The Environmental Impact of Internet Regulation

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Abstract

We address the need to regulate Internet infrastructure usage to take into account environmental externalities. We model the interactions between a monopoly ISP and different types of content providers in settings where the former chooses the network size and the latter influences congestion on the network. We first show that current net neutrality regulation does not provide agents the right incentives to cope with the environmental externality issue. Then, we study several alternatives, including laissez-faire, price-based regulation, and norm-based regulation. We derive conditions under which these alternatives fare better than net neutrality. In particular, the two types of regulations are useful tools to accommodate consumer interest and environmental concerns.

1 Introduction

In recent decades, the telecom and digital sector has witnessed growth that is unprecedented in economic history. A whole new world of social media, online advertising and online sales has emerged, and this world has relied on investments

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by some actors of the traditional sectors, in particular the construction and telecom industries. However, the increase in energy prices and the negative environmental impact of digital industries have put into question the sustainability of the current trend. Indeed, the digital sector is said to account for 34% of worldwide greenhouse gas output with annual growth of approximately 8% (see ARCEP, 2020). The objective of this paper is to address the need to regulate the use of infrastructures to mitigate the environmental cost of internet usage.

To date, digital debates concerning infrastructures have mainly focused on the net neutrality issue, i.e., the extent to which content providers (mostly social media but, more globally, businesses operating on the Internet) should pay Internet service providers (ISPs) to reach consumers. On the one hand, most ISP have claimed that content providers have to pay to use their infrastructures to provide them the right incentives to invest in capacity. On the other hand, content providers have claimed that ISPs are already being paid by their subscribers and that any additional fee would only limit the variety of content available, at the expense of consumers. In this debate, most of the arguments are assessed on the impact (positive or negative) on the total level of investment. The environmental crisis should lead us to consider how to make the best use of the network as least as much as the best way to increase total capacity.

The reason for considering the total capacity of the network as a key variable is threefold. First, the capacity of the network puts an upper bound on the use of the internet; that is, the flow of data, by all undertakings. Therefore, if the actions of the agents generate environmental damages, they are related to network capacity. Second, building extra capacity directly induces some direct environmental costs (e.g., raw material, use of energy). Third, the management of this capacity is very costly. According to a recent report (see France Stratégie, 2020), 75% of Telco electricity consumption is driven by network operations, so limiting its growth is a direct way to limit this consumption (and the associated externalities). However, for a given size of the network, how it is used no longer depends on telecom operators but rather on content providers and consumers. Concerning content providers, many of them can decide to use the network in a more or less effective way. Indeed, they can use data compression techniques or store the data closer to consumers in order to minimize their use of network capacity. Regarding consumers, they can reduce the intensity of internet usage either by altering the video parameters or their time allocation across websites. But neither content providers nor consumers directly benefit from these actions. Therefore, as for any other externality, this change of behavior will only happen if the agents are given the proper incentives either by the terms of the contract they sign with the ISPs or by adequate regulation. In this article, we discuss the possible solutions to the presence of these congestion and environmental externalities.

More precisely, we develop the idea, in line with some proposals of the French telecom regulator (see ARCEP, 2020) that more should be done to incentivize the content producers. Indeed, the amount of data they generate is one of the main drivers of the capacity choice by the ISP, and therefore of the carbon footprint of the industry. To that end, we develop a model in which an ISP allows consumers to access the service of some ad-financed content providers (CP). For this service to be provided, the ISP must choose a level of network capacity that has both a private cost and an environmental cost. But the necessary size of the network depends on the characteristics and technology of the CPs. In line with the real world, we consider different types of CP that differ according to their impact on the network, i.e., how much capacity they need, but also on their ability to reduce the needed capacity. Some CPs are capacity-intensive and can, at a small cost, significantly reduce their traffic load, whereas other CPs require less capacity but can barely further reduce their traffic load. In this setting, the level of consumption, the actions of the CPs, and size of the network should take into account the impact on consumers, the ad revenues generated by the CP, and the cost of the capacity, both for the ISP and the environment. This ideal outcome is first compared with a situation in which the only monetary transaction is between consumers and the ISP. This case, called net neutrality, suffers from many inefficiencies. First, the CPs are never incentivized to reduce the load they generate on the network. Second, the ISP does not take into account the revenues generated by the CP or the negative environmental externalities. Therefore, both the productive and allocative efficiencies are distorted from the optimal situation. This outcome is compared to several alternatives.

First, we analyze the laissez-faire situation in which the ISP can freely charge CPs. If, as it is generally assumed in the literature, the ISP uses a fixed price per unit of traffic, it will generally results in higher investment levels. Indeed, the ISP can capture the revenues generated by the CPs, so its marginal gain from expanding the network size is increased. Nevertheless, we show that laissez-faire could lead to reducing the network size and therefore the environmental cost of the sector in two cases. First, if the CPs differ in the revenues they generate, then the ISP can choose to exclude some CPs, and the equilibrium size of the network decreases. Second, and more interestingly for us, when the ISP can charge the CPs for the congestion they create, it may give them some incentives to reduce the congestion on the network, although its incentives to do so are reduced compared to that of a benevolent planner. However, even in this case, the prospect of capturing ad revenues may drive the ISP to choose a larger network size than in the net neutrality case.

Second, we consider the possibility that a regulator sets some congestion-based prices for the CPs. This new regulatory tool gives the right incentives to the CPs without directly affecting the ISP's incentives to increase the size of the network. Moreover, the price level is optimally chosen to take into account not only consumer surplus and the cost of building the network but also environmental externalities. One may wonder how the ISP reacts to the decrease in the congestion the CPs generate. The ISP incentives are driven by two forces that were already at play in the characterization of the optimal allocation. First, when congestion decreases, for a given consumer level of usage, the capacity need is decreased (a pure *congestion-based effect*). Second, a lower congestion cost increases the incentives to propose higher usage levels to consumers (a *consumption-based effect*). When consumer marginal utility is quite sensitive to changes in quantity, which could be justified by a standard rachet effect in consumption, the first effect dominates, and decreasing congestion leads the ISP to decrease its capacity investment.

Finally, we look at the possibility that the regulator opts for a system of norms—congestion caps—to control the behavior of the CPs. We show that this case may replicate the outcome generated by the congestion-based prices, although not as efficiently due to the CPs' heterogeneity.

This paper is related to the net neutrality debate and, therefore, to the literature that addresses the impact of allowing or preventing the ISP to discriminate, in price or quality, among the content providers for accessing their subscribers. In this debate, net neutrality has been defined either as the ban on prioritization—see, for example, Choi and Kim (2010) or Bourreau, Kourandi, and Valletti (2015)—or as a restriction of the way ISPs can charge CPs. We take the second approach, which was first developed by Economides and Tåg (2012). Whereas this article emphasizes the two-sided nature of the industry, we are more concerned by the impact on investment and congestion of regulation. In particular, we develop the idea that the need for investment is the joint result of the actions of all the undertakings, i.e., those by consumers, ISP, and content providers. The role of consumers in limiting the congestion of the network has been developed by Jullien and Sand-Zantman (2018). Here, we focus on the supply side, by looking at the extent to which various regulations can give CPs more or fewer incentives to exert some congestion-reducing effort. The idea that CPs could influence the congestion of the network was discussed by Peitz and Schuett (2016). In their article, this action was designed to guarantee that the content was delivered in case of congestion, and the lack of coordination was at the source of what the authors called the "inflation of traffic". In our approach, the effort of content providers reduces congestion instead of increases it. We show that, as in Choi, Jeon, and Kim (2018), there can be some complementarity or substitutability between the effort exerted by the content providers to reduce the congestion on the network and the ISP's investment decision. This interplay between the ISP and the content providers' action is analyzed in a setting of heterogeneity across content providers. A last important aspect of our work is related to the environmental impact of the industry; for this, we assume that the environmental damage depends on the size of the network. This is an important element of our study since, in contrast to the standard net neutrality debate, increasing the size of the network may not be the goal of a regulatory change.

In Section 2, we describe our model, whereas Section 3 presents the two main benchmarks, i.e., the optimal allocation and the outcome under net neutrality. In Section 4, we discuss the laissez-faire approach and compare it to the previous cases focusing on incentives CPs have to make congestion-reducing efforts and on the equilibrium environmental footprint. In Section 5, we study two forms of regulation—price-based and norm-based—and show when they contribute to improving the allocation compared to the net neutrality case. Section 6 concludes. All proofs are relegated to the Appendix.

2 Model

Internet Service Provider (ISP) We consider a model in which a monopoly ISP contracts with consumers and allows some content providers to access its consumers. The ISP owns a network for which it must choose the total capacity K. This capacity can be built or extended at an increasing and nonconcave cost $C(K)$ and, with loss of generality, we assume that $C(K) = cK$. The investment process in broadband capacities generates negative environmental impacts. More precisely, we assume that a network of capacity K generates CO2-GHG emissions equal to δK. Here δ is a factor that measures the environmental impact of investments in broadband. Two important remarks. In the digital industry, most of the detrimental environmental effects (between 70% and 80%) originate from the cost of building the devices (e.g., computers, screens, mobile phones). Second, the environmental impact that is generated by the network (between 5% and 15%) is very country-dependent, as it is linked to the electricity generation process. As this can hardly be controlled by the ISP, we will take the environmental impact δ per unit of capacity as given.

Content providers (CPs) The capacity chosen by the ISP is used by many CPs to reach consumers. But different CPs have different effects on the network, some of them needing more capacity than others. More precisely, the link between the amount of content consumed (and therefore the satisfaction consumers can derive) and the impact on the network is not uniform across CPs . We model this by assuming that when a quantity q of content is consumed, the ISP capacity usage is θq. Here, the parameter θ represents the gross congestion impact of consumption and varies across CPs. To keep the analysis simple, we assume that there are two types of CPs, that is $\theta \in \{\underline{\theta}, \overline{\theta}\}$, where $\overline{\theta} > \underline{\theta}$ and there is a share of $\mu \in (0, 1)$ of CPs with a factor $\underline{\theta}$ and $(1 - \mu)$ with a factor $\overline{\theta}$. Moreover, we denote by $\mathbb{E}(\theta)$ the expected value of θ.

An important feature of our model lies in the fact that the CPs can affect the congestion impact. Indeed, the gross congestion impact can be reduced by some data compression techniques, modeled by an effort $e \in \{0, \hat{e}\}$ chosen by the CPs. Therefore, the net congestion per unit of consumption is given by $z(\theta) = (\theta - e)$ for a type-θ usage. Reducing the congestion is costly and this unit cost is type-dependent, $\psi(\theta)$. To represent the fact that load reduction is easier for capacity-intensive CPs, we assume that $\psi(\overline{\theta}) < \psi(\underline{\theta})$. We denote $\psi(\overline{\theta}) = \overline{\psi}$ and $\psi(\underline{\theta}) = \underline{\psi}$, so our assumption boils down to $\underline{\psi} > \overline{\psi}$. Therefore, the ISP faces a capacity constraint that writes as

$$\mu z\left(\underline{\theta}\right) q\left(\underline{\theta}\right) + (1-\mu)\, z\left(\overline{\theta}\right) q\left(\overline{\theta}\right) \leq K$$

where $q(\theta)$ denotes the consumption of θ content.

Finally, the CP business model is based on online ads with a monetary value of $b\,(\theta)$ per unit of consumption q. For most of the paper, we will assume that b is the same across CPs, as we want to emphasize CP heterogeneity in another dimension. Still, we will discuss how some of our future results could be affected when the revenues per unit differ across CPs.

We make two assumptions on the ad revenues of the CPs. First, we assume that $c\theta - b(\theta) > 0$ for all c, θ, b. This means that, if consumers do not derive any utility from consumption, it is not optimal to built any capacity regardless of the congestion generated by the CPs. This assumption is necessary in our setting to avoid an infinite choice of capacity even when consumers do not care about content. Second, we assume that $b(\theta) \geq \min\{\underline{\theta}\underline{\psi}, \bar{\theta}\bar{\psi}\}$, i.e., the revenues generated by the ads are not too small. This does not mean that all CPs can finance the cost of reducing congestion but that all CPs could finance the lowest cost of congestion. This assumption is not crucial to derive most of our results but limits the number of cases to study.

Consumers We assume that consumers subscribe to the network at a fixed price T and derive a utility $u(q(\theta))$ when they consume a quantity $q(\theta)$ of a content proposed by a type-θ content provider. We assume that $u'(0) > 0$ and that $u'' < 0$. When choosing how much to consume, it is reasonable to consider that consumers do not take into account the impact of each type of CP on network capacity. Therefore, we model their choice as a simple problem of determining consumption levels of each content subject to a content capacity k supplied by the ISP for their usage. An important point is that k differs from K. In this setting, when consumers can freely choose their usage levels, the consumptions denoted $\underline{q}$ and $\overline{q}$ for each type of CP, are such that

$$\max_{q} U = \mu u\left(\underline{q}\right) + (1-\mu)\, u\left(\overline{q}\right) - T$$

subject to

$$\mu \underline{q} + (1-\mu)\,\overline{q} \leq k$$

Direct computations lead to $\underline{q} = \overline{q} = k$.

In this case, the ISP will face the following constraint:

$$\left[\mu\left(\underline{\theta} - \underline{e}\right) + (1-\mu)\left(\overline{\theta} - \overline{e}\right)\right] k \leq K \tag{1}$$

where $\underline{e}$ and $\overline{e}$ correspond to the congestion-reducing effort of each CP. In this article, we do not investigate in depth how the ISP sets its price for subscribers or how different incentive schemes could make consumers more responsive to the congestion issues (see Jullien and Sand-Zantman, 2018, on this point). Therefore, to focus on the relationship between the CPs and the ISP, we assume that the ISP can extract the whole surplus from consumers by setting a tariff $T = u(k)$.[1]

3 Benchmarks

In this section, we derive two useful benchmarks. First, we compute the allocation that maximizes social welfare, and then we derive what happens under net neutrality.

