Peer-to-Peer Energy Platforms: Incentives for Prosuming*

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Abstract

We analyse how new models of peer-to-peer exchange in the electricity sector may be effective and could yield incentives to invest in decentralized domestic production units based on renewable energy sources. We model a local exchange system for electricity, designed as a dealing platform, which determines purchase and selling prices on a continuous time basis. This allows us to question the participation of prosumers in peer-to-peer energy exchanges and their willingness to invest in local energy production. Compared to a no-platform configuration, we show that a pure dealing welfare maximizing platform creates at least as much incentive to install domestic production units. Then we challenge this main result considering several relevant features for peer-to-peer energy exchange.

JEL classification: L14, L81, L94, Q4
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1 Introduction

The European Parliament adopted at first reading on 13 November 2018 with a view to the adoption of a new Directive of the European Parliament and of the Council to promote the use of energy from renewable sources. This legal process should favour the development of new trading arrangements and new technological improvements in energy systems.[1] By 30 June 2021, national governments will need to transpose the laws (and the community energy rights) into their legal system. A part of the energy transition, the development of smart grids seems to be a major challenge: thanks to new technologies, it will be possible to increase the share of renewable energy and reduce energy consumption.

The development of self-consumption has now modified traditional economic models based on a clear distinction between consumers and producers. A new type of agent has appeared, the prosumer, who is an "active" consumer that both consumes and produces electricity based on distributed renewable energy sources (DRES). Smart grids open up new perspectives and constitute a revolution in the energy field. The emergence of peer-to-peer (P2P) electricity trading systems could support these changes.

Indeed, these P2P trading systems allow consumers and prosumers to trade energy in real time within a local group of agents, i.e. a community. As DRES are small-scale systems, they constitute decentralized ways to generate electricity.[2] They are mainly based on hybrid or combined technologies such as solar power or small wind electric system,,[3] but can also be made up of a single technology, like a diesel or gas genset.

The rise of P2P electricity trading, using exchange platforms, like the Airbnb and Uber platforms, is the basis for significant societal changes that will make it possible to achieve the objectives of the energy transition. According to Rifkin (2011), the Internet technology could allow these changes to arise, and could help households to share their energy surplus with neighbours or to sell it back to the grid. Mengelkamp *et al.* (2018) argues that small-scale energy consumers and prosumers may be empowered by P2P electricity trading, as investments in local generation are promoted, and the development of self-sustainable microgrid communities would be easier.

Nowadays, the number of P2P trading systems is growing, even if they are sometimes still at the stage of R&D projects, but their economic significance may be questioned. For

[1]Thus, Article 21-2a indicates that Member States shall ensure that renewable self-consumers, individually or through aggregators, are entitled: "*to generate renewable energy, including for their own consumption, store and sell their excess production of renewable electricity, including through renewables power purchase agreements, electricity suppliers and peer-to-peer trading arrangements*".

[2]DRES may require energy storage systems to maintain the stability of the grid. However we do not consider energy storage in our analysis.

[3]Doshi (2021) estimates that, between 2017 and 2030, the annual average growth rate of these technologies is projected to reach of 7.2% for wind turbines and 7.4% for solar PV modules. According to Frost & Sullivan (2020), the global annual investments in distributed energy resources will increase by 75% by 2030.

example, what are the favourable economic conditions for a prosumer to join such a P2P trading platform? More precisely, could P2P platforms create more incentives to promote investments in solar panel or small wind turbines than the present centralized system? What are the impacts of the platform design on these incentives?

The economic literature on smart grids has focused on costs and prices, in particular the design of tariffs as a tool for reducing electricity demand during peak periods (peak-load pricing, capacity trading), thus allowing the reduction of CO2 and GHG emissions. However, relatively little attention has been paid to the economics of peer-to-peer exchanges in the electricity sector. In this paper, we aim the fill this gap.

The objective of this paper is to address these issues, by providing an economic analysis of P2P energy trading systems when they are organized through a platform that acts a market-dealer. We investigate under which conditions these organizations may be effective and may yield sufficient incentives to invest in a fixed domestic production units (hereafter DPU) based on renewable energy sources. We provide a simple model by considering prosumers, connected to the national grid, who purchase or sell energy among themselves or to the grid. We aim to take into account the heterogeneity of agents within the community with respect to their energy needs (or load profiles) and also the inner intermittency of the DRES used to produce their local electricity. Heterogeneity is a necessary feature to allow for effective P2P trades within the platform at any time, and intermittency is a standard assumption when the electricity generation is based on DRES.

In this setting, we first look at the benchmark case where prosumers can install a DPU when no platform exists. Consequently, they face a simple cost-benefit trade-off. A prosumer installs the DPU when the expected opportunity cost savings from purchasing from the grid exceed the installation expenditure of the DPU. Then, we determine the equilibrium price levels for a welfare maximizing dealing platform, showing that the purchase prices are always greater than the central grid price, and selling prices are always lower. Yet despite this, we show that the expected net gains from being a trader within the platform are always greater than the expected average price on the grid. Indeed, investing in DPU's helps to increase the expected net gains from trading, as they are always greater than the incentives when there is no platform. As a result, a welfare maximizing platform would always be profitable for a community of prosumers as it can boost the installation process of DPUs.

In order to challenge these results, we focus on several alternative designs for the platform, such as zero pricing, for-profit platforms and matching platforms. First, we show that a zero pricing scenario creates less incentives to invest than the optimal outcome, but more than those without a platform. Second, if the dealing platform is for-profit, i.e., with market power on both buyer and seller sides, purchase prices are further increased and selling prices are reduced. This has a dampening effect on the incentives to adopt DPU compared to the non-profit case. Consequently, some prosumers do not invest any more whereas they would have done so without a platform. Third, relying on key features on digital markets, we study a matching platform rather than a dealer one. This affects the market outcomes and incentives to adopt DPU, in the sense that the more the matching technology is efficient to match sellers and buyers, the more the incentives to adopt DPU may increase, compared to a pure dealer.

