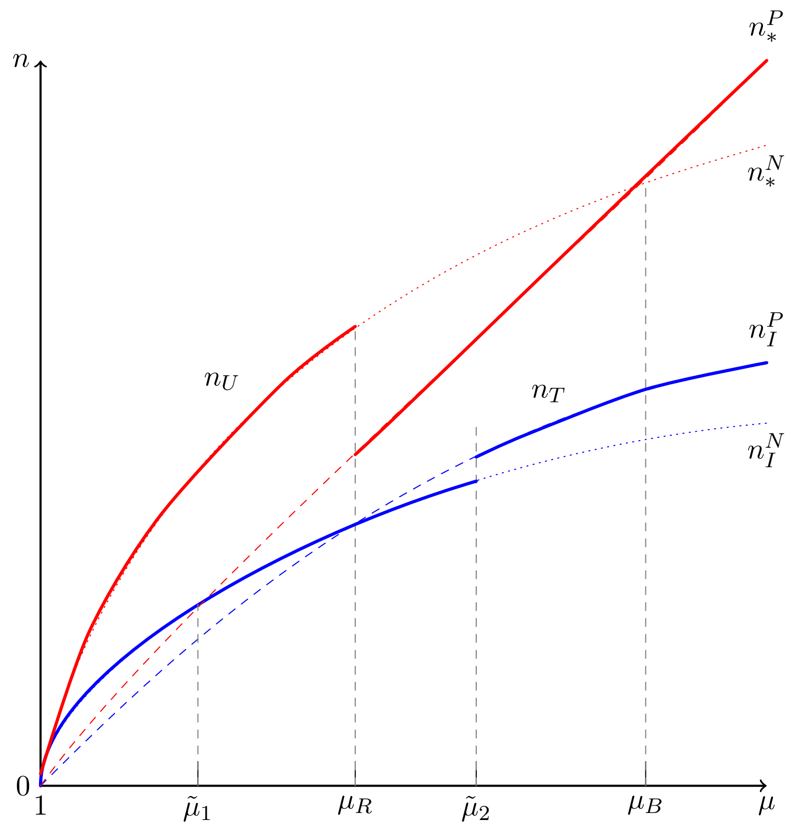

Figure 4: Proposition 6. n_T: blue thick line, n_U: red thick line

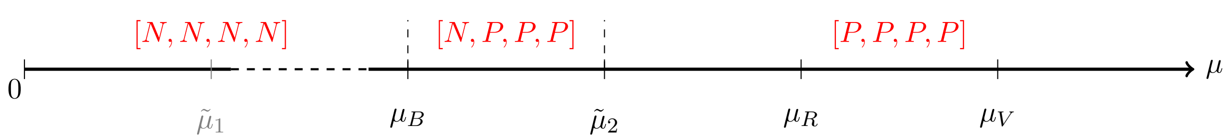

Figure 5: Criteria with ISP low bargaining power