3.1 Optimal Allocation

As a starting point, it is important to derive what the optimal allocation could be. In our model, this means 1) whether the CPS should invest in order to reduce the capacity on the network, 2) what size of network should be chosen, and 3) how much time consumers should spend on each type of CP. Formally, we define the first-best allocation as the actions $\{q(\theta), e(\theta), K\}$ that maximize social welfare subject to the ISP capacity constraint

$$W = \mu\left[u\left(q(\underline{\theta})\right) + \left(b - \overline{\psi} e(\underline{\theta})\right) q(\underline{\theta})\right] + (1-\mu)\left[u\left(q(\overline{\theta})\right) + \left(b - \overline{\psi} e(\overline{\theta})\right) q(\overline{\theta})\right] - (\delta + c)K$$

subject to

$$\mu\left(\underline{\theta} - e(\underline{\theta})\right) q(\underline{\theta}) + (1-\mu)\left(\overline{\theta} - e(\overline{\theta})\right) q(\overline{\theta}) \leq K \tag{2}$$

[1] We could also assume that the ISP captures only a share of this surplus, without changing the main message of this article.

In the above objective, all the direct and indirect consequences of investment, effort and consumption are taken into account. Note also that the constraint clearly shows an interplay between congestion and network size, an important aspect we will comment on later. Note also that the social optimum should account for the environmental impact of the network size (or expansion).

Solving this problem leads to this first proposition.

Proposition 1. *Let us define for all θ $q_0(\theta)$ and $q_1(\theta)$ by*

$$\begin{aligned} u'(q_0(\theta)) &= (\delta + c)\,\theta - b \\ u'(q_1(\theta)) &= (\delta + c)\,\theta + (\psi(\theta) - (\delta + c))\,\hat{e} - b \end{aligned}$$

Then, the optimal allocation is such that

- *If $\underline{\psi} > \overline{\psi} > \delta + c$, then*

$$e(\theta) = 0 \text{ for all } \theta,\ q(\theta) = q_0(\theta), \text{ and } K_0^* = \mu\underline{\theta}q_0(\underline{\theta}) + (1-\mu)\,\overline{\theta}q_0(\overline{\theta}).$$

- *If $\underline{\psi} > \delta + c > \overline{\psi}$, then*

$$e(\theta) = 0 \text{ and } e(\overline{\theta}) = \hat{e},\ q(\underline{\theta}) = q_0(\underline{\theta}),\ q(\overline{\theta}) = q_1(\overline{\theta}), \text{ and } K_{01}^* = \mu\underline{\theta}q_0(\underline{\theta}) + (1-\mu)\left(\overline{\theta} - \hat{e}\right)q_1(\overline{\theta}).$$

- *If $\delta + c > \underline{\psi} > \overline{\psi}$, then*

$$e(\theta) = \hat{e},\ q(\theta) = q_1(\theta), \text{ and } K_1^* = \mu\left(\underline{\theta} - \hat{e}\right)q_1(\underline{\theta}) + (1-\mu)\left(\overline{\theta} - \hat{e}\right)q_1(\overline{\theta}).$$

The optimal allocation internalizes all the externalities that any action can generate. First, the optimal efforts by the CPs reflect their impacts not only on the ISP cost but also on the climate. For each CP, the first-best level of effort depends on a trade-off between the private cost of effort and these two impacts (c and δ). When these impacts are low, the optimal solution entails zero efforts for both CPs. As the impacts increase, it is optimal to request an effort from the high-type CPs and then, when the impacts are very strong, an effort from the low-type CPS. Second, the usage levels depend on their effects on all the agents. Indeed, this level increases with the consumers' marginal utility and the ad revenues generated by the CPs, whereas it decreases with the cost of building the capacity and the environmental damage. It is important to remark that there is an interplay between the effort of the CPs and the optimal consumption level. Indeed, when the CPs exert an effort to reduce congestion, it also reduces the cost of building some extra capacity, thereby increasing the usage levels. This first result is important, as it will be at play in the following sections.

Finally, it is interesting to examine whether the effort exerted by the CP will increase or decrease the optimal level of capacity.

Corollary 1. *If u is concave enough, i.e., the marginal benefit of consumers decreasing sharply, the optimal size of the network decreases with the congestion-reducing effort of the CPs.*

The effort exerted by the CPS has two effects on the optimal allocation. First, for a given level of consumption, this decreases the need for a large network. This leads the benevolent planner to choose a lower K. We will refer to this as the *congestion-based effect*. Second, this effort decreases the perceived cost of increasing the network size, and therefore the marginal cost of consumption; so, the optimal consumption levels increase, and we will refer to this as the *consumption-based effect*. When the marginal utility is strongly decreasing, the optimal consumption level does not vary much when the building cost increases, so this latter effect is offset by the former and induces more effort from the CPs to lead the planner to decrease the network size. This first case is illustrated in Figure 1 (with $\eta = \psi - (\delta + c) > 0$). When the marginal utility is weakly decreasing, the reverse holds and a reduction in the level of congestion is associated with an increase in the optimal size of the network. This second case is illustrated in Figure 2.

In the rest of this article, we study to what extent a monopoly ISP, whether unregulated or regulated, has incentives to make the same (or relatively similar) choices as a planner.

3.2 Net Neutrality

As a second benchmark, we look at a decentralized situation in which not only there is no central planner to control the actions of the agents but also the ISP is forbidden to charge any fee to the CPs. This situation, called net neutrality, can be considered

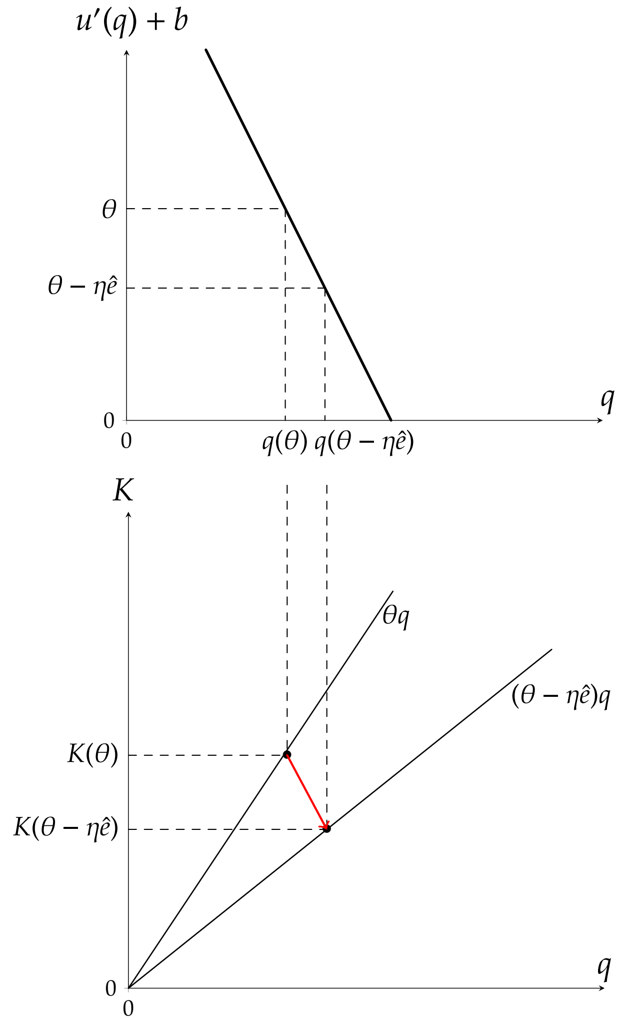

Figure 1: Substitutability between congestion-reducing effort and network size

to be the current situation in Europe and in the U.S. Indeed, even if CPs play a key role in bringing some content to consumers and therefore use the network more than the standard internet user, ISPs are not allowed to charge them termination fees for their connection to the ISP's subscribers. What are the resulting allocations and inefficiencies in this case?

In this setting, the revenue the ISP earns come only from its subscribers. As explained in Section 2, the ISP can extract the whole consumer surplus. Since consumers do not internalize the impact of their consumption on the network, they

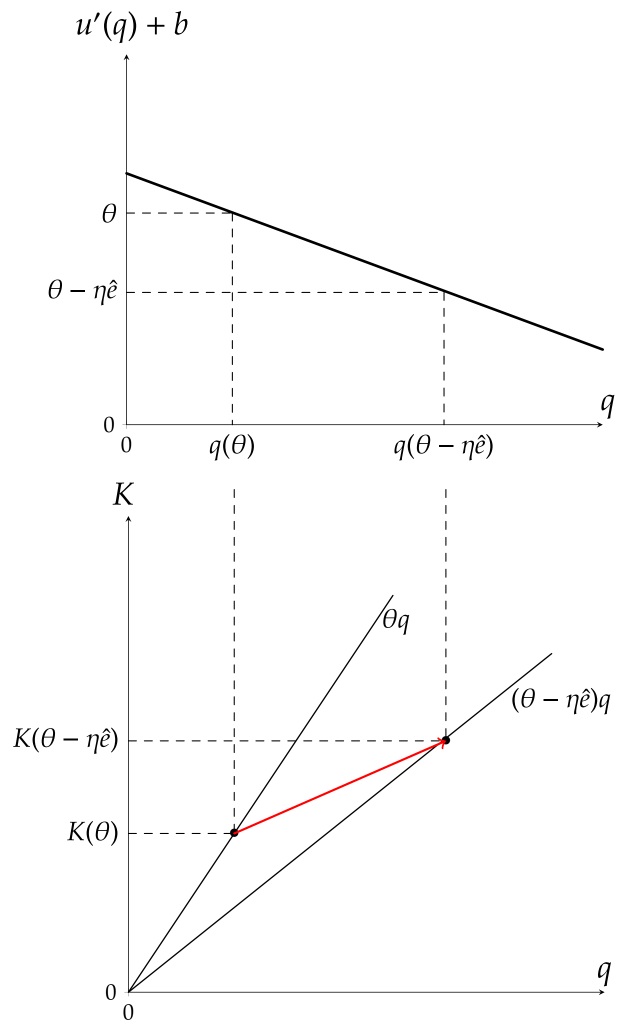

Figure 2: Complementarity between congestion-reducing effort and network size

consume the same quantity k for any type of CP.[2] This implies that the ISP will establish a subscription fee equal to $u(k)$ and its profit are simply given by

$$\Pi = u(k) - cK.$$

The only choice the ISP has is to set the size of the network, which allows consumers to obtain a level of usage k. The link between k and K depends on the congestion factor that is not controlled by the ISP. Indeed, this congestion is a function of the

[2]In fact, consumption could differ from one type of CP to another according to the user's taste. It is only to simplify the analysis, and without loss of generality, that we assume consumers have the same satisfaction from every type of content.

type of content available and consumed, and of the effort exerted by the CPs to reduce the load they generate. As reducing the load is costly, and the CPs have no personal or financial incentives to bear this cost, there will be no effort on their side. This implies that the ISP will maximize its profit under the constraint

$$k\mathbb{E}(\theta) \leq K$$

This problem is easily solved and leads to the following proposition.

Proposition 2. *In a net neutrality regime, the equilibrium consumption and investment levels are given by*

$$u'(k^n) = \mathbb{E}(\theta)\, c; \;\; q^n(\theta) = k^n \text{ and } K^n = \mathbb{E}(\theta)\, k^n$$

In this net neutrality situation, neither the consumption level (in general), nor the choice of effort by the CPs nor the choice of investment by the ISP are optimal. Indeed, the CPs do not exert any congestion-reducing effort. This may be optimal, say when the cost of both types of CP are high, but this is not the case in general. Second, consumers do not adjust their consumption to the load, as there are no reasons to do so. Their consumption patterns only depend on their taste, not on the different cost/load they could generate on the network by consuming, for example, more capacity-intensive content. Finally, the ISP, when choosing its network size, only takes into account the congestion factor for each type of content, but not the benefit that accrues to CPs when there is more consumption (the b) nor the marginal environmental cost of the capacity (δ). One cannot tell, in general, whether this leads to an upward distortion or a downward distortion of the network size. But the situation is very far from optimal, as none of the many externalities (on the load, or on the environment) are considered.

4 Laissez-faire

In this section, we investigate two laissez-faire options: the first corresponds to a situation in which the ISP can charge the CPs uniform prices regardless of the congestion CPs generate on the network, and in the second, we allow the ISP to fine tune its pricing to take into account CP heterogeneity.

4.1 Uniform prices

We assume that a *uniform price p* can be set by the ISP to allow CPs be connected to their network. This pricing can be justified when the ISP cannot perfectly observe (or contract on) the level of congestion the CPs generate. In this setting, the ISP profit writes as

$$\Pi = u(k) + pk - cK.$$

To maximize this profit, the ISP then chooses the price and the size of the network, taking into account the congestion constraint (2) and the CPs profitability constraint

$$\pi(\theta) = (b(\theta) - p - \psi(\theta)\, e(\theta))\, k \geq 0 \tag{3}$$

As in the case of net neutrality, the CPs have no incentives to exert any congestion-reducing effort, so $e(\theta) = 0$. Assume first that CPs do not differ in the revenue they generate from their business model ($\overline{b} = \underline{b} = b$), so, we can state the following proposition

Proposition 3. *When the ISP can charge the CPs, the consumption and equilibrium investment are given by*

$$\begin{aligned} q^u(\theta) &= k^u : u'(k^u) + b = \mathbb{E}(\theta)\, c \\ \textit{and } K^u &= \mathbb{E}(\theta)\, k^u \end{aligned}$$

As a result, $K^u > K^n$ *and* $k^u > k^n$.

The result of Proposition 3 shows the standard result of the laissez-faire regime.

When the ISP can charge the CPs , it takes into account the revenues generated from ads when choosing the size of the network. As more consumption generates more ad revenues, a higher network size allows increased consumption, and the ISP chooses to invest more than in the net neutrality regime. When the ISP can charge the CPs, it internalizes the money CPs can generate, an externality that was not internalized in the net neutrality regime. But by comparison with the first-best allocation, there are still two unsolved issues. First, the CPs are given no incentive to exert any congestion-reducing effort. Second, the negative environmental impact is not internalized by the ISP. It is difficult to say if this situation is welfare-improving compared to the net neutrality case but if one only focused on the environmental side, this laissez-faire situation creates more damages than the net neutrality situation.

In most of this article, we assume that the ad revenues generated by the CPs are the same regardless of their type. What is the impact of laissez-faire when the CPs also differ in this dimension? The new element is that the ISP must decide whether to set a price compatible with the business model of all types of CPs or, on the contrary, to set a price that excludes the less-profitable content. This potential exclusion would have some impact on the network congestion, and, therefore, on the equilibrium network size. When the less profitable CPs are the capacity-economical CPs ($\underline{b} < \overline{b}$), their exclusion has a limited impact on the load unless they constitute the majority of CPs. Instead, when the less profitable CPs are the capacity-intensive CPs, ($\underline{b} > \overline{b}$), exclusion is more likely to have a positive impact on network congestion and, therefore, induce a fall in the ISP investment level. The following proposition discusses these case.