Finally we also provide two extensions of our analysis. First, we relax our basic assumption that the DPU capacity is fixed. With variable capacities, the main result does not always hold. Indeed, an agent connected to a dealing platform might not have strictly superior marginal incentives to install DPU than in the case of not being connected. Second, in the main setting, it was assumed that intermittency affects all prosumers in the same way. If individual shocks exist, the main result still holds (in a weak version): if an agent would have installed a DPU if there were no platform, then that agent will do so and will not be worse off when a dealing platform is active.

The present paper is related to three strands of the literature. First, it touches upon the literature on energy communities and decentralised energy systems. There is a growing economic literature on those topics which is particularly well exposited in Abada *et al.* (2020a,b). To sum up, this literature is mainly oriented in terms of an engineering or optimization perspective, depicting the optimal technical performance of decentralized generation on energy communities and micro-grids. However, some papers adopt an economic point of view. Abada *et al.* (2020a) study the viability of the community by using a cooperative game approach and find that inadequate gain sharing may jeopardize the stability of a community but if aggregation benefits can compensate for coordination costs, the community may be stable. Abada *et al.* (2020b) also find that the development of such energy communities is dependent on the grid tariff structure, which can lead to over-investment in decentralised energy systems (mainly rooftop PVs). Instead, in the present paper we build a non-cooperative framework to analyse the significance of energy communities based on local exchanges.

Second, this paper is closely related to the literature on P2P economics. The recent economic literature applied to digital platforms has mainly been developed on the basis of questions raised by the emergence of service platforms such as eBay, Uber and Airbnb. The main objective of these platforms is to facilitate the exchange of commodities, services and cultural goods between a large number of heterogeneous buyers and sellers. There arise new economic issues concerning the economic and business models of the actors, their pricing strategies, and how these activities could be regulated. Krishnan *et al.*(2003) argue that P2P networks could be perceived either as public goods or as club goods. They provide an overview of P2P networks, focusing on the agent behavior, such as free-riding, that is, when users consume network resources without providing resources to the network. Basically, with such behavior, a P2P network could collapse. This risk can be mitigated if the users' participation is conditioned by altruism, or if the viability of P2P networks is based on trust and reputation. In our paper, we rely also on some intrinsic preferences to be engaged in P2P energy trading. Einav *et al.* (2016) consider elements common to all these P2P platforms, such as the role of intermediation for the platform owner, monitoring agents via technology, sophisticated pricing mechanisms and so on. They highlight the issue of matching heterogeneous buyers and sellers and determine the conditions under which P2P markets arise and are efficient. Among these conditions, the choice of pricing mechanism or market design is essential. For example, by using data about eBay, Einav *et al.* (2018) provide an empirical and theoretical analysis about the trade-offs between online auctions and posted prices. In our analysis, we will take stock of these ideas by comparing different platform designs.

Lastly, a new branch of literature deals with energy digital platforms that use blockchain and distributed ledger technologies. Sousa *et al.* (2019) and Soto *et al.* (2021) provide broad overviews of P2P energy trading markets, focusing on technical, optimization and engineering issues. The development of such technologies has made possible decentralized exchanges with automated management systems that are essential for the balancing of supply and demand within microgrids, without intermediation (or aggregators), as noted by Mengelkamp *et al.* (2018). From an economic point of view, Gautier and Salem (2021) show that the social efficiency of P2P trading may depend on the strength of the negative externalities created by too generous feed-in tariffs. In our paper, we rather assume that no feed-in tariffs support the prosumer investments and we analyse the performance of the platform design from the perspective of local efficiency.

The rest of this paper is organized as follows. In the first section, we depict some of the experiences with P2Ps: first we propose a benchmark, i.e. without platform, in which we focus on the incentives for installing a fixed size DPU capacity. We go on with a simple dealing platform and determine and compare those incentives. Lastly, we consider several extensions in order to challenge our basic framework and results. We study zero pricing schemes, market power for the dealing platform, a matching process and we extend our basic framework to allow for variable DPU size and individual shocks. Details and proofs are given in the Appendix.

2 P2P Electricity Trading: Some Experiences

In practice, P2P electricity trading systems rely on physical and virtual layers which embody an energy management system, so P2P trading is facilitated by the existence of digital platforms connecting a large number of peers. According to IRENA (2020), smart meters, broadband communication infrastructure, as well as network digitalisation may be fundamental enablers for P2P electricity trading models. As a result, they may be seen as a real opportunity for integrating these technologies into the electric system.

Several implementations of P2P energy trading systems based on platforms have been achieved in the world, and we give a brief presentation of the most significant experiences with microgrids and smart grids. A lot of papers and reports describe the technological details of these P2P trading systems, that we briefly review here.

Mainly, these are located in Europe and United States and are are supported by public research programs.[4] However, there are a lot of independent projects worldwide For instance, as mentioned in IRENA (2020), P2P microgrids can constitute home electrification solutions in developing countries such as Bangladesh, Malaysia and Colombia, e.g. the Transactive Energy Colombia project implemented in Medellin. The key point of these projects is to connect low-income prosumers, equipped with photovoltaic roofs, and unequipped richer consumers. In Bangladesh, the Solshare company has developed similar technological solutions based on connected objects, such as smart phones.

Zhang *et al.* (2017) and Soto *et al.* (2021) propose a comparison of several projects or start-ups based on the network size or their scope and on the information and communication technologies (hereafter ICT). Hence, as explained by Zhang *et al.* (2017), Piclo (UK), Vandebron (Netherland), SonnenCommunity (Germany) and Litchtblick Swarm Energy (Germany) have national scope, whereas Smart Watts (Germany), Yeloha Mosaic[5] (US) are regional. The smallest sized platforms, such as TransActive Grid/LO3 Energy (US) and Electron (UK) correspond to a local P2P market in which blockchain technology is used in order to simplify the metering and billing system (see Soto *et al.*, 2021). Some studies have shown that by trading power with peers, prosumers can achieve overall savings on their bills (up to 20% on average), such as with the German project Lition tested in 2018 (GJETC, 2020).