Proposition 4. *When CPs differ in their business models,*

1. *to have $K^u \leq K^n$, it requires the exclusion of one type of CP.*
2. *there exist weights $\hat{\mu}$ and μ^* such that for $\hat{\mu} < \mu \leq \mu^*$, ISP will charge a uniform price $p^* = \max\{\overline{b}, \underline{b}\}$, and this leads to $K^u \leq K^n$.*

The first point states that the reduction of the network size can only be achieved by excluding some CPs. Indeed, the ISP has no means of inducing CPs to exert a congestion-reducing effort. As a consequence, the only way to reduce congestion is to exclude some types of CPs. The second point states some conditions under which the ISP may be willing to exclude some CPs and that this exclusion is good for the environment. The first condition is satisfied when the share of excluded CPs is not too small. For the second, the total capacity needed to serve the remaining CP should not be too large, which means that the share of excluded CPs should be high enough. These two conditions are quite demanding and more likely to be satisfied when the less profitable CPs (those excluded) are also the capacity-intensive CPs. Finally, even if the environmental impact of exclusion is positive, the effect on the consumer's gross surplus is likely to be negative. Indeed, consumers will be deprived of one type of content. They will consume more of the remaining content but, with their taste for variety, they are likely to be worse off compared to the net neutrality case.

4.2 Tailored prices

We now assume that the ISP can use the ex post CPs' impact $z = \theta - e(\theta)$ to adjust prices. Thus, the ISP can propose two-part tariffs $T(z) = p(\theta)z + t(\theta)$. This new pricing system creates some incentives for the CPs to exert their congestion-reducing effort. Indeed, the CPs profit is given by

$$\pi(\theta) = (b(\theta) - p(\theta - e) - t - \psi(\theta) e) k.$$

It is direct to see that $\frac{\partial \pi(\theta)}{\partial e} = p - \psi(\theta)$. Hence if $p \geq \psi(\theta)$ then $e(\theta) = \hat{e}$ and if $p < \psi(\theta)$ then $e(\theta) = 0$. Moreover, with the fee $t(\theta)$, the ISP is then able to capture the remaining profit of the CPs by setting $t(\theta) = b(\theta) - p(\theta)(\theta - e) - \psi(\theta) e$. We now discuss in detail what choice the ISP will make in this context.

It is important to see that the ISP may not be interested in inducing the CPs to exert congestion-reducing effort. Indeed, even if this effort reduces network expansion, it deprives the ISP from some revenue. This situation is quite similar to that faced by the benevolent planner in Section 3, the only difference being that the ISP does not take into account the environmental externality. We can therefore state the following lemma.

Lemma 1. *When the ISP can set some congestion-based prices, a type-θ CP will be incentivized to exert some congestion-reducing effort if and only if $c \geq \psi(\theta)$.*

Note that we have assumed that the ad revenues are not too small, or more precisely, that $b \geq \min\{\underline{\theta\psi}, \overline{\theta\psi}\}$. This implies that the ISP will be able to induce some effort of at least one type of CP (otherwise, we would return to the uniform-price case studied above). Suppose, first, that for all CPs, the ad revenues are large enough to finance the cost of the congestion-reducing effort, i.e., if $b > \theta\psi(\theta)$ for all θ. then the ISP always wants to induce this effort, as by assumption $c\theta > b$, which leads to $c > \underline{\psi}$. Consequently the ISP profit writes as

$$\Pi = u(k) + \mathbb{E}(b - \theta\psi) k - cK \text{ s.t. } k(\mathbb{E}(\theta) - \hat{e}) \leq K$$

where

$$\mathbb{E}(b-\theta\psi)=\mu\left(\underline{b}-\underline{\theta\psi}\right)+(1-\mu)\left(\overline{b}-\overline{\theta\psi}\right)>0$$

Proposition 5. *Suppose that $b>\theta\psi(\theta)$ for all θ and that the ISP can use congestion-based prices. Then, if $c>\underline{\psi}$, all the CPs are induced to exert some congestion-reducing effort, the ISP will capture all the CPs' profits and the equilibrium consumption and investment levels are given by*

$$\begin{aligned} k^t &: \ u'\left(k^t\right)=(\mathbb{E}(\theta)-\hat{e})\,c-\mathbb{E}(b-\theta\psi) \\ K^t &= k^t(\mathbb{E}(\theta)-\hat{e})\,. \end{aligned}$$

Moreover, if u is concave enough and $0<\mathbb{E}(b-\theta\psi)\leq\overline{B}$, then

$$k^t\geq k^n \text{ and } K^t\leq K^n$$

where $\overline{B}=(\mathbb{E}(\theta)-\hat{e})c-u'\left(k^n\frac{\mathbb{E}(\theta)}{\mathbb{E}(\theta)-\hat{e}}\right)$.

Here, the ISP can capture all the rents after congestion reduction due to the CPs' effort. Consequently, it may prefer to induce these efforts and capture the remaining rents instead of investing in costly capacities. If expected ex post rents are not too high ($\mathbb{E}(b-\theta\psi)<\overline{B}$), this has a dampening effect on the network size. Moreover, this is not done at the expense of consumption, which increases thanks to the reduction in congestion. As a result, tailored price discrimination can be environmentally friendly. However, if the expected ex post rents are high ($\mathbb{E}(b-\theta\psi)>\overline{B}$) or if consumer utility is not concave enough, the ISP has more incentives to increase the consumer's demand, boosting capacity investments above the net neutrality level.

Let us suppose now that if the ISP cannot induce all CPs to exert some effort, i.e., that there exist $\tilde{\theta}$ such that $b\left(\tilde{\theta}\right)-\tilde{\theta}\psi\left(\tilde{\theta}\right)<0$. Then, for this type of CP, the ISP will choose not to induce any effort by setting $p\left(\tilde{\theta}\right)=0<\psi\left(\tilde{\theta}\right)$ such as $e\left(\tilde{\theta}\right)=0$ and capture all its profit through the fee $t\left(\tilde{\theta}\right)=b$. Now the ISP profit writes as

$$\Pi=u(k)+\mathbb{E}(b)\,k-\theta\psi(\theta)\,k-cK \ \text{ s.t. } k(\mathbb{E}(\theta)-m(\theta)\,\hat{e})\leq K$$

with $\theta\neq\tilde{\theta}$ and $m(\theta)$ is the share of CPs that exert the congestion-reducing effort.

Corollary 2. *Suppose that there exists one type $\tilde{\theta}$ such that $b-\tilde{\theta}\psi\left(\tilde{\theta}\right)<0$ and that the ISP can use congestion-based prices. Then only those CPs with $\theta\neq\tilde{\theta}$ are induced to exert some* *congestion-reducing effort, but the ISP will still capture all the CPs' profit. The equilibrium consumption and investment levels are given by*

$$\begin{aligned} k^{\tau} &: \quad u'(k^{\tau}) = c(\mathbb{E}(\theta) - m(\theta))\hat{e} - [\mathbb{E}(b) - \theta\psi(\theta)] \\ K^{\tau} &= (\mathbb{E}(\theta) - m(\theta)\hat{e})k^{\tau} \end{aligned}$$

Moreover, if u is concave enough and $\mathbb{E}(b) - \theta\psi(\theta) \leq \underline{B}$ then $K^u > K^n \geq K^{\tau}$ and $k^{\tau} > k^n$ where $\underline{B} = (\mathbb{E}(\theta) - \hat{e})c - u'\left(k^n \frac{\mathbb{E}(\theta)}{\mathbb{E}(\theta) - m(\theta)\hat{e}}\right)$.

When some CPs cannot afford to exert congestion-reducing effort, the ISP still captures their profit and induces the other CPs to exert some effort. This situation can still induce less investment in capacity while increasing gross consumer surplus, but the conditions to obtain this results are more stringent than in Proposition 5. Note that the same result is obtained if $c \in [\overline{\psi}, \underline{\psi}]$. In this case, only the CPs with type $\overline{\theta}$ will be an incentive to exert congestion-reducing effort. At last, for $c < \underline{\psi}$, the ISP will set $p = 0$ for all types and uses only uniform prices as in the previous section.

What we can say about the impact of this laissez-faire regulation on the environment? First, the ISP should be given enough flexibility to be able to induce CPs to exert some congestion-reducing effort. In this respect, tailored prices are more efficient than uniform prices. Second, the revenues generated by the CPs (or at least the share the ISP can capture) should not be too high. Otherwise, the ISP will be tempted to focus on revenue maximization instead of incentivizing the CPs to reduce the externalities they generate on the network and, therefore, on the environment. When these two conditions are satisfied and consumer utility is concave enough, laissez-faire generates less environmental damage than under net neutrality.

5 Environmental-based Regulated prices

In the previous section, we showed that laissez-faire could lead to different outcomes, some of them quite satisfactory compared to net neutrality, and others less satisfactory. One key element is that laissez-faire does not guarantee that the ISP will not induce the CPs to exert some congestion-reducing effort. In so far as this is a necessary condition to reconcile the environmental requirement (smaller network capacity) and the consumers' needs (i.e., constant or increasing consumption levels), we consider some alternative options. In this section, we study how a regulator could meet these two goals using price regulation.

More precisely, we assume that the regulator relaxes the internet rules, allowing that a price be charged to the CPs. In contrast to the previous section, however, this price must be based only on the ex post congestion impact $z = \theta - e$, and second it will be set by the regulator. The amount paid by a CP is then written as $t(z) = pz$, where p is the same across all CPs, and the CP profitability constraint is given by

$$\pi(\theta) = (b - p(\theta - e) - \psi e) k \geq 0.$$

This price regulation creates some incentives for the CPs to exert some congestion-reducing effort. As before, for a price per unit of congestion p, we have $\frac{\partial \pi(\theta)}{\partial e} = p - \psi$. Hence, if $p \geq \psi$ then $e(\theta) = \hat{e}$.

We assume that the money collected is not captured by the ISP. Moreover, as we do not want to introduce additional motives for taxation, we do not consider any cost of public funds (see Browning, 1976). Note that the financial burden imposed on the CPs could lead some of them to exit the market (we referred to this as exclusion in the previous section). This would be the case if their revenues (b) were small enough. We will look first at the situation in which exclusion is not an issue—the most interesting and relevant case—and then consider this possibility and how it influences the regulator optimal policy.

5.1 No exclusion

Regulation aims at choosing a price to increase efforts from CPs. Therefore, we will study two possible regulated prices given by $p^r = \underline{\psi}$ and $p^r = \overline{\psi}$. If $p^r = \underline{\psi}$, all CPs exert the effort $\hat{e}$ and the ISP problem writes as

$$\max_{k,K} \Pi = u(k) - cK \ \text{ s.t. } k(\mathbb{E}(\theta) - \hat{e}) \leq K$$

This price is feasible if capacity-intensive CPs remain profitable. A condition for this to be true is $b \geq \underline{\psi}\overline{\theta} + \left(\underline{\psi} - \overline{\psi}\right)\hat{e} > \underline{\psi}\overline{\theta}$.

Now, if $p = \overline{\psi}$, the capacity-economical CPs ($\underline{\theta}$) do not exert the effort but are still profitable. The ISP profit is unchanged, but its constraint is now written as

$$k\left(\mathbb{E}\left(\theta\right) - (1-\mu)\hat{e}\right) \leq K \tag{4}$$

The next proposition describes the consequence of setting these two prices.

Proposition 6. *When the regulator uses prices, then*
(a) *for $p^r = \underline{\psi}$, the consumption and equilibrium investment are given by*

$$u'\left(\underline{k}^r\right) = \left(\mathbb{E}\left(\theta\right) - \hat{e}\right)c \;\; and \;\; \underline{K}^r = \underline{k}^r\left(\mathbb{E}\left(\theta\right) - \hat{e}\right)$$

(b) *for $p^r = \overline{\psi}$, the consumption and equilibrium investment are given by*

$$u'\left(\overline{k}^r\right) = \left(\mathbb{E}\left(\theta\right) - (1-\mu)\hat{e}\right)c \;\; and \;\; \overline{K}^r = \overline{k}^r\left(\mathbb{E}\left(\theta\right) - (1-\mu)\hat{e}\right)$$

(c) *If u is concave enough, then $\underline{k}^r > \overline{k}^r > k^n$, $\underline{K}^r < \overline{K}^r < K^n$, and $\underline{\Pi} > \overline{\Pi} > \Pi^n$.*

Price regulation is intended to create direct incentives for CPs to exert some effort. If the regulated price is based on the highest cost $\underline{\psi}$, all CPs reduce their impacts on the network (as long as they remain profitable), and when the consumer marginal benefit is high, the ISP reduces its investment level as the congestion-based effect dominates the consumer-based effect (see Section 3). The ISP constraint is relaxed, which, compared to net neutrality, allows for savings in capacity investments and an increase in consumer traffic. If the regulated price is based on the lowest cost $\overline{\psi}$, no congestion-reduction incentives are given to the capacity-economical CPs. Therefore, the congestion-based effect still applies but is weakened.

What would be the choice of a consumerist-environmentalist regulator? In such a case, the welfare function only takes into account the consumer surplus and both the building and environmental costs of the capacity, that is $W^r(p) = u(k) - (\delta + c)K$. Applying directly the results in Proposition 6, one can state the following.[3]

Corollary 3. *If the revenues generated by the CPs are high enough to avoid exclusion, i.e., if $b \geq \underline{\psi}\overline{\theta} + \left(\underline{\psi} - \overline{\psi}\right)\hat{e}$, a consumerist-environmentalist regulator chooses $p^r = \underline{\psi}$.*

[3]The result of the following corollary extends to any welfare function $\gamma(u(k) - cK) - (1-\gamma)\delta K$ for any $\gamma \in [0,1]$.

A consumerist-environmentalist regulator has social preferences oriented toward dual objectives: to increase consumer traffic and to decrease the network size. As long as all CPs remain profitable when the congestion regulation is implemented, the regulator prefers to make the congestion-based effect fully effective, as it is strongly aligned with its preferences. To that end, choosing $p^r = \underline{\psi}$ is the optimal policy and clearly improves upon the current net neutrality situation.

5.2 Potential Exclusion

The above results must be qualified insofar as it is assumed that all CPs were profitable when they exerted congestion-reducing efforts. In this subsection, we consider cases in which ad revenues are lower than before, and thus may prevent some CPs from exerting effort in a profitable way.

Indeed, if $\underline{\psi}\overline{\theta} + \left(\underline{\psi} - \overline{\psi}\right)\hat{e} > b$, capacity-intensive CPs cannot exert a profitable effort with the highest regulated price. Then, if the regulator sets the $p = \underline{\psi}$, only type-$\underline{\theta}$ CPs will exert congestion-reducing effort, while the other CPs still operate on the market but do not make any effort. In this case, the ISP profit writes as before but the constraint is changed to:

$$k\left(\mathbb{E}\left(\theta\right) - \mu\hat{e}\right) \leq K \tag{5}$$

The equilibrium consumption and investment level are now given by

$$u'\left(\underline{k}^r\right) = \left(\mathbb{E}\left(\theta\right) - \mu\hat{e}\right)c \;\text{ and }\; \underline{K}^r = \underline{k}^r\left(\mathbb{E}\left(\theta\right) - \mu\hat{e}\right)$$

As before, for sufficiently concave u, it is direct to see that $\underline{k}^r > k^n$; $\underline{K}^r < K^n$ and $\underline{\Pi} > \Pi^n$.

Moreover, the congestion constraints (4) and (5) are identical for $\mu = \frac{1}{2}$, i.e., when the share of both types of CPs are the same. So if $\mu \geq \frac{1}{2}$, the total congestion is lower with $p = \underline{\psi}$ than with $p = \overline{\psi}$ and conversely. Hence, we obtain the same results as in Proposition 7c ($\underline{k}^r \geq \overline{k}^r$ and $\underline{K}^r \leq \overline{K}^r$) and Corollary if and only if $\mu \geq \frac{1}{2}$. This means that setting a high regulated price is still optimal when most of the CPs are $\underline{\theta}$-type. As a result, we can state the following.

Corollary 4. *If* $\underline{\psi}\overline{\theta} + \left(\underline{\psi} - \overline{\psi}\right)\hat{e} > b \geq \underline{\psi}\overline{\theta}$, *and there are mostly capacity-economical CPs, i.e.,* $\mu \geq \frac{1}{2}$, *a consumerist-environmentalist regulator will choose* $p^r = \underline{\psi}$. *Otherwise, the regulator will choose* $p^r = \overline{\psi}$.

In this setting, the regulator implements its price regulation in order to obtain the most effective *congestion-based effect*. When there are mostly type-$\underline{\theta}$ CPs, it is too harmful for society not to incentivize them. Indeed, in this case, the impact of the regulation on the ISP constraint would be too limited, stalling the congestion-based effect. As a result, the regulator opts for $p^r = \underline{\psi}$.

With the above price, the $\overline{\theta}$-type were not excluded from the market. Suppose now that the ad revenues are even lower, and more precisely that $\underline{\psi}\overline{\theta} > b \geq \max\{\psi\theta\}$. Then, when $p = \underline{\psi}$, the type-$\overline{\theta}$ CPs cannot make any positive profit in this market, regardless of their choice of effort. If those CPs are excluded from the market, the ISP problem changes to

$$\max_{k,K} \Pi = \mu u\left(k\right) - cK \text{ s.t. } k\mu\left(\underline{\theta} - \hat{e}\right) \leq K$$

Then, the equilibrium consumption and investment levels are now given by

$$u'\left(\underline{k}^r\right) = \left(\underline{\theta} - \hat{e}\right)c \text{ and } \underline{K}^r = \underline{k}^r\mu\left(\underline{\theta} - \hat{e}\right) \tag{6}$$

And, for a sufficiently concave u, we still obtain $\underline{k}^r > \overline{k}^r > k^n$ and $\underline{K}^r < \overline{K}^r < K^n$.

The optimal choice of regulatory price by a consumerist-environmentalist regulator is now based on the welfare function $\hat{W}^r\left(p\right) = \mu(p)u\left(k\right) - \left(\delta + c\right)K$, with $\mu(\underline{\psi}) = \mu$ and $\mu(\overline{\psi}) = 1$. Then, there exists a threshold μ^r such that $\hat{W}^r\left(\underline{\psi}\right) \geq W^r\left(\overline{\psi}\right)$ if $\mu \geq \mu^r$.

Corollary 5. *If* $\underline{\psi}\overline{\theta} > b \geq \max\{\psi\theta\}$, *and there are mainly type-*$\underline{\theta}$ *CPs* ($\mu \geq \mu^r$), *a consumerist-environmentalist regulator will choose* $p^r = \underline{\psi}$, *and conversely.*

This result is reminiscent of Corollary 4. Indeed, the consumerist-environmentalist regulator will choose a price that allows the most common CPs to exert some congestion-reducing effort. This shows that even a low regulated price can be welfare optimal, as long as it induces most of the CPs to exert some effort.

Finally, let us consider the lowest admissible values for the ad revenues, i.e., the case in which $\max_\theta\{\psi\left(\theta\right)\theta\} > b \geq \min_\theta\{\psi\left(\theta\right)\theta\}$. Now, the exclusion of CPs can occur for the two possible regulated price levels. Indeed, suppose first that $\underline{\psi}\underline{\theta} > \overline{\psi}\overline{\theta}$. Then, setting a regulated price $p^r = \underline{\psi}$ will exclude all CPs so it is not feasible and the only possible policy is to set $p^r = \overline{\psi}$. Suppose instead that $\overline{\psi}\overline{\theta} > \underline{\psi}\underline{\theta}$. Then, for both prices $p^r = \underline{\psi}$ or $p^r = \overline{\psi}$, the $\overline{\theta}$-type CPs will be excluded. However, the regulator wants to induce the remaining CPs to exert some effort so it will choose $p^r = \underline{\psi}$ and obtains $\hat{W}^r\left(\underline{\psi}\right) = \mu u\left(\underline{k}^r\right) - (\delta + c)\underline{K}^r$ where the consumption and investment levels are defined in (6).

6 Environmental-based norms

Another approach for environmental policy is to opt for command-and-control tools. Here, the consumerist-environmentalist regulator targets some technical standards to reduce congestion and, therefore, emissions due to capacity building. Then, he defines a cap on congestion z_m imposed on all CPs such that

$$z(\theta) = \theta - e(\theta) \leq z_m$$

We focus on caps that can be achieved by both types of CPs, which implies that this policy must verify that

$$z_m \geq \overline{\theta} - \hat{e} > \underline{\theta} - \hat{e}$$

This regulation, even when technically feasible, may not be profitable for all CPs. Indeed, the regulation imposes some cost on the CPs, leading capacity-intensive CPs to exit the market. Note that the participation for at least one type of CP is guaranteed as, by assumption, we have $b \geq \min_\theta\{\psi(\theta)\theta\}$.

In such a setting, the ISP problem writes as

$$\max_{k,K} \Pi = u(k) - cK \text{ s.t. } \left[\mu z\left(\underline{\theta}\right) + (1-\mu) z\left(\overline{\theta}\right)\right] k \leq K$$

To simplify the exposition, we focus here on the case such that $b > \max\{\underline{\theta}\underline{\psi}, \overline{\theta}\overline{\psi}\}$. With price-based regulation, this condition was not enough to prevent some exclusion based on financial motives. In contrast, with norm-based regulations, this condition is sufficient to ensure that no CPs will be excluded from the market.

For a consumerist-environmentalist regulator, the objective is to maximize $W(z_m) = u(k) - (\delta + c)K$, then we can state

Proposition 7. *If $b > \max\{\underline{\theta\psi}, \overline{\theta\psi}\}$, the regulator sets a congestion norm $z_m^* = \overline{\theta} - \hat{e}$, such that*
(a) *if efforts are such that $\overline{\theta} - \underline{\theta} \geq \hat{e}$, only capacity-intensive CPs exert some effort, and the outcome is equivalent to price regulation when $p^r = \overline{\psi}$, defined in Proposition 6b.*
(b) *if efforts are such that $\hat{e} > \overline{\theta} - \underline{\theta}$, all CPs make some effort and the equilibrium consumption and investment levels are given by*

$$u'(k^e) = \left(\overline{\theta} - \hat{e}\right)c \ \text{ and } \ K^e = k^e\left(\overline{\theta} - \hat{e}\right)$$

and for a sufficiently concave u, we have $\underline{k}^r > k^e > k^n$, $\underline{K}^r < K^e < K^n$ and $\underline{\Pi} > \Pi^e > \Pi^n$, where $\left(\underline{k}^r, \underline{K}^r, \underline{\Pi}\right)$ is the price-regulation outcome when $p^r = \underline{\psi}$ defined in Proposition 6a.

Whether the maximum effort is high or low, setting a norm, i.e., a congestion cap, can relax the ISP constraint and trigger the congestion-based effect when u is sufficiently concave. This implies savings in capacity investments and increases in consumers' traffic. Compared to price-based regulation, norms can achieve the same outcome as $p^r = \overline{\psi}$, when the maximal effort is low ($\hat{e} \leq \overline{\theta} - \underline{\theta}$), as only the capacity-intensive CPs makes an effort. However, when the maximal effort is high ($\hat{e} > \overline{\theta} - \underline{\theta}$), all CPs are making the effort, but this norm achieves a worse result than price-based regulation would. Indeed, with a price $p^r = \underline{\psi}$ all CPs are exerting their maximal effort $\hat{e}$, but with a norm-based regulation, the capacity-economical CPs exert less effort.

When considering cases in which congestion costs are such that $\max_\theta\{\psi(\theta)\theta\} > b \geq \min_\theta\{\psi(\theta)\theta\}$, the results in Proposition 7 are altered in two directions: potential exclusion and/or norm relaxation.[4] First, the regulator still chooses the norm $z_m^* = \overline{\theta} - \hat{e}$, but the most costly CPs are excluded when they are not predominant. As with price regulation, exclusion occurs when the impact of the norm on the ISP constraint is limited. Second, the regulator chooses a less stringent norm $z_m^* = \overline{\theta} - \frac{b}{\psi} > \overline{\theta} - \hat{e}$, in order to induce some effort by the type-$\overline{\theta}$ CPs when they are predominant.

In this section, we have assumed that the regulator never wanted to use the technical requirement to exclude some CPs. Suppose, instead, that the norm set by

[4]The details are provided in the Appendix, following the proof of Proposition 7

the regulator could be more stringent than the one type-$\overline{\theta}$ CPs could achieve, with the only constraint that $z_m \geq \underline{\theta} - \hat{e}$. Then, the regulator can choose between a high cap equal to $\overline{\theta} - \hat{e}$ and a lower cap equal to $\underline{\theta} - \hat{e}$. This choice is exactly the same as that analyzed in Corollary 5 to adapt to this context of norm-based regulation. Therefore, the optimal norm will be either $\overline{\theta} - \hat{e}$ when most of the CPs are capacity intensive or $\overline{\theta} - \hat{e}$ otherwise.

In summary, norm-based regulation can improve upon net neutrality, for consumers, the ISP and the environment, by forcing CPs to exert some congestion-reducing effort. However, its lack of flexibility does not accommodate CP heterogeneity as well as price-based regulation. Therefore, the latter form of regulation tends to dominate the former.

7 Conclusions

For years, the debate about the regulation of the Internet has been limited to actors in the industry, mostly Internet service providers and content providers. Recent awareness of the industry's environmental footprint has made this question much more global. It has also changed the perspective one should have on the issue at stake; whereas the original question was, "how to provide the best incentives to increase the size of the network", it has become, "how to provide the right incentives to limit the environmental impact of the sector". In this respect, we have shown that the current situation is far from optimal. The lack of both regulation and prices between CPs and ISPs leads those players, as well as consumers, to overlook the negative externalities their activity generates on the environment. The freedom that was gained thanks to the implementation of net neutrality has created a moral hazard issue whose consequences can be measured by a sharp rise in CO_2 emissions linked to the Internet.

We presented some possible remedies to this issue that all point to the need for more incentives for the players that can reduce the need for a larger network. In particular, we showed what a regulator could do by setting prices or norms to induce CPs to account for their impact on the environment. These solutions rely on the idea that consumers could benefit from the same quality of service at a lower cost for the industry. Introducing some prices—or some norms—on the supply side would allow more coordination and lead to a lower negative environmental impact of the Internet.

Another possibility that we do not explore in this article would be to influence the demand side. For example, allowing ISPs to propose some plans with limited load would be useful to smooth total consumption and limit the need for network expansion. It is difficult to know whether it would be more efficient to play on the supply side or on the demand side. However, in either case, a move from the current situation of net neutrality is needed to seriously tackle the environmental impact of the digital and telecom industries.

References

ARCEP, "Pour un Numérique Soutenable," 2020, available at: https://www.arcep.fr/uploads/tx_gspublication/rapport-pour-un-numerique-soutenable_dec2020.pdf, accessed November 2021.

Bourreau, Marc, Frago Kourandi, and Tommaso Valletti, "Net neutrality with competing internet platforms," *The Journal of Industrial Economics*, 63 (2015), 30–73.

Browning, Edgar K, "The marginal cost of public funds," *Journal of Political Economy*, 84 (1976), 283–298.

Choi, Jay Pil, Doh-Shin Jeon, and Byung-Cheol Kim, "Net neutrality, network capacity, and innovation at the edges," *The Journal of Industrial Economics*, 66 (2018), 172–204.

Choi, Phil Jay, and Byung-Cheol Kim, "Net neutrality and investment incentives," *The RAND Journal of Economics*, 41 (2010), 446–471.

Economides, Nicholas, and Joacim Tåg, "Network neutrality on the Internet: A two-sided market analysis," *Information Economics and Policy*, 24 (2012), 91–104.

France Stratégie, "Maitriser la consommation du numérique: le progrès technologique n'y suffira pas," 2020, available at: https://www.strategie.gouv.fr/sites/strategie.gouv.fr/files/atoms/files/

fs-2020-dt-consommation-metaux-du-numerique-juin.pdf, accessed November 2021.

Jullien, Bruno, and Wilfried Sand-Zantman, "Internet regulation, two-sided pricing, and sponsored data," *International Journal of Industrial Organization*, 58 (2018), 31–62.

Peitz, Martin, and Florian Schuett, "Net neutrality and inflation of traffic," *International Journal of Industrial Organization*, 46 (2016), 16–62.