For Zhang *et al.* (2017), P2P energy trading systems are also based on three levels. The first level represents a P2P energy trading within a eco-neighborhood, as, for example, the iconic Brooklyn microgrid (TransActive Grid/LO3 Energy) or Lyon Confluence in France. The second level is characterized by trading between several microgrids (Multi-Microgrids, or so-called P2P within Cell[6]). This is the case for two connected microgrids, Walqa and Atenea, located in Spain, separated by 150 km.[7] Energy trading is also possible between them, organized around industrial laboratories of small tertiary companies. Finally, the third level corresponds to P2P among Cells (Multi-Cells). As explained by Zhang *et al.*

[4]See also Gangale *et al.* (2017) for an overview of European smart grid projects.

[5]This project aimed to create a solar-like Airbnb, without success in the end. In Spain, Sotysolar aims to do the same.

[6]According Zhang *et al.* (2018), a "Cell" defines a wider area of network than a single microgrid in which a collection of DRES may operate in either grid-connected or islanded mode.

[7]They are included in the P2P-SmartTest R&D project, see the report available online: https://cordis.europa.eu/project/id/646469/reporting

(2018), Multi-Cells corresponds to a region, a large city or a metropolitan area including multiple Cells. In this case a "peer" is a microgrid, a Cell or a region that may trade with each other. The two last levels raise the question of a conducive regulatory framework allowing such interconnection, but also of the structure of the pricing scheme. The third level is mostly hypothetical at this time but may be occur in the future.

Sousa *et al.* (2019) classify R&D projects for P2P energy trading considering 1) the market design and business models and 2) the implementation of local control and ICT platforms for prosumers. Projects such as Enerchain, NRGcoin, and Energy Collective are most advanced along the first dimension, whereas Empower and P2P-SmartTest focus on the second. Some, such as Lumenaza (Germany), aim to cover both dimensions.

Therefore it appears from these various experiments, that research organizations, entrepreneurs, or consumers consider the implementation of P2P energy trading systems based on platforms to be significant. These experiments show that several challenges have to be taken up. From the consumers' point of view, the main issue is participation and their incentives to invest in DPU. Indeed, as illustrated by the Brooklyn microgrid project, participants developed shared values when trading within the platform and, they focused on understanding how it works. Consequently, this has impacts on network dimensioning, as Zhang et al. (2017) have shown. For research organizations, they face a technological challenge. The innovative digital technologies implementation, such as blockchain processes, and ICT infrastructures allow real-time energy exchanges. They also lead to better matches between participants. Last, from firms' and entrepreneurs' points of view, the market design of P2P trading systems and the choice of a business model are key issues. Indeed, price formation and pricing schemes (including zero pricing), metering, and billing have been essential concerns for the experiments described above. In the following, we provide an economic analysis of the relevant issues raised by the emergence of P2P energy trading systems. For that purpose, we present a model that mainly studies prosumers' participation, platform design, and matching technologies. In the next section, we detail the elements of this modeling.

3 Model

We now develop a simple stylized model where heterogeneous agents aim to exchange the excess energy flows they produce using renewable decentralized production units. Our main goal is to see how such P2P trading arrangements can be viable for all participants. In our model, prosumers, who are consumers and producers of energy goods, can offer these goods in competition with professionals producers (i.e. companies or local communities) and interact with possible pure consumers on a dedicated platform. In the first step, the platform is just considered as a dealer that purchases excess energy from some prosumers and resells it to consumers or through the central grid.

Suppose there is a mass n of agents with a load factor (state of demand) of $\phi \in [\underline{\phi}, \overline{\phi}]$ distributed according to a cdf $G(\phi)$ where $G'(\phi) = g(\phi)$ and $G(\bar{\phi}) = n$. This state describes the level of consumption they desire to achieve in all periods. This corresponds to their standard energy needs in relation to the size of the agent's household (i.e. dwelling area, number of people, installed power). We assume that the surplus derived from this baseline level of consumption is $u(\phi)$, where $u(\cdot)$ is an increasing concave function.

To satisfy their needs, each agent has the choice to install or not a domestic production unit of energy, here represented by a maximal production capacity of $q > 0$ kWp at a capacity up-front cost $k > 0$. We assume that q is fixed and later relax this assumption in Section 6. For example, this can be the case if the agent acquires a dwelling in a connected residential area where residential cells are standardized and so is the DPU.

With this capacity installed, an agent can be a prosumer in the sense that they can use it at will, to self-consume it or to sell it if it is possible according to the excess capacity observed at each time $\phi - qx$. Here, the variable qx represents the available amount of the renewable capacity q that is actually dispatchable in state $x \in [0, 1]$; they are distributed according to a cdf $F(x)$, with $F'(x) = f(x)$. The state of nature x represents weather conditions or the occurrence of failures, that is, all external conditions that drive the intermittency of DPUs. Then, in a given state of nature x, a prosumer with an installed a capacity q, may be either a pure consumer if $\phi - qx \geq 0$ or a potential seller if $\phi - qx < 0$. For the sake of simplicity, let us assume that $\bar{\phi} \geq q > \underline{\phi}$, which means that in favourable conditions ($x = 1$), there are always some buyers (those with load factors near the upper bound $\bar{\phi}$) and sellers (those with small load factors near the lower bound $\underline{\phi}$).

Note that heterogeneity of agents is a key assumption in our analysis. Indeed in each state, without heterogeneity on load factors, all agents would be either pure consumers or pure sellers. In this case, P2P trades would be impossible and a platform would have no reason to exist.

Figure 1 depicts the heterogeneous consumption model. The sloping dotted lines represent the net consumption/production for the extremal agents, the sloping thick line is that of a given agent with a load factor ϕ.

We will now describe the supply side. First, we assume that there is always a centralized professional supplier who can provide unlimited energy volumes to all agents that demand them at a given price $a(x)$.[8] This price may include the energy wholesale prices and volumetric parts of grid access tariffs. We also assume that a fixed periodic (non-volumetric) tariff τ is charged to pure consumers served from the grid. However, as our focus is on the

[8]To embrace the possibilities of dynamic pricing or time-of-use pricing on the grid side, we assumed a state-dependent grid price. However, taking a constant price does not change our results.