8 Appendix

A useful Lemma

In the different proofs above, we will need to invoke the following useful intermediate result.

Lemma 2. *The solution in K of the following equation where $x, y, a > 0$:*

$$u'\left(\frac{K}{ax}\right) = cx - y$$

is an implicit function is $\mathcal{K}(x, y)$ such that

1. $\mathcal{K}'_x(x, y) > 0$ *for all a, y if u is concave enough such that $\rho(\mathcal{K}/ax) > \frac{cx}{cx-y} \geq 1$ where $\rho(k) = -\frac{u''(k)k}{u'(k)} > 0$.*
2. $\mathcal{K}'_y(x, y) > 0$ *for all a, x*

Proof of Lemma. First, differentiating with respect to $x \geq 0$ leads to

$$u''\left(\frac{\mathcal{K}}{ax}\right)\left(\frac{1}{a}\frac{\mathcal{K}'_x\, x - \mathcal{K}}{x^2}\right) = c > 0$$

so

$$\mathcal{K}'_x = a\frac{cx}{u''(\mathcal{K}/x)} + \frac{\mathcal{K}}{x}$$

which can be rearranged as:

$$\begin{aligned}\mathcal{K}'_x &= a\frac{cx}{u''(\mathcal{K}/ax)}+\frac{\mathcal{K}}{x}\\ &= a\frac{u'(\mathcal{K}/ax)+y}{u''(\mathcal{K}/ax)}+a\frac{\mathcal{K}}{ax}\\ &= \frac{\mathcal{K}}{x}\left[1+\frac{u'(\mathcal{K}/ax)+y}{\frac{\mathcal{K}}{ax}u''(\mathcal{K}/ax)}\right]\\ &= a\mathcal{K}\left(1-\frac{1}{\rho(\mathcal{K}/ax)}\frac{u'(\mathcal{K}/ax)+y}{u'(\mathcal{K}/ax)}\right)\end{aligned}$$

where ρ is an "Arrow–Pratt of relative risk aversion"-like measure (relative curvature index); here, there is no uncertainty but heterogeneity, so ρ is a heterogeneity elasticity for consumers

$$\rho(k)=-\frac{u''(k)\,k}{u'(k)}>0$$

So, we have

$$\rho(\mathcal{K}/ax)>\frac{u'(\mathcal{K}/ax)+y}{u'(\mathcal{K}/ax)}\geq 1\Rightarrow \mathcal{K}'_x>0 \text{ for all } y\geq 0$$

This means that $u(k)$ is sufficiently concave. If $\rho(\mathcal{K}/ax)<1$, then $\mathcal{K}'_x<0$, i.e., $u(k)$ is not too concave

Second, differentiating with respect to $y\geq 0$ leads to

$$\mathcal{K}'_y\,u''\left(\frac{\mathcal{K}}{ax}\right)=-ax<0$$

so $\mathcal{K}'_y>0$ for all $x\geq 0$. Last, differentiating with respect to $1\geq a\geq 0$ leads to

$$\mathcal{K}'_a=\frac{\mathcal{K}}{a}>0$$

Proof of Proposition 1

Let the Lagrangian $L=W+\lambda\left(K-\mu\left(\underline{\theta}-\underline{e}\right)\underline{q}-(1-\mu)\left(\overline{\theta}-\overline{e}\right)\overline{q}\right)$ where

$$W=\mu\left[u\left(\underline{q}\right)+\left(b-\underline{\psi}\underline{e}\right)\underline{q}\right]+(1-\mu)\left[u\left(\overline{q}\right)+\left(b-\overline{\psi}\overline{e}\right)\overline{q}\right]-\delta K-C(K)$$

Khun-Tucker conditions yield

$$\begin{aligned} u'\left(\underline{q}\right) + b - \underline{\psi}\underline{e} &= \lambda\left(\underline{\theta} - \underline{e}\right) \\ u'\left(\overline{q}\right) + b - \overline{\psi}\overline{e} &= \lambda\left(\overline{\theta} - \overline{e}\right) \end{aligned}$$

$$\begin{aligned} \lambda &> \underline{\psi} > \overline{\psi} \Rightarrow \underline{e} = \overline{e} = \hat{e} \\ \underline{\psi} &> \lambda > \overline{\psi} \Rightarrow \underline{e} = 0 \text{ and } \overline{e} = \hat{e} \\ \underline{\psi} &> \overline{\psi} > \lambda \Rightarrow \underline{e} = \overline{e} = 0 \end{aligned}$$

$$\lambda = \delta + c > 0$$

As $\lambda > 0$ then $K = \mu\left(\underline{\theta} - \underline{e}\right)\underline{q} + (1-\mu)\left(\overline{\theta} - \overline{e}\right)\overline{q}$

- If $\underline{\psi} > \overline{\psi} > \delta + c \Rightarrow \underline{e} = \overline{e} = 0$

  $$\begin{aligned} u'\left(\underline{q}\right) &= (\delta + c)\,\underline{\theta} - b \\ u'\left(\overline{q}\right) &= (\delta + c)\,\overline{\theta} - b \\ \underline{q}_0 &> \overline{q}_0 \end{aligned}$$

  then

  $$K_0^* = \mu\underline{\theta}\underline{q}_0 + (1-\mu)\,\overline{\theta}\overline{q}_0$$

- If $\underline{\psi} > \delta + c > \overline{\psi} \Rightarrow \underline{e} = 0$ and $\overline{e} = \hat{e}$

  $$\begin{aligned} u'\left(\underline{q}_0\right) &= (\delta + c)\,\underline{\theta} - b \\ u'\left(\overline{q}_1\right) &= (\delta + c)\,\overline{\theta} + \left(\overline{\psi} - (\delta + c)\right)\hat{e} - b \\ \overline{q}_1 &> \overline{q}_0 \end{aligned}$$

  then

  $$K_{01}^* = \mu\underline{\theta}\underline{q}_0 + (1-\mu)\left(\overline{\theta} - \hat{e}\right)\overline{q}_1$$

- If $\delta + c > \underline{\psi} > \overline{\psi} \Rightarrow \underline{e} = \overline{e} = \hat{e}$

  $$\begin{aligned} u'\left(\underline{q}_1\right) &= (\delta + c)\,\underline{\theta} + \left(\underline{\psi} - (\delta + c)\right)\hat{e} - b \\ u'\left(\overline{q}_1\right) &= (\delta + c)\,\overline{\theta} + \left(\overline{\psi} - (\delta + c)\right)\hat{e} - b \end{aligned}$$

  then

  $$K_1^* = \mu\left(\underline{\theta} - \hat{e}\right)\underline{q}_1 + (1-\mu)\left(\overline{\theta} - \hat{e}\right)\overline{q}_1$$

Proof of Corollary 1

Note first that

$$K_0^* \geq K_{01}^* \Leftrightarrow \overline{\theta} q_0(\overline{\theta}) \geq (\overline{\theta} - \hat{e}) q_1(\overline{\theta}).$$

Let us define $A(e) = (\theta - e)q(e)$ with $q(e)$ implicitly defined by

$$u'(q(e)) + b - (c+\delta)\theta - (\psi(\theta) - (\delta + c))\, e = 0.$$

Then, if $A(e)$ increases in e, we will have K_1^* will be larger than K_0^*. Then $A'(e) = -q(e) + (\theta - e)\frac{dq(e)}{de}$, with $\frac{dq}{de} = \frac{\psi(\theta)-(\delta+c)}{u''(q(e))}$. Therefore,

$$A'(e) \leq 0 \Leftrightarrow u''(q(e)) \leq \frac{(\theta - e)(\psi(\theta) - (\delta + c))}{q(e)}$$

Recall that $u'' < 0$ by assumption. Therefore, if the slope of u' is not too small, then the optimal capacity level increases with the level of effort of the Content Providers.

Proof of Proposition 2

The ISP problem writes as

$$\max_{(k,K)} \Pi = u(k) - C(K) \quad \text{s.t. } k\mathbb{E}(\theta) \leq K$$

then as $\frac{\partial \Pi}{\partial k} = u'(k) > 0$ and $\frac{\partial \Pi}{\partial K} = -c < 0$, the constraint is binding and

$$q^n(\theta) = k^n : u'(k^n) = c\mathbb{E}(\theta) \quad \text{and} \quad K^n = k^n \mathbb{E}(\theta)$$

Proof of Proposition 3

When $\overline{b} = \underline{b} = b$, the CP profit is $\pi(\theta) = (b - p)\,k \geq 0$. So, the ISP problem is then

$$\max_{(p,k,K)} u(k) + pk - C(K) \quad \text{s.t. } k\mathbb{E}(\theta) \leq K \text{ and } b \geq p$$

then

$$\begin{aligned} p^u &= b \\ q^u(\theta) &= k^u = \frac{K^u}{\mathbb{E}(\theta)} \\ K^u &: \; u'\left(\frac{K^u}{\mathbb{E}(\theta)}\right) = c\mathbb{E}(\theta) - b < c\mathbb{E}(\theta) = u'\left(\frac{K^n}{\mathbb{E}(\theta)}\right) \end{aligned}$$

so by concavity of u, this leads to $\frac{K^u}{\mathbb{E}(\theta)} > \frac{K^n}{\mathbb{E}(\theta)}$, and we always have:

$$k^n < k^u \text{ and } K^n < K^u$$

Proof of Proposition 4

When $\overline{b} > \underline{b}$. So, if $p^* = \underline{b}$ no exclusion occurs and the ISP problem is then

$$\max_{(k,K)} u(k) + \underline{b}k - C(K) \quad \text{s.t. } k\mathbb{E}(\theta) \leq K$$

then

$$\begin{aligned} q^u(\theta) &= \underline{k}^u = \frac{\underline{K}^u}{\mathbb{E}(\theta)} \\ \underline{K}^u &: u'\left(\frac{K^u}{\mathbb{E}(\theta)}\right) = c\mathbb{E}(\theta) - \underline{b} \end{aligned}$$

If so, if $p^* = \overline{b}$, CP $\underline{\theta}$ is excluded and then

$$\max_{(k,K)} (1-\mu)\left[u(k) + \overline{b}k\right] - C(K) \quad \text{s.t. } k(1-\mu)\overline{\theta} \leq K$$

then

$$\begin{aligned} \underline{q}^u &= 0 \text{ and } \overline{q}^u = \overline{k}^u = \frac{\overline{K}^u}{(1-\mu)\overline{\theta}} \\ \overline{K}^u &: u'\left(\frac{K^u}{(1-\mu)\overline{\theta}}\right) = c\overline{\theta} - \overline{b} \end{aligned}$$

and we see that we always have:

$$\begin{aligned} u'\left(\frac{K^n}{\mathbb{E}(\theta)}\right) &= c\mathbb{E}(\theta) > u'\left(\frac{\underline{K}^u}{\mathbb{E}(\theta)}\right) = c\mathbb{E}(\theta) - \underline{b} \\ &\Rightarrow k^n < \underline{k}^u \text{ and } K^n < \underline{K}^u \end{aligned}$$

Therefore, it is not possible to reduce capacity without exclusion, that is, $p^* = \overline{b} = \max\{\overline{b}, \underline{b}\}$ is needed, which proves Part 1 of the Proposition when $\overline{b} > \underline{b}$.

To prove Part 2 of the Proposition (when $\overline{b} > \underline{b}$), we have to determine if it is optimal for the ISP to adopt $p^* = \overline{b}$. Optimal levels of profit are such that

$$\begin{aligned} \Delta\Pi(\mu) &= \overline{\Pi}^u(\mu) - \underline{\Pi}^u(\mu) \\ &\Leftrightarrow \Delta\Pi(\mu) = (1-\mu)\left[u\left(\overline{k}^u\right) - \left(c\overline{\theta} - \overline{b}\right)\overline{k}^u\right] - \left[u\left(\underline{k}^u\right) - \left(c\mathbb{E}(\theta) - \underline{b}\right)\underline{k}^u\right] \geq 0 \\ &\Leftrightarrow \Delta\Pi(\mu) = (1-\mu)V\left(\overline{k}^u\right) - V\left(\underline{k}^u\right) \geq 0 \end{aligned}$$

where $V(k) = u(k) - u'(k)k$ is an increasing function of k as $V'(k) = -u''(k)k < 0$ by concavity of u. So, if $\underline{k}^u \geq \overline{k}^u$ i.e., if $0 < \overline{b} - \underline{b} \leq c\left(\overline{\theta} - \mathbb{E}(\theta)\right) \Delta\Pi(\mu) \leq 0$, and the ISP cannot choose $p^* = \overline{b}$. Hence, if $\overline{b} - \underline{b} > c\left(\overline{\theta} - \mathbb{E}(\theta)\right) \Leftrightarrow \mu < \underline{\mu} = \frac{\overline{b} - \underline{b}}{c\Delta(\theta)}$ we have

$$\overline{k}^u = \frac{\overline{K}^u}{(1-\mu)\overline{\theta}} > \frac{\underline{K}^u}{\mathbb{E}(\theta)} = \underline{k}^u \Rightarrow \overline{K}^u > \frac{(1-\mu)\overline{\theta}}{\mathbb{E}(\theta)} \underline{K}^u$$

So, when $\overline{k}^u > \underline{k}^u$

$$\Delta\Pi'(\mu) = -V\left(\overline{k}^u\right) + c\left(\overline{\theta} - \underline{\theta}\right)\underline{k}^u \leq -\left[u\left(\overline{k}^u\right) - \left(c\underline{\theta} - \overline{b}\right)\overline{k}^u\right] < 0$$

so $\Delta\Pi(\mu) \leq 0$ for all μ and $\Delta\Pi(0) = V\left(\overline{k}^u\right) - V\left(\underline{k}^u\right) > 0$. So, it exists $\mu^* : \Delta\Pi(\mu^*) = 0$, such that $\mu \leq \mu^* < \underline{\mu}$, $\Delta\Pi(\mu) \geq 0$. To have $\overline{K}^u \leq K^n$ we need to verify $\overline{b} \geq c\left(\overline{\theta} - \mathbb{E}(\theta)\right) \Leftrightarrow \mu \leq \overline{\mu} = \frac{\overline{b}}{c\Delta(\theta)}$. Now when $\mu < \underline{\mu} < \overline{\mu}$:

$$\begin{aligned}
\overline{K}^u &\leq K^n \Leftrightarrow (1-\mu)\overline{\theta}\overline{k}^u \leq \mathbb{E}(\theta) k^n \\
K^n &= \mathbb{E}(\theta)\gamma(c\mathbb{E}(\theta)) \geq \overline{K}^u = (1-\mu)\overline{\theta}\gamma\left(c\overline{\theta} - \overline{b}\right)
\end{aligned}$$

where $\gamma = (u')^{-1}$. Let us form $G(\mu) = \left(\overline{\theta} - \mu\Delta(\theta)\right)\gamma\left(c\overline{\theta} - \mu c\Delta(\theta)\right)$ with

$$\begin{aligned}
G(0) &= \overline{\theta}\gamma\left(c\overline{\theta}\right) < \overline{\theta}\gamma\left(c\overline{\theta} - \overline{b}\right) \\
G(1) &= \underline{\theta}\gamma\left(c\underline{\theta}\right) > 0
\end{aligned}$$

Then it exists $\hat{\mu} : G(\hat{\mu}) = (1-\hat{\mu})\overline{\theta}\gamma\left(c\overline{\theta} - \overline{b}\right)$, such that $\overline{K}^u \leq K^n$ iff $\mu \geq \hat{\mu}$. So, whenever $\hat{\mu} < \mu^*$ when $\mu \in [\hat{\mu}, \mu^*]$, we have the result that $\Delta\Pi(\mu) \geq 0$ and $\underline{K}^u \leq K^n$.

When $\underline{b} > \overline{b}$. So, if $p^* = \overline{b}$ no exclusion arises and the ISP problem is then

$$\max_{(k,K)} u(k) + \overline{b}k - cK \text{ s.t. } k\mathbb{E}(\theta) \leq K$$

then

$$\begin{aligned}
q^u(\theta) &= \overline{k}^u = \frac{\overline{K}^u}{\mathbb{E}(\theta)} \\
\overline{K}^u &: u'\left(\frac{K^u}{\mathbb{E}(\theta)}\right) = c\mathbb{E}(\theta) - \overline{b}
\end{aligned}$$

If so, if $p^* = \underline{b}$, CP $\overline{\theta}$ is excluded and then

$$\max_{(k,K)} \mu\left[u(k) + \underline{b}k\right] - cK \text{ s.t. } k\mu\underline{\theta} \leq K$$

then

$$\begin{aligned}
\overline{q}^u &= 0 \text{ and } \underline{q}^u = \underline{k}^u = \frac{\underline{K}^u}{\mu\underline{\theta}} \\
\underline{K}^u &: u'\left(\frac{\underline{K}^u}{\mu\underline{\theta}}\right) = c\underline{\theta} - \underline{b}
\end{aligned}$$

As a result,

$$\begin{aligned}
\underline{k}^u &= \frac{\underline{K}^u}{\mu\underline{\theta}} > \frac{\overline{K}^u}{\mathbb{E}(\theta)} = \overline{k}^u \text{ if } \underline{b} > \overline{b} - c\left(\mathbb{E}(\theta) - \underline{\theta}\right) \\
&\Rightarrow \underline{K}^u > \frac{\mu\underline{\theta}}{\mathbb{E}(\theta)}\overline{K}^u
\end{aligned}$$

and we see that we always have:

$$\begin{aligned}
u'\left(\frac{K^n}{\mathbb{E}(\theta)}\right) &= c\mathbb{E}(\theta) > u'\left(\frac{\overline{K}^u}{\mathbb{E}(\theta)}\right) = c\mathbb{E}(\theta) - \overline{b} \\
&\Rightarrow \underline{k}^n < \overline{k}^u \text{ and } \underline{K}^n < \underline{K}^u
\end{aligned}$$

Therefore, it is not possible to reduce the capacity without exclusion, that is, $p^* = \underline{b} = \max\{\overline{b}, \underline{b}\}$, which finishes to prove Part 1 of the proposition. To complete the proof of Part 2 of the Proposition (when $\overline{b} < \underline{b}$), we must determine if it is optimal for the ISP to adopt $p^* = \underline{b}$, when optimal levels of profit are such that

$$\begin{aligned}
\underline{\Pi}^u(\mu) - \overline{\Pi}^u(\mu) &\Leftrightarrow \mu\left[u\left(\underline{k}^u\right) - \left(c\underline{\theta} - \underline{b}\right)\underline{k}^u\right] - \left[u\left(\overline{k}^u\right) - \left(c\mathbb{E}(\theta) - \overline{b}\right)\overline{k}^u\right] \geq 0 \\
&\Leftrightarrow \mu V\left(\underline{k}^u\right) - V\left(\overline{k}^u\right) \geq 0
\end{aligned}$$

So, if $\mu \geq \mu^*$ such that $\mu^* = \frac{V\left(\underline{k}^u\right)}{V\left(\overline{k}^u\right)}$. Moreover,

$$\begin{aligned}
u'\left(\frac{K^n}{\mathbb{E}(\theta)}\right) &> u'\left(\frac{\underline{K}^u}{\mu\underline{\theta}}\right) = c\underline{\theta} - \underline{b} \Rightarrow \underline{k}^u > k^n \\
\underline{K}^u &> \frac{\mu\underline{\theta}}{\mathbb{E}(\theta)}K^n
\end{aligned}$$

so it can be possible that $K^n \geq \underline{K}^u > \frac{\mu\underline{\theta}}{\mathbb{E}(\theta)}K^n$ when

$$\begin{aligned}
\underline{K}^u &\leq K^n \Leftrightarrow K^n = \mathbb{E}(\theta)k^n \geq \mu\underline{\theta}\underline{k}^u \\
K^n &= \mathbb{E}(\theta)\gamma(c\mathbb{E}(\theta)) \geq \underline{K}^u = \mu\underline{\theta}\gamma\left(c\underline{\theta} - \underline{b}\right)
\end{aligned}$$

Now

$$\begin{aligned} G(0) &= \overline{\theta}\gamma\left(c\overline{\theta}\right) > 0 \\ G(1) &= \underline{\theta}\gamma\left(c\underline{\theta}\right) < \underline{\theta}\gamma\left(c\underline{\theta} - \underline{b}\right) \end{aligned}$$

Then, it exists $\hat{\mu} : G(\hat{\mu}) = \hat{\mu}\underline{\theta}\gamma\left(c\underline{\theta} - \underline{b}\right)$, such that $\underline{K}^u \leq K^n$ iff $\mu \leq \hat{\mu}$. So, whenever $\hat{\mu} > \mu^*$ when $\mu \in [\mu^*, \hat{\mu}]$, we have the result that $\Delta\Pi(\mu) \geq 0$ and $\underline{K}^u \leq K^n$. So, this completes the proof.

Proof of Lemma 1

For tariff $t(\theta), p(\theta)$, the ISP profit writes as

$$\Pi = u(k) + \mu\left(t(\underline{\theta}) + p(\underline{\theta})(\underline{\theta} - e(\underline{\theta}))\right) + (1-\mu)\left(t(\overline{\theta}) + p(\overline{\theta})(\overline{\theta} - e(\overline{\theta}))\right)k - cK$$

under the congestion constraint. In this case, the ISP will capture the whole CP surplus. Therefore, for any e, we have $t = b - p(\theta - e) - \psi e$. Moreover, e depends on p. The ISP must choose k, K and the price p that pins down the effort chosen by the CP. The ISP profit now writes as

$$\Pi = u(k) + \mu\left(b - \underline{\psi}e(\underline{\theta})\right)k + (1-\mu)\left(b - \overline{\psi}e(\overline{\theta})\right)k - cK$$

subject to $\mu\left(\underline{\theta} - e(\underline{\theta})\right)k + (1-\mu)\left(\overline{\theta} - e(\overline{\theta})\right)k \leq K$.

Optimizing wrt to $p(\underline{\theta})$ leads to $\frac{de}{dp}(-\underline{\psi} + \lambda)\mu k$. It is direct to see, optimizing with respect to K, that the multiplier associated with the congestion constraint is $\lambda = c$. Therefore, the sign of the derivative will depend on $c - \underline{\psi}$ (and similarly $c - \overline{\psi}$ for the $\overline{\theta}$-CPs). Since $\frac{de}{dp} \geq 0$, the result follows directly.

Proof of Proposition 5

If $b(\theta) > \theta\psi(\theta)$ for all θ, then the ISP profit writes as $\Pi = u(k) + \mathbb{E}(b - \theta\psi)k - cK$, so the problem is

$$\max_{k,K} \Pi \text{ s.t. } k(\mathbb{E}(\theta) - \hat{e}) \leq K$$

and equilibrium is given by

$$\begin{aligned} u'\left(k^t\right) &= c\left(\mathbb{E}\left(\theta\right) - \hat{e}\right) - \mathbb{E}\left(b - \theta\psi\right) \\ K^t &= \left(\mathbb{E}\left(\theta\right) - \hat{e}\right) k^t \end{aligned}$$

If u is concave enough by invoking Lemma 2, we have

$$\mathcal{K}(\mathbb{E}\left(\theta\right) - \hat{e}, \mathbb{E}\left(b - \theta\psi\right)) = K^t \text{ and } \; K^n = \mathcal{K}\left(\mathbb{E}\left(\theta\right), 0\right)$$

Then

$$K^n = \mathcal{K}\left(\mathbb{E}\left(\theta\right), 0\right) > \mathcal{K}(\mathbb{E}\left(\theta\right) - \hat{e}, 0)$$

and since $\mathbb{E}\left(b - \theta\psi\right) > 0$ as $\mathcal{K}'_y\left(x, y\right) > 0$:

$$\mathcal{K}\left(\mathbb{E}\left(\theta\right) - \hat{e}, 0\right) < \mathcal{K}(\mathbb{E}\left(\theta\right) - \hat{e}, \mathbb{E}\left(b - \theta\psi\right)) = K^t$$

Hence, as there exists a level $\overline{B}$ of $\mathbb{E}\left(b - \theta\psi\right)$ such that $K^t \leq K^n$ when $\mathbb{E}\left(b - \theta\psi\right) \leq \overline{B}$. More precisely,

$$\overline{B} = c\left(\mathbb{E}\left(\theta\right) - \hat{e}\right) - u'\left(\frac{k^n \mathbb{E}\left(\theta\right)}{\mathbb{E}\left(\theta\right) - \hat{e}}\right).$$

Proof of Corollary 5

Indeed, if for *only one* given CPθ, $b\left(\theta\right) - \theta\psi\left(\theta\right) > 0$, and

$$\begin{aligned} k^\tau &: \; u'\left(k^d\right) = c\mathbb{E}\left(\theta\right) - cm\left(\theta\right)\hat{e} - \left(\mathbb{E}\left(b\right) - \theta\psi\left(\theta\right)\right) < cm\left(\theta\right)\hat{e} \\ K^\tau &= \left(\mathbb{E}\left(\theta\right) - m\left(\theta\right)\hat{e}\right) k^\tau \end{aligned}$$

Invoking Lemma 2, we have $K^\tau = \mathcal{K}\left(\mathbb{E}\left(\theta\right) - m\left(\theta\right)\hat{e}, \mathbb{E}\left(b\right) - \theta\psi\left(\theta\right)\right) > \mathcal{K}\left(\mathbb{E}\left(\theta\right) - m\left(\theta\right)\hat{e}, 0\right)$ but $\mathcal{K}\left(\mathbb{E}\left(\theta\right) - m\left(\theta\right)\hat{e}, 0\right) < \mathcal{K}\left(\mathbb{E}\left(\theta\right), 0\right) = K^n$. Hence, it exists a level $\underline{B}$ of $\mathbb{E}\left(b\right) - \theta\psi\left(\theta\right)$ such that $K^\tau = \left(\mathbb{E}\left(\theta\right) - m\left(\theta\right)\hat{e}\right) k^\tau \leq K^n$ when $\mathbb{E}\left(b\right) - \theta\psi\left(\theta\right) \leq \underline{B}$. Then

$$\underline{B} = (\mathbb{E}(\theta) - \hat{e})c - u'\left(k^n \frac{\mathbb{E}(\theta)}{\mathbb{E}(\theta) - m(\theta)\hat{e}}\right).$$

Proof of Proposition 6 and Corollaries

For Proposition 6. **(a)** If $p = \underline{\psi} > \overline{\psi}$ then

$$u'\left(\underline{k}^r\right) = c\left(\mathbb{E}\left(\theta\right) - \hat{e}\right) \quad \text{and} \quad \underline{K}^r = \underline{k}^r\left(\mathbb{E}\left(\theta\right) - \hat{e}\right) \tag{7}$$

so as $c(\mathbb{E}(\theta) - \hat{e}) < c\mathbb{E}(\theta)$ by concavity of $\underline{k}^r < k^n$.
(b) If $\underline{\psi} > p = \overline{\psi}$ then

$$u'\left(\overline{k}^r\right) = (\mathbb{E}(\theta) - (1-\mu)\hat{e})\,c \text{ and } \overline{K}^r = \overline{k}^r(\mathbb{E}(\theta) - (1-\mu)\hat{e})$$

Straightforwardly, : $k^n > \overline{k}^r$.
(c) Invoking Lemma 2, when u is concave, we have

$$\underline{K}^r = \mathcal{K}(\mathbb{E}(\theta) - \hat{e}, 0) < \overline{K}^r = \mathcal{K}(\mathbb{E}(\theta) - (1-\mu)\hat{e}, 0) < K^n = \mathcal{K}(\mathbb{E}(\theta), 0)$$

Moreover, $\underline{k}^r > \overline{k}^r$ as

$$u'\left(\underline{k}^r\right) = c(\mathbb{E}(\theta) - \hat{e}) < (\mathbb{E}(\theta) - (1-\mu)\hat{e})\,c = u'\left(\overline{k}^r\right)$$

Consequently,

$$\underline{\Pi} = u\left(\underline{k}^r\right) - c\underline{K}^r > \overline{\Pi} = u\left(\overline{k}^r\right) - c\underline{K}^r > \Pi^n = u(k^n) - cK^n$$