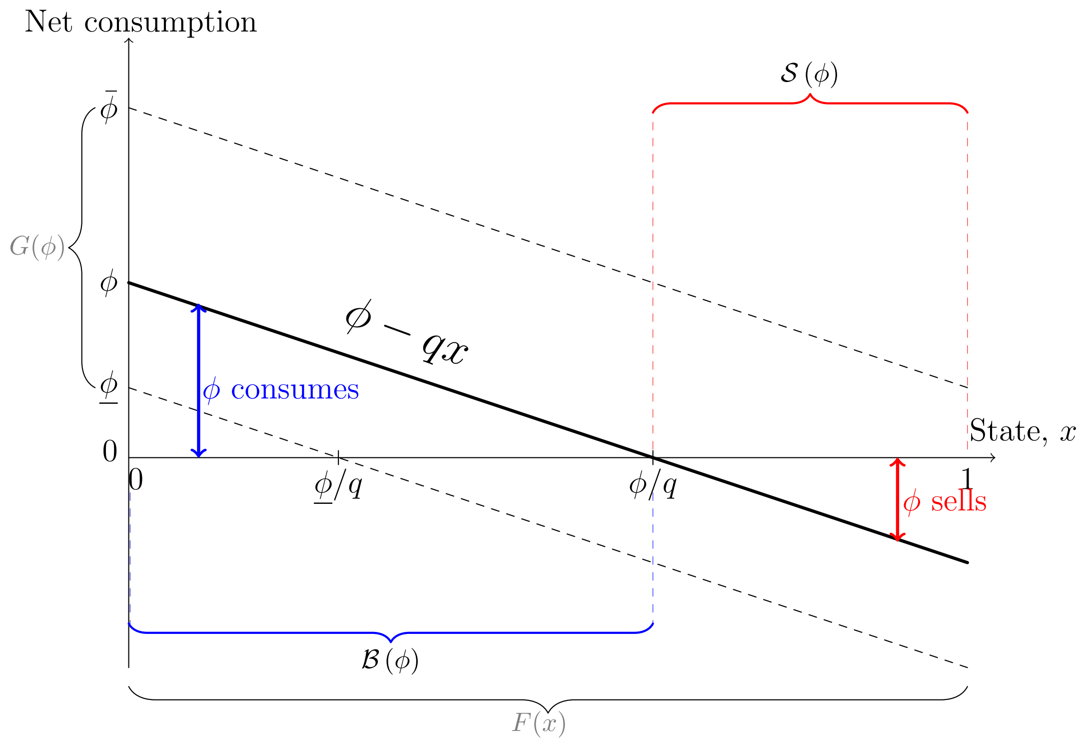

Figure 1: Net consumptions

P2P exchanges, we take the (centralized) grid to be the external supply, as, in some sense, the grid supply is the outside option for all agents, whether or not they are prosumers. Second, we analyse the viability of a dealing platform through which all prosumers may want to trade their excess/lack energy volumes in any state of nature.

The basic business model for this platform is to resell excess energy to connected consumers, or, if no internal deals are closed, to the central grid. We assume that the platform cannot affect the price $a(x)$, based on the wholesale market price and access tariffs, so the platform cannot make profits on this external side. We denote by $p(x) \geq 0$ the platform purchase price and $r(x) \geq 0$ the platform selling price.[9] Then if agent ϕ is a consumer in state x, they will have to pay an amount $p(x)(\phi - qx) \geq 0$ if purchasing the needed energy through the platform. In contrast, if this agent is a seller in state x, a profit of $r(x)(qx - \phi) \geq 0$ will be received if selling their excess energy through the platform. We also assume that agents participating in the platform have an intrinsic preference for being served through this channel, which is represented by a parameter $\delta \geq 0$. For instance it represents the surplus of being in sharing relationships with identified agents (neighbours, flatmates, members of an dedicated association). This parameter can be identified with a social value or altruism of belonging to a group or community. It also represents a part of the surplus from avoiding power cuts due to outdated or flawed distribution grids, or the reduction compared to the costs of transactions with the professional suppliers. It can also

[9]In such a model with vertical differentiation for participating in the platform, negative prices would be possible. However, we assume that faced with a negative price, a seller does not trade.

be related to the gain from some ancillary local or specific services provided by the platform that are valuable to the connected consumers. Furthermore, it can be alternatively assimilated to the environmental preference of an agent who produces with residential renewable sources (i.e. "fossil fuel freedom").

As a result, an agent fulfilling their needs through the local channel or the platform derives an utility of $u(\phi+\delta)$. So for an agent with a load factor ϕ, the utility from trading through the platform in state x is

$$U(\phi, x, q) = \begin{cases} u(\phi+\delta) - p(x)(\phi - qx) & \text{if } \phi \geq qx \\ u(\phi+\delta) + r(x)(qx - \phi) & \phi < qx \end{cases}$$

The utility from trading through using the grid is

$$\underline{U}(\phi, x, q) = \begin{cases} u(\phi) - a(x)(\phi - qx) - \tau & \phi \geq qx \\ u(\phi) + a(x)(qx - \phi) - \tau & \text{if } \phi < qx \\ u(\phi) - a(x)\phi - \tau & q = 0 \end{cases}$$

So for each x, there may exist a $\hat{\phi}_x = qx$ such that the agent is a pure self-consumer (if $x > 0$).