For the Corollary 3. As $W^r(p) = u(k) - (\delta + c)K$ and using results in Proposition 6, we get:

$$W^r\left(\underline{\psi}\right) = u\left(\underline{k}^r\right) - (\delta + c)\underline{K}^r > u\left(\overline{k}^r\right) - (\delta + c)\underline{K}^r > u\left(\overline{k}^r\right) - (\delta + c)\overline{K}^r = W^r\left(\overline{\psi}\right)$$

so $p^r = \underline{\psi}$ is preferred by the regulator.
For the Corollary 4. As when $\mu \geq \frac{1}{2}$ we have $\underline{k}^r \geq \overline{k}^r$ and $\underline{K}^r \leq \overline{K}^r$

$$W^r\left(\underline{\psi}\right) = u\left(\underline{k}^r\right) - (\delta + c)\underline{K}^r \geq u\left(\overline{k}^r\right) - (\delta + c)\overline{K}^r = W^r\left(\overline{\psi}\right)$$

so $p^r = \underline{\psi}$ is preferred by the regulator. Conversely, when $\mu < \frac{1}{2}$.
For the Corollary 5. As now invoking Lemma 2, $\underline{K}^r = \mathcal{K}\left(\mu\underline{\theta} - \mu\hat{e}, 0\right) < \overline{K}^r = \mathcal{K}(\mathbb{E}(\theta) - (1-\mu)\hat{e}, 0)$ as

$$\mu\underline{\theta} - \mu\hat{e} < \mu\underline{\theta} \leq \mathbb{E}(\theta) - (1-\mu) = \mu\underline{\theta} + (1-\mu)\left(\overline{\theta} - \hat{e}\right)$$

then

$$\hat{W}^r\left(\underline{\psi}\right) = \mu u\left(\underline{k}^r\right) - (\delta + c)\underline{K}^r \geq u\left(\overline{k}^r\right) - (\delta + c)\overline{K}^r = W^r\left(\overline{\psi}\right)$$

if

$$\mu \geq \mu^r = \frac{u\left(\overline{k}^r\right) - (\delta + c)\left(\overline{K}^r - \underline{K}^r\right)}{u\left(\underline{k}^r\right)} < 1 - (\delta + c)\frac{\overline{K}^r - \underline{K}^r}{u\left(\underline{k}^r\right)}$$

Proof of Proposition 7

Therefore, these norms induce no efforts for CP if $z_m > \overline{\theta}$, and net neutrality applies. Now if $\overline{\theta} > z_m > \underline{\theta}$ CP $\underline{\theta}$ is making no effort but CP $\overline{\theta}$ reduces congestion to z_m so she makes an effort $\overline{\theta} - z_m$.

A. *Consider first* that $\overline{\theta} > z_m \geq \overline{\theta} - \hat{e} > \underline{\theta}$, then CP $\overline{\theta}$ makes an effort $\overline{\theta} - z_m$.

- When $b > \max\{\underline{\psi\theta}, \overline{\psi\theta}\}$ or if $\underline{\psi\theta} > b \geq \overline{\psi\theta}$, the CP $\underline{\theta}$ is making no effort so $\pi\left(\underline{\theta}\right) = bk > 0$ and $\pi\left(\overline{\theta}\right) \geq \left(b - \overline{\psi\theta} + \overline{\psi} z_m\right) k > 0$ Then the ISP solves

$$\max_{K,k} \Pi(k, K) = u(k) - cK \text{ s.t. } kE(z_m) \leq K$$

where $E(z_m) = \mu\underline{\theta} + (1-\mu) z_m = \mathbb{E}(\theta) - (1-\mu)\left(\overline{\theta} - z_m\right) < \mathbb{E}(\theta)$. Note that $E'(z_m) = 1 - \mu > 0$, as a result, capacities state as

$$k^e > k^n : u'(k^e) = E(z_m) c < \mathbb{E}(\theta) c \text{ and } K^e = E(z_m) k^e$$

Invoking Lemma 2, $K^e = \mathcal{K}(E(z_m), 0) < K^n = \mathcal{K}(\mathbb{E}(\theta), 0)$ as $E(z_m) < \mathbb{E}(\theta)$. Hence, for a consumerist-environmentalist regulator with a welfare function:

$$W(z_m) = u(k^e) - (c+\delta) E(z_m) k^e = \Pi(k^e, K^e) - \delta E(z_m) k^e$$

the problem is to $\max_{z_m \geq \overline{\theta} - \hat{e}} W(z_m)$ then, by the envelope theorem,

$$\begin{aligned} W'(z_m) &= -\delta E'(z_m) k^e - \delta E(z_m) \frac{\partial k^e}{\partial z_m} \\ &= -\delta E'(z_m) k^e \left(1 + \frac{u'(k^e)}{u'(k^e) k^e}\right) \\ &= -\delta E'(z_m) k^e \left(1 - \frac{1}{\rho(k^e)}\right) < 0 \end{aligned}$$

if $\rho(k^e) > 1$. Then we have $z_m^* = \overline{\theta} - \hat{e}$ so $E\left(\overline{\theta} - \hat{e}\right) = \mathbb{E}(\theta) - (1-\mu)\hat{e}$. Then $K^e = E\left(\overline{\theta} - \hat{e}\right) k^e = \overline{K}^r$ and $k^e = \overline{k}^r$. In this case, the ISP profit equals $\Pi(k^e) = u(k^e) - cE(z_m^*) k^e = u(k^e) - u'(k^e) k^e = V(k^e)$. So, the ISP is always better off with such an environmental policy than with NN as $V(k^e) > V(k^n) = \Pi^n$. This proves point (*a*) of Proposition 7 .

- Now if $\overline{\psi\theta} > b > \underline{\psi\theta}$, then the CP $\overline{\theta}$ profit is $\pi\left(\overline{\theta}\right) = \left(b - \overline{\psi\theta} + \overline{\psi} z_m\right) k$, and if $z_m < \overline{\theta} - \frac{b}{\overline{\psi}}$, he is excluded. However, as $z_m \geq \overline{\theta} - \hat{e}$, it depends on the difference $\frac{b}{\overline{\psi}} - \hat{e}$.

  - If $\hat{e} \leq \frac{b}{\overline{\psi}}$ then $z_m \geq \overline{\theta} - \hat{e}$, we have same results as above.
  - If $\hat{e} > \frac{b}{\overline{\psi}}$ then if $z_m \geq \overline{\theta} - \frac{b}{\overline{\psi}} > \overline{\theta} - \hat{e}$, no exclusion occurs as above, so $z_m^* = \overline{\theta} - \frac{b}{\overline{\psi}}$. But if $\overline{\theta} - \frac{b}{\overline{\psi}} > z_m \geq \overline{\theta} - \hat{e}$, then the CP $\overline{\theta}$ is excluded and the ISP solves

$$\max_{K,k} \Pi(k, K) = \mu u(k) - cK \text{ s.t. } k\mu\underline{\theta} \leq K$$

    then

$$\hat{k}^e > k^n : u'\left(\hat{k}^e\right) = \underline{\theta} c < \mathbb{E}(\theta) c \quad \text{and} \quad \hat{K}^e = \mu\underline{\theta}\hat{k}^e$$

    Invoking Lemma 2, $\hat{K}^e = \mathcal{K}(\underline{\theta}, 0) < K^e = \mathcal{K}(E(z_m), 0) < K^n = \mathcal{K}(\mathbb{E}(\theta), 0)$ as $\mu\underline{\theta} < E(z_m) < \mathbb{E}(\theta)$. Hence, for a consumerist-environmentalist regulator with a welfare function:

$$\hat{W}(z_m) = \mu\left[u\left(\hat{k}^e\right) - (c+\delta)\,\underline{\theta}\hat{k}^e\right] = \Pi\left(\hat{k}^e, \hat{K}^e\right) - \delta\mu\underline{\theta}\hat{k}^e$$

    the problem is to $\max_{z_m \geq \overline{\theta} - \hat{e}} \hat{W}(z_m)$ then using same arguments as above we have $z_m^* = \overline{\theta} - \hat{e}$. In this case the ISP profit equals $\Pi\left(\hat{k}^e, \hat{K}^e\right) = V\left(\hat{k}^e\right)$. So again, the ISP is always better off with such an environmental policy than with NN as $V(k^e) > V(k^n) = \Pi^n$. In this case, the regulator prefers to exclude the CP $\overline{\theta}$ if $\hat{W}\left(\overline{\theta} - \hat{e}\right) \geq W\left(\overline{\theta} - \frac{b}{\overline{\psi}}\right)$, that is if

$$\mu \geq \mu^w = \frac{u(k^e) - (c+\delta)\,E\left(\overline{\theta} - \frac{b}{\overline{\psi}}\right)k^e}{u\left(\hat{k}^e\right) - (c+\delta)\,\underline{\theta}\hat{k}^e}$$

B. *Consider that* $\overline{\theta} > \underline{\theta} > \overline{\theta} - \hat{e}$ then $\underline{\theta} > z_m \geq \overline{\theta} - \hat{e}$. That is, $\hat{e} > \overline{\theta} - \underline{\theta}$.

- When $b > \max\{\underline{\psi\theta}, \overline{\psi\theta}\}$, no exclusion arises, but all CPs are making efforts and the constraint is $kz_m \leq K$, as a result quantities state:

$$k^e > k^n : u'(k^e) = z_m c < \mathbb{E}(\theta) c \;\text{ and }\; K^e = z_m k^e$$

Then, using the same arguments as above, we have $z_m^* = \overline{\theta} - \hat{e}$ and $K^e = \left(\overline{\theta} - \hat{e}\right) k^e < K^n$. Again, the regulator objective is $W\left(\overline{\theta} - \hat{e}\right)$, the ISP is always better off with such an environmental policy than with NN. Moreover as $\overline{\theta} - \hat{e} > \mathbb{E}(\theta) - \hat{e}$, from (7), we immediately have: $K^e > \underline{K}^r$ and $k^e > \underline{k}^r$. This completes the proof of Proposition 7.

- When $\underline{\psi\theta} > b > \overline{\psi\theta}$, the CP $\underline{\theta}$ is excluded if $z_m < \underline{\theta} - \frac{b}{\underline{\psi}}$.

  - Then, if $\hat{e} \leq \overline{\theta} - \underline{\theta} + \frac{b}{\underline{\psi}}$ then $z_m \geq \overline{\theta} - \hat{e}$, all CPs are making the efforts and the constraint is $kz_m \leq K$, as a result, the quantities state:

    $$k^e > k^n : u'(k^e) = z_m c < \mathbb{E}(\theta) c \text{ and } K^e = z_m k^e$$

    Then, using the same arguments as above, we have $z_m^* = \overline{\theta} - \hat{e}$ and $K^e = z_m^* k^e < K^n$. Again, the regulator objective is $W\left(\overline{\theta} - \hat{e}\right)$, the ISP is always better off with such an environmental policy than with NN.

  - If $\hat{e} > \overline{\theta} - (\underline{\theta} - \frac{b}{\underline{\psi}})$ then $\underline{\theta} - \frac{b}{\underline{\psi}} > z_m \geq \overline{\theta} - \hat{e}$, so the CP $\underline{\theta}$ is excluded, then the ISP solves

    $$\max_{K,k} \Pi(k, K) = (1-\mu) u(k) - cK \text{ s.t. } k(1-\mu) z_m \leq K$$

    then

    $$\hat{k}^e > k^n : u'\left(\hat{k}^e\right) = z_m c < \mathbb{E}(\theta) c \text{ and } \hat{K}^e = (1-\mu) z_m \hat{k}^e < K^n$$

    Invoking Lemma 2, $\hat{K}^e = \mathcal{K}((1-\mu) z_m, 0) < K^e = \mathcal{K}(z_m, 0) < K^n = \mathcal{K}(\mathbb{E}(\theta), 0)$ as $(1-\mu) z_m < z_m < \mathbb{E}(\theta)$. Hence, for a consumerist-environmentalist regulator with a welfare function:

    $$\hat{W}(z_m) = (1-\mu)\left[u\left(\hat{k}^e\right) - (c+\delta) z_m \hat{k}^e\right] = \Pi\left(\hat{k}^e, \hat{K}^e\right) - \delta z_m \hat{k}^e$$

    Of course, $z_m^* = \overline{\theta} - \hat{e}$.

- When $\overline{\psi\theta} > b > \underline{\psi\theta}$, the CP $\overline{\theta}$ is excluded if $z_m < \overline{\theta} - \frac{b}{\overline{\psi}}$.

  - If $\hat{e} \leq \frac{b}{\overline{\psi}}$ then $z_m \geq \overline{\theta} - \hat{e}$, all CPs are making efforts and $z_m^* = \overline{\theta} - \hat{e}$.

– If $\hat{e} > \frac{b}{\underline{\psi}}$ then if $z_m \geq \overline{\theta} - \frac{b}{\underline{\psi}} > \overline{\theta} - \hat{e}$, all CPs are making efforts, but $z_m^* = \overline{\theta} - \frac{b}{\underline{\psi}}$. Finally, if $\overline{\theta} - \frac{b}{\underline{\psi}} > z_m \geq \overline{\theta} - \hat{e}$, then the CP $\overline{\theta}$ is excluded. In this case, the regulator prefers to exclude this CP $\overline{\theta}$ with $z_m^* = \overline{\theta} - \hat{e}$ if

$$\hat{W}(z_m) = \mu\left[u\left(\hat{k}^e\right) - (c+\delta)\left(\overline{\theta} - \hat{e}\right)\hat{k}^e\right] \geq W(z_m) = u(k^e) - (c+\delta)\left(\overline{\theta} - \frac{b}{\underline{\psi}}\right)k^e$$

that is if

$$\mu \geq \tilde{\mu}^w = \frac{u(k^e) - (c+\delta)\left(\overline{\theta} - \frac{b}{\underline{\psi}}\right)k^e}{u\left(\hat{k}^e\right) - (c+\delta)\left(\overline{\theta} - \hat{e}\right)\hat{k}^e}$$

A specific example

We provide an example to illustrate all our results and to show the existence of all the cases we discuss in the paper. We assume $u(q) = \alpha q - \frac{\beta}{2}q^2$ with $\beta > 1$ then $\overline{q} = \frac{\alpha}{\beta}$.