4 No platform

Consider first the common situation in which there is not platform. The central grid is viewed as an aggregator that purchases or sells energy at a given price $a(x)$. The only decision for each agent is to install or not the DPU capacity q at cost k. A prosumer ϕ installs the DPU if (expectations are taken over x):

$$\mathbb{E}[U_0] - k \geq \mathbb{E}[\underline{U}|q = 0] = u(\phi) - \mathbb{E}[a(x)\phi] - \tau$$

where

$$\mathbb{E}[U_0] = u(\phi) - \mathbb{E}_{\mathcal{B}(\phi)}[a(x)(\phi - qx)] + \mathbb{E}_{\mathcal{S}(\phi)}[a(x)(qx - \phi)] - \tau \tag{1}$$

and

$$\begin{aligned} \mathcal{B}(\phi) &= \{x \in [0,1] : 0 \leq x \leq \phi/q\} \\ \mathcal{S}(\phi) &= \{x \in [0,1] : 1 \geq x \geq \phi/q\} \end{aligned}$$

which are the set of states of nature in which the prosumer ϕ is a buyer, respectively, a seller. Note that $\mathcal{S}(\phi)$ may be eventually *empty*, as for instance when $\phi = \bar{\phi}$, $x \leq 1 < \bar{\phi}/q$. In Figure 1, both sets are depicted.

Looking for the indifferent prosumer ϕ_0 such that $\mathbb{E}[U_0] = \mathbb{E}[\underline{U}|q=0]$, we have

$$\phi_0 : q\mathbb{E}[a(x)x] - k = 0$$

which does not depend on the value of ϕ. As a result, with no platform, the amount $I_0 = \max\{q\mathbb{E}[a(x)x] - k, 0\}$ is the gain from installing the DPU. It represents the incentives to invest in DPU for agent ϕ. Thus, the indifferent prosumer $\phi_0 = \bar{\phi}$ if $q\mathbb{E}[a(x)x] < k$ and $\phi_0 = \underline{\phi}$ if $q\mathbb{E}[a(x)x] > k$.

Lemma 1 *With no platform, all agents are prosumers and install capacity $q > 0$ iff $q\mathbb{E}[a(x)x] > k$, and there are no prosumers otherwise.*

The result in this lemma is just the cost–benefit trade-off for each prosumer. On the one hand, the amount $q\mathbb{E}[a(x)x]$ represents the opportunity benefits of the grid purchase cost savings expected for a prosumer ϕ that had installed capacity to the amount of q. On the other hand, k is the fixed expenditure to have access to this capacity. As a result, a prosumer actually invests in this capacity if this benefit outweighs the cost. Moreover, as these cost savings are independent of the load factor ϕ, then either all agents are prosumers or else all are pure consumers.

5 Simple dealing platform

A dealing platform has the ability to identify the supplies of the prosumers and their demands and ensure their equilibrium. In the context of an electricity system, the platform is also an aggregator that dispatches the power within the local grid and towards the central grid. It can purchase prosumers' supplies, if any, at the price $r(x) \geq 0$ in state x and resell these electricity flows to connected consumers at the price $p(x) \geq 0$.

The objective of the platform can be profit-oriented or welfare maximizing. To start with, let us suppose that the platform has a local welfare objective.[10] Indeed a first step, one could imagine that in the future "turnkey digital technologies" and ready-made microgrids[11] may be quite easily installed by energy communities. In that sense, up the installation cost, the trading platform could be socially managed and zero-pricing be even desired by users.

So if in state x, the total supply to the platform in order to be resold within is $S(r(x))$, it must match the total demand $D(p(x))$ from prosumers that are in lack of power with

[10] Alternatively, one could argue that, due to the technicality dimension of microgrids technology which may involves costly investments, the dealing platform would be a for profit organization. This for-profit configuration is treated in section 6.

[11] For instance Howland (2021) reports that in Nebraska, Lincoln Electric System, a power utility, has set up a microgrid with up to 29 MW of load at near-zero cost.

regard to their domestic production at that state. However, some agents may prefer not to purchase or resell to the platform but to the grid. The platform cannot make money from them.

Demand and supply to the dealing platform The platform will implement choices that are individually preferable for each participants. So agent ϕ will be consumer within the platform if they prefer to purchase the energy needed or to sell the energy in excess in some state x, to the platform whereas to the grid.

Concerning purchases, that is for agents such that $\phi \geq qx$, this implies $U(\phi, x, q) \geq \underline{U}(\phi, x, q)$ and writes (omitting the argument x)

$$u(\phi + \delta) - p(\phi - qx) \geq u(\phi) - a(\phi - qx) - \tau$$

put differently:

$$\Delta(\phi) = u(\phi + \delta) - u(\phi) + \tau \geq (p - a)(\phi - qx) \tag{2}$$

Here $\Delta(\phi)$ represents the direct periodic gains for a prosumer both from being connected to the community through the platform and also due to saving from the fixed access costs. Note that $\Delta'(\phi) < 0$ by concavity of u.

This implies that if $p > a$, some buyers prefer to purchase their electricity from the grid. Only those who have a load factor lower than a certain level, denoted $\beta(p)$, will consume their electricity within the platform. This load factor level $\beta(p)$ represents the indifferent buyer from purchasing from the platform or from the grid, using (2) it is defined as $\Delta(\beta(p)) = (p - a)(\beta(p) - qx)$. This is the highest load value for which the price-sensitive platform demand, denoted $d(p)$,[12] peaks at a given price p. So there exists a floor price level $\underline{p} > a$ such that all buyers prefer the platform i.e. $\beta(\underline{p}) = \bar{\phi}$ and then $d(\underline{p}) = \bar{d}$, the highest potential demand in state x. Hence if $p \leq a$, the platform purchase price is so low that all potential buyers in state x always prefer to purchase their electricity among peers, the demand becomes rigid and equals $\bar{d}$. Then the aggregate demand at state x can be written as:

$$D(p) = \begin{cases} \bar{d} & \text{if } p \leq \underline{p} \\ d(p) & \quad\; p > \underline{p} \end{cases} \tag{3}$$

The determination of the platform supply is done in the same spirit but for agents such that $\phi \leq qx$. Then, if $r < a$, some sellers prefer to sell their electricity to the grid. Only those who have a load factor greater than a level $\sigma(r)$ will sell their electricity in excess to the platform. Here $\sigma(r)$ represents the indifferent seller from selling to the platform or to the grid, defined as $\Delta(\sigma(r)) = (a - r)(qx - \sigma(r))$. As a result, the price-sensitive platform supply, denoted $s(r)$, peaks at when $\phi = \sigma(r)$, at a given price r. So there exists a ceiling

[12]More details are provided in the Appendix.

price level $\bar{r} < a$ such that all sellers prefer the platform, i.e., $\sigma(\bar{r}) = \underline{\phi}$ and $s(\bar{r}) = \overline{s}$. If $r > a$, all sellers always prefer the platform and the supply is always rigid and equals $\overline{s}$. Then the aggregate supply at state x is such that

$$S(r) = \begin{cases} \overline{s} & \text{if } r \geq \bar{r} \\ s(r) & \phantom{\text{if }} r < \bar{r} \end{cases} \tag{4}$$

Market clearing and platform pricing In some state, $x > \underline{\phi}/q$, it may exist platform exchanges in the sense that the above demand and supply may meet. The market clearing prices are then a couple which balances demand and supply on the platform:

$$(p, r) : D(p) = S(r)$$

As the grid is a default option, the non served demands and supplies through the platform are served by the central grid. As a result, in any time, all energy flows are balanced. Let us now assume that, for each state, the platform chooses the prices (p, r) that maximize the total welfare of participants. This welfare is just the sum of prosumer' surpluses and the dealer's profit:

$$W(x) = \int_{\underline{\phi}}^{\bar{\phi}} U(\phi, x, q)\, dG + \pi(x) = \int_{\sigma(r)}^{\beta(p)} u(\phi + \delta)\, dG$$

subject to $D(p) = S(r)$ and where $\pi(x) = pD(p) - rS(r)$ is the platform's profit. This leads to corner solutions[13] as depicted in the following Lemma, where $\hat{x} = \frac{\mathbb{E}[\phi]}{nq}$.

Lemma 2 *Optimal prices (p^*, r^*) are such that*

1. *$r^* = \bar{r}$ and $p^* > \underline{p}$ whenever $\overline{s} < \bar{d}$ that is for $x < \hat{x}$,*
2. *$p^* = \underline{p}$ and $r^* \leq \bar{r}$ whenever $\overline{s} \geq \bar{d}$ that is for $x \geq \hat{x}$.*
3. *and $p^* > a > r^*$ for all x*

In unfavourable conditions of availability, i.e. x low, the aggregate demand to the platform is structurally high and the supply low, so the selling price is set at least to its maximum value[14] in order to attract all sellers to the platform. As a result, the demand price is the one that just clears the market. In favourable conditions of availability, i.e.

[13]Indeed, there are multiple solutions as they are depicted in the proof in the Appendix. We pick down the less favourable for prosumers in order to give as little chance as possible to the platform to dominate and not to favor the platform situation in an artificial way. However, if we chose others prices further results are unchanged.

[14]This is also equivalent is terms of demands or supplies to set alternatively the price equal to $a(x)$ or lower. But it is not in terms of net welfare as the platform generates a additional utility.

x high, the aggregate supply to the platform is structurally high and the demand low, so the demand price is set at its minimum value to push possible local buyers to be active on the platform. As a result, the selling price just clears the market.[15] The optimal market

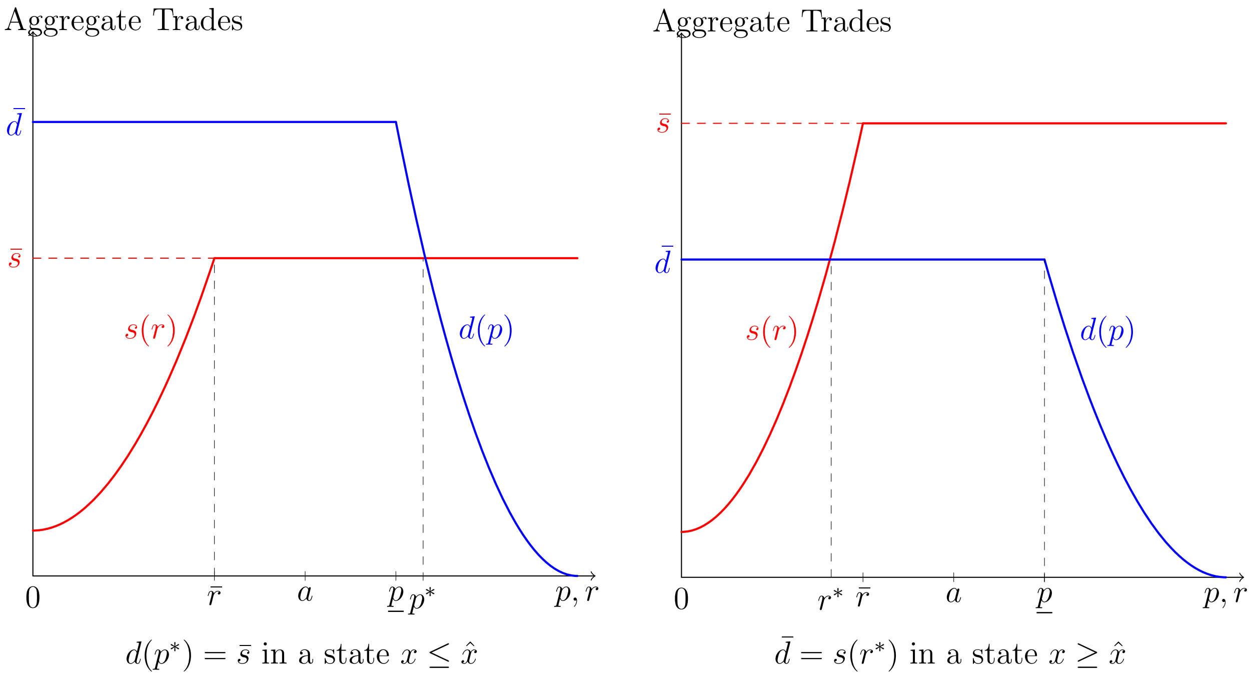

Figure 2: Market clearing

clearing is depicted in Figure 2. These optimal equilibrium prices put the agent in a trade set representing the states of nature in which the prosumer ϕ is a buyer from the grid ($\mathcal{B}^G$) and from the platform ($\mathcal{B}^P$), a seller to the platform ($\mathcal{S}^P$) and as well as a seller to the grid ($\mathcal{S}^G$). They write[16]