In this case, strong concavity means that

$$q \geq \frac{\alpha}{2\beta}$$

as

$$u'(q) = \alpha - \beta q \leq \frac{\alpha}{2}$$

Optimal allocation

$$\begin{aligned} q_0(\theta) &= \frac{\alpha + b - (\delta + c)\theta}{\beta} \\ q_1(\theta) &= \frac{\alpha + b - (\delta + c)\theta - (\psi(\theta) - (\delta + c))\hat{e}}{\beta} \end{aligned}$$

- If $\underline{\psi} > \overline{\psi} > \delta + c$, then $\underline{e} = \overline{e} = 0$, $q(\theta) = q_0(\theta)$ and

$$\begin{aligned} K_0^* &= \mu\underline{\theta}\frac{\alpha + b - (\delta + c)\underline{\theta}}{\beta} + (1-\mu)\overline{\theta}\frac{\alpha + b - (\delta + c)\overline{\theta}}{\beta} \\ &= \mathbb{E}(\theta)\frac{\alpha + b}{\beta} - \frac{\delta + c}{\beta}\mathbb{E}\left(\theta^2\right) \end{aligned}$$

- If $\underline{\psi} > \delta + c > \overline{\psi}$, then $\underline{e} = 0$, $\overline{e} = \hat{e}$, $q(\underline{\theta}) = q_0(\underline{\theta})$, $q(\overline{\theta}) = q_1(\overline{\theta})$, and

$$
\begin{aligned}
K_{01}^{*} &= \mu\underline{\theta}\frac{\alpha + b - (\delta + c)\,\underline{\theta}}{\beta} + (1-\mu)\left(\overline{\theta} - \hat{e}\right)\frac{\alpha + b - (\delta + c)\,\overline{\theta} - \left(\overline{\psi} - (\delta + c)\right)\hat{e}}{\beta} \\
&= K_0^{*} - (1-\mu)\left[\alpha + b + \overline{\theta}\left(\overline{\psi} - 2(\delta + c)\right) - \left(\overline{\psi} - (\delta + c)\right)\hat{e}\right]\frac{\hat{e}}{\beta}
\end{aligned}
$$

- If $\delta + c > \underline{\psi} > \overline{\psi}$, then $\underline{e} = \overline{e} = \hat{e}$, $q(\theta) = q_1(\theta)$ and

$$
\begin{aligned}
K_1^{*} &= K_0^{*} - \left[\alpha + b + \mu\underline{\theta}\underline{\psi} + (1-\mu)\,\overline{\theta}\overline{\psi} + 2\mu\underline{\theta}\left(\delta + c - \underline{\psi}\right) + 2(1-\mu)\,\overline{\theta}\left(\delta + c - \overline{\psi}\right)\right]\frac{\hat{e}}{\beta} \\
&\quad + \left[\delta + c - \mu\underline{\psi} - (1-\mu)\,\overline{\psi}\right]\frac{\hat{e}^2}{\beta}
\end{aligned}
$$

One can see that the result in Corollary 1 is true here

$$K_{01}^{*} \leq K_0^{*}$$

when

$$\hat{e} \geq \frac{2\overline{\theta}\,(\delta + c) - \overline{\psi} - (\alpha + b)}{\delta + c - \overline{\psi}}$$

and

$$K_1^{*} \leq K_0^{*}$$

when

$$\hat{e} \geq \frac{2\mu\underline{\theta}\left(\delta + c - \underline{\psi}\right) + 2(1-\mu)\,\overline{\theta}\left(\delta + c - \overline{\psi}\right) - \left(\alpha + b + \mu\underline{\theta}\underline{\psi} + (1-\mu)\,\overline{\theta}\overline{\psi}\right)}{\delta + c - \mu\underline{\psi} - (1-\mu)\,\overline{\psi}}$$

Net Neutrality

$$k^n = \frac{\alpha - \mathbb{E}(\theta)\,c}{\beta} \text{ and } K^n = \frac{\alpha - \mathbb{E}(\theta)\,c}{\beta}\mathbb{E}(\theta)$$

Laissez-faire. Uniform prices

Proposition 3 is readily illustrated.

$$
\begin{aligned}
q^u(\theta) &= k^u = \frac{\alpha + b - \mathbb{E}(\theta)\,c}{\beta} = \frac{b}{\beta} + k^n \\
K^u &= \frac{\alpha + b - \mathbb{E}(\theta)\,c}{\beta}\mathbb{E}(\theta) = \frac{b}{\beta}\mathbb{E}(\theta) + K^n > K^n
\end{aligned}
$$

Proposition 4 is also true. If $\overline{b} - \underline{b} > 0$: Equilibrium capacities are

$$\begin{aligned}
\underline{K}^u &= \frac{\mathbb{E}(\theta)}{\beta}\left(\alpha + \underline{b} - \mathbb{E}(\theta)c\right) \text{ and } \underline{k}^u = \frac{1}{\beta}\left(\alpha + \underline{b} - \mathbb{E}(\theta)c\right) \\
\overline{K}^u &= (1-\mu)\overline{\theta}\frac{\alpha + \overline{b} - \overline{\theta}c}{\beta} \text{ and } \overline{k}^u = \frac{\alpha + \overline{b} - \overline{\theta}c}{\beta}
\end{aligned}$$

We see that if $\overline{b} - \underline{b} > c\left(\overline{\theta} - \mathbb{E}(\theta)\right) > 0$ then

$$k^n = \frac{\alpha - \mathbb{E}(\theta)c}{\beta} < \underline{k}^u = \frac{1}{\beta}\left(\alpha + \underline{b} - \mathbb{E}(\theta)c\right) < \overline{k}^u = \frac{\alpha + \overline{b} - \overline{\theta}c}{\beta}$$

So

$$\overline{K}^u = (1-\mu)\overline{\theta}\frac{\alpha + \overline{b} - \overline{\theta}c}{\beta} \leq K^n = \frac{\mathbb{E}(\theta)}{\beta}(\alpha - \mathbb{E}(\theta)c)$$

when

$$\overline{b} \leq \frac{\mathbb{E}(\theta)(\alpha - \mathbb{E}(\theta)c)}{\overline{\theta}(1-\mu)} - \left(\alpha - \overline{\theta}c\right) = b_0$$

Profits write as

$$\underline{\Pi}^u = \frac{1}{2\beta}(\alpha + \underline{b} - \mathbb{E}(\theta)c)^2 \text{ and } \overline{\Pi}^u = \frac{1-\mu}{2\beta}(\alpha + \overline{b} - c\overline{\theta})^2$$

so

$$\frac{\overline{\Pi}^u}{\underline{\Pi}^u} \geq 1 \Leftrightarrow \overline{b} \geq b_1 = \frac{\alpha + \underline{b} - \mathbb{E}(\theta)c}{\sqrt{1-\mu}} - \left(\alpha - \overline{\theta}c\right)$$

We have

$$b_0 - b_1 = (\alpha - \mathbb{E}(\theta)c)\left[\frac{1 - \mu\frac{\Delta_\theta}{\overline{\theta}}}{1-\mu} - \frac{1}{\sqrt{1-\mu}}\right] - \frac{\underline{b}}{\sqrt{1-\mu}}$$

We see that this difference is not always negative (positive between brackets), so $b_0 \geq b_1$ is possible and the result $\overline{K}^u \leq K^n$ is possible as the firm chooses $p^* = \overline{b}$.
When $\underline{b} - \overline{b} > 0$. Equilibrium capacities are

$$\begin{aligned}
\underline{K}^u &= \mu\underline{\theta}\left(\alpha + \underline{b} - \underline{\theta}c\right) \text{ and } \underline{k}^u = \frac{1}{\beta}\left(\alpha + \underline{b} - \underline{\theta}c\right) \\
\overline{K}^u &= \frac{\mathbb{E}(\theta)}{\beta}\left(\alpha + \overline{b} - \mathbb{E}(\theta)c\right) \text{ and } \overline{k}^u = \frac{\alpha + \overline{b} - \mathbb{E}(\theta)c}{\beta}
\end{aligned}$$

so

$$K^n = \frac{\alpha - \mathbb{E}(\theta)c}{\beta}\mathbb{E}(\theta) \leq \underline{K}^u = \mu\underline{\theta}\left(\alpha + \underline{b} - \underline{\theta}c\right)$$

when

$$\underline{b} \leq \frac{\mathbb{E}(\theta)(\alpha - \mathbb{E}(\theta) c)}{\underline{\theta}\mu} - \left(\alpha - \underline{\theta} c\right) = b_2$$

Profits

$$\begin{aligned} \underline{\Pi}^u &= \frac{\mu}{2\beta}(\alpha + \underline{b} - \underline{\theta} c)^2 \\ \overline{\Pi}^u &= \frac{1}{2\beta}(\alpha + \overline{b} - \mathbb{E}(\theta) c)^2 \end{aligned}$$

so $\underline{\Pi}^u \geq \overline{\Pi}^u$

$$\frac{\underline{\Pi}^u}{\overline{\Pi}^u} \geq 1 \Leftrightarrow \underline{b} \geq b_3 = \frac{\alpha + \overline{b} - \mathbb{E}(\theta) c}{\sqrt{\mu}} - \left(\alpha - \underline{\theta} c\right)$$

We have

$$b_2 - b_3 = (\alpha - \mathbb{E}(\theta) c)\left[1 + \frac{\overline{\theta}}{\underline{\theta}} \frac{(1-\mu)}{\mu} - \frac{1}{\sqrt{\mu}}\right] - \frac{\overline{b}}{\sqrt{\mu}}$$

We see that this difference is not always negative (positive between brackets) so $b_2 \geq b_3$ is possible and the result $\overline{K}^u \leq K^n$ is possible as the firm chooses $p^* = \overline{b}$.

Tailored prices

Suppose that $b > \psi(\theta)\theta$ for all θ. One can recast the result in the Proposition 5 as

$$\begin{aligned} k^t &= \frac{\alpha - (\mathbb{E}(\theta) - \hat{e}) c}{\beta} + \frac{\mathbb{E}(b - \theta\psi)}{\beta} \\ K^t &= \frac{\alpha - (\mathbb{E}(\theta) - \hat{e}) c}{\beta} (\mathbb{E}(\theta) - \hat{e}) + \frac{\mathbb{E}(b - \theta\psi)}{\beta} (\mathbb{E}(\theta) - \hat{e}) \end{aligned}$$

We see that

$$k^t = k^n + \frac{\hat{e}c + \mathbb{E}(b - \theta\psi)}{\beta} > k^n$$

Denote $\hat{\kappa}(x) = (\alpha + y - xc)\frac{x}{\beta}$ we have $\hat{\kappa}(\mathbb{E}(\theta)) = K^n$ with $y = 0$ and $\hat{\kappa}(\mathbb{E}(\theta) - \hat{e}) = K^t$ when $y = \mathbb{E}(b - \theta\psi) > 0$. Then

$$\kappa'(x) = \frac{1}{\beta}(\alpha + y - 2cx)$$

so there is a maximum $x^* : \kappa'(x^*) = 0$ hence, if

$$x \leq x^*(y) = \frac{\alpha + y}{2c} \Rightarrow \kappa'(x) \geq 0$$

Then $K^n > K^t$ as

$$\mathbb{E}(\theta) - \hat{e} < \mathbb{E}(\theta) < \frac{\alpha}{2c} < \frac{\alpha + \mathbb{E}(b - \theta\psi)}{2c}$$

and

$$\mathbb{E}(b - \theta\psi) \leq (\alpha - \mathbb{E}(\theta)c)\frac{\mathbb{E}(\theta)}{\mathbb{E}(\theta) - \hat{e}} - (\alpha - (\mathbb{E}(\theta) - \hat{e})c) = \overline{B}$$

Environmental-based Regulated prices

If $p^r = \underline{\psi} > \overline{\psi}$,

$$\begin{aligned} \alpha - \beta k^r &= (\mathbb{E}(\theta) - \hat{e})c < \mathbb{E}(\theta)c = \alpha - \beta k^n \\ k^n &< k^r = \frac{\alpha - (\mathbb{E}(\theta) - \hat{e})c}{\beta} \end{aligned}$$

$$K^r = \frac{\mathbb{E}(\theta) - \hat{e}}{\beta}(\alpha - (\mathbb{E}(\theta) - \hat{e})c)$$

Denote $\kappa(x) = (\alpha - xc)\frac{x}{\beta}$ we have $\kappa(\mathbb{E}(\theta)) = K^n$ and $\kappa(\mathbb{E}(\theta) - \hat{e}) = K^r$. Then

$$\kappa'(x) = \frac{1}{\beta}(\alpha - 2xc)$$

so there is a maximum $x^* : \kappa'(x^*) = 0$ hence, if

$$x \leq x^* = \frac{\alpha}{2c} \Rightarrow \kappa'(x) \geq 0$$

If $\frac{\alpha}{2c} \geq \mathbb{E}(\theta)c > (\mathbb{E}(\theta) - \hat{e})c$ then $K^n > K^r$

(b) If $\underline{\psi} > p^r = \overline{\psi}$,

$$\begin{aligned} \alpha - \beta k^r &= (\mathbb{E}(\theta) - \mu\hat{e})c < \mathbb{E}(\theta)c = \alpha - \beta k^n \\ k^n &< k^r = \frac{\alpha - (\mathbb{E}(\theta) - \mu\hat{e})c}{\beta} \end{aligned}$$

$$K^r = \frac{\mathbb{E}(\theta) - \mu\hat{e}}{\beta}(\alpha - (\mathbb{E}(\theta) - \mu\hat{e})c)$$

Here we have $\kappa(\mathbb{E}(\theta)) = K^n$ and $\kappa(\mathbb{E}(\theta) - \mu\hat{e}) = K^r$. If $\frac{\alpha}{2c} \geq \mathbb{E}(\theta)c > (\mathbb{E}(\theta) - \mu\hat{e})c$ then $K^n > K^r$

Environmental based congestion norms

In general, we have in each case

$$
\begin{aligned}
k^e &= \frac{\alpha - E(z_m)\,c}{\beta} > k^n = \frac{\alpha - \mathbb{E}(\theta)\,c}{\beta} \\
K^e &= E(z_m)\,k^e
\end{aligned}
$$

where $z_m \leq E(z_m) = \mu\underline{\theta} + (1-\mu)\,z_m \leq \mathbb{E}(\theta)$. Then

$$
K^e \leq K^n \Leftrightarrow \frac{\alpha - E(z_m)\,c}{\beta} E(z_m) \leq \frac{\alpha - \mathbb{E}(\theta)\,c}{\beta} \mathbb{E}(\theta)
$$

Here we have $\kappa(\mathbb{E}(\theta)) = K^n$ and $\kappa(E(z_m)\,c) = K^r$.Then if $\frac{\alpha}{2c} \geq \mathbb{E}(\theta)\,c > E(z_m)\,c$ then $K^n > K^e$