$$
\begin{aligned}
\mathcal{B}^G &= \{x \in [0,1] : x \leq \xi_b(\phi)\} \\
\mathcal{B}^P &= \{x \in [0,1] : \xi_b(\phi) \leq x \leq \phi/q\} \\
\mathcal{S}^P &= \{x \in [0,1] : \xi_s(\phi) \geq x \geq \phi/q\} \\
\mathcal{S}^G &= \{x \in [0,1] : x \geq \xi_s(\phi)\}
\end{aligned}
$$

[15] One might expect there to be a cost associated with managing the platform, and this cost would be increasing and convex in the number of suppliers and consumers on the platform (there would be a need for huge servers). However, if platforms are ICT based, at least for microgrids those costs may be negligible. Nevertheless, if we introduce such a cost, the optimal pricing is now changed in such a way that the prices are now interior: $p^* > \underline{p} > a > \bar{r} > r^*$. In the end, this does not much change Proposition 1 as this create the same effect on prices as the for-profit platform described in Section 6.

[16] These sets could be further subdivided to take into account the pricing structure of the platform, as is shown is Figure 3.

and when[17] $x = \xi_b(\phi) : \beta(p^*) = \phi$ and $x = \xi_s(\phi) : \sigma(r^*) = \phi$. Note by definition that $\xi_b(\bar{\phi}) = \xi_s(\underline{\phi}) = \frac{\mathbb{E}[\phi]}{nq}$ as when $\beta(p^*) = \bar{\phi}$ and $\sigma(p^*) = \underline{\phi}$ then $\bar{d} = \bar{s}$, which occurs when $\hat{x} = \frac{\mathbb{E}[\phi]}{nq}$. Figure 3 represents the equilibrium trade sets in the (ϕ, x) plane, where red/blue areas are such that agents buy/sell on the platform. We see that for a given state of intermittency (a given point on the x-axis), depending on their load profile ϕ, a prosumer may be a seller on the platform (red hatched area) or to the grid (grey hatched area on the right), or may be a buyer on the platform (blue hatched area) or to the grid (grey hatched area on the left).

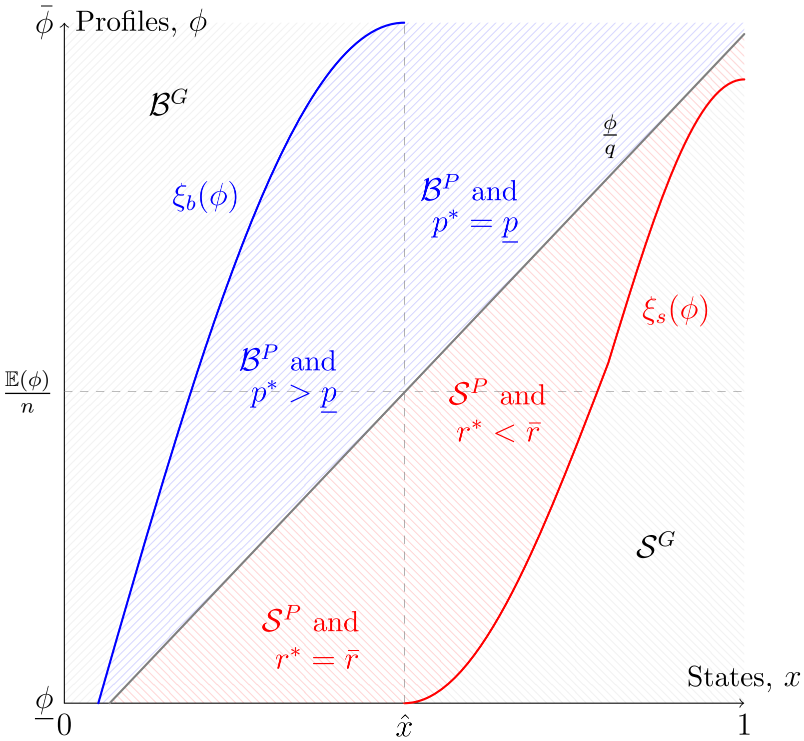

Figure 3: Trade regions

Incentives to install DPU Now, we analyse the incentives to install DPU created by the existence of the exchange platform. For an agent ϕ, the expected surplus from participating in the platform is then

$$\begin{aligned} \mathbb{E}[U] &= u(\phi) - \mathbb{E}_{\mathcal{B}^G}[a(\phi - qx)] + \mathbb{E}_{\mathcal{B}^P}[\Delta(\phi) - p^*(\phi - qx)] \\ &\quad + \mathbb{E}_{\mathcal{S}^P}[\Delta(\phi) + r^*(qx - \phi)] + \mathbb{E}_{\mathcal{S}^G}[a(qx - \phi)] \end{aligned} \tag{5}$$

[17]Indeed we always have $\xi_b(\phi) \le \phi/q$ as $\xi_b(\phi) = \frac{\phi}{q} - \frac{\Delta(\phi)}{q(p^*-a)} < \phi/q$, identically for $\xi_s(\phi)$, when $\xi_s(\phi) \ge \phi/q$. Moreover they are both increasing in ϕ.

Actually an agent installs capacity q when $\mathbb{E}[U] - k \geq [\underline{U}|q = 0]$ and looking for the indifferent prosumer ϕ^* such that $\mathbb{E}[U] - k = u(\phi) - \mathbb{E}[a\phi]$. Rearranging the terms, this leads to the equality

$$\begin{aligned}
\mathbb{E}[U] - k - (u(\phi^*) - \mathbb{E}[a\phi^*]) &= q\mathbb{E}[ax] - k \\
&\quad + \mathbb{E}_{\mathcal{B}^P}[\Delta(\phi^*) - (p^* - a)(\phi^* - qx)] \\
&\quad + \mathbb{E}_{\mathcal{S}^P}[\Delta(\phi^*) + (r^* - a)(qx - \phi^*)] = 0.
\end{aligned}$$

First, we now see that *in general*, not all consumers are willing to participate in the platform and install DPU. Indeed, we see that the load factor now is involved in the decision. Here the amount, $I_P(\phi) = \max\{\mathbb{E}[U] - k - (u(\phi) - \mathbb{E}[a\phi]), 0\}$ represents the incentives to invest in DPU for agent ϕ. However, assume that $q\mathbb{E}[ax] = k - \varepsilon$, so that no agent would be a prosumer in the benchmark case (without platform). Then in that case we see that no agent is worse off for being a prosumer connected to the platform, as

$$\mathbb{E}[U] - (u(\phi) - \mathbb{E}[a\phi]) = \mathbb{E}_{\mathcal{B}^P}[\Delta(\phi) - (p^* - a)(\phi - qx)] + \mathbb{E}_{\mathcal{S}^P}[\Delta(\phi) + (r^* - a)(qx - \phi)] \geq 0 \tag{6}$$

Indeed, depending on the price levels, mainly if the spread $p^* - r^*$ is large, the sets $\mathcal{B}^P$ and $\mathcal{S}^P$ may be empty and the agents are in the same conditions as in the no platform case. But when the sets $\mathcal{B}^P$ and $\mathcal{S}^p$ are not empty, for an agent with a load factor ϕ, both terms on the RHS of (6) are not negative. So for these states of nature, an agent with a load factor ϕ has a greater surplus trading with peers on the platform than with the grid, so (6) holds.

Assume now that $q\mathbb{E}[ax] > k$, so that all agents install a DPU without a platform. They have an incentive to invest as long as $I_0 = q\mathbb{E}[ax] - k > 0$. However, when connected to the platform, $I_P(\phi)$ is never less than I_0 as $I_P(\phi) = I_0 + \mathbb{E}[U] - (u(\phi) - \mathbb{E}[a\phi])$ and (6) holds. The following proposition sums up the previous discussion.

Proposition 1 *If all agents would install a DPU if there were not platform, then they will not be worse off if a dealing platform is active. They will also invest in DPU in this case.*

Even if the energy prices are less favourable, the intrinsic and differentiated services provided by the platform (safer distribution, local trades, traceability or just sharing renewable sources) as well as the grid cost savings, lead some prosumers to use the platform to trade their domestic production. The intuition that drives Proposition 1 is that on top of the cost–benefit trade-off for any prosumer to install the DPU (being a trader on the platform or not), there are now further gains and costs to participating in P2P trading for some agents. These gains come from the intrinsic values of participation and grid cost savings. The costs are market based: the prices on the platform for purchasing or selling electricity are less favourable than those from the grid. However, installing a DPU for trading with peers allows triggering these gains and avoiding those costs at least for some states of nature. In the end, the platform cost–benefit trade-off is positive for all agents.

Indeed, if no agent would be a prosumer without a platform, i.e. if $q\mathbb{E}[ax] < k$, then, with a dealing platform, there is room for some agents to install the DPU, that is, for which the platform cost–benefit trade-off is positive. So there exists a set of agents $\Phi^* \subset [\underline{\phi}, \bar{\phi}]$, for which $I_P(\phi) > 0 > I_0$. However, one cannot state generally what kind of agents will be concerned (low or high load profile).

Corollary 1 *(i) If no agent would install a DPU when there is no platform, then when there is a dealing platform, there are some agents who do install DPU and are not worse off than they would have been without the platform.*
(ii) Without further information on the distribution of the intermittency state of nature, one cannot assess which set of load profiles will be better off.

To understand this result, let us analyse the shape of such incentives to install capacity with respect to the load profile of agents. Indeed, the variations of those incentives are a non monotonic function of ϕ:

$$I_P'(\phi) = \Delta'(\phi) - \mathbb{E}_{\mathcal{B}^P}[p^* - a] + \mathbb{E}_{\mathcal{S}^P}[a - r^*]$$

This depends first on the marginal utility from being "more" served within the platform Δ' which is negative (as u is concave). Second, it relies on the relative price spreads $p^* - a$ and $a - r^*$ at each state and also on the skewness of the distribution of the states of nature. On the one hand, agents with higher load profiles will be buyers more often (at the margin) and accordingly on the platform, then will have to pay the premium $p^* - a$ as a cost of sourcing. This reduces their incentive to invest at the margin, i.e. $-\mathbb{E}_{\mathcal{B}^P}[p^* - a] < 0$. On the other hand, agents with higher load profiles will be sellers on the platform less often, and so they will not have to bear shortfalls resulting from selling to the platform, which increases their incentive to invest, i.e. $\mathbb{E}_{\mathcal{S}^P}[a - r^*] > 0$ at the margin.

When $I_P'(\phi) < 0$, for all ϕ, then $\Phi^* = [\underline{\phi}, \phi^*]$, prosumers connected to the platform are those who have low load profiles (i.e. small consumers), and they are motivated by a *selling argument* to participate and install DPU: the shortfall $a - r^*$ is not so important for them. Big consumers are not interested in participating, for which the premium $p^* - a$ is too costly for them. When $I_P'(\phi) > 0$, then $\Phi^* = [\phi^*, \bar{\phi}]$, so prosumers connected to the platform are those who have high load profiles (i.e. big consumers), who are motivated by a *consumption argument* to participate.

To finish, in order to go beyond point *(ii)* in Corollary 1 and to illustrate the previous discussion on the shape of the incentive to invest in DPU with respect to the load profile ϕ, we give a specific example of our model. Mainly it shows how some kind of bell shapes can be found for $I_P(\phi)$ using uniform distributions for ϕ and x, which can be viewed as neutral configurations with respect to variability. Indeed, let us consider the following specifications: $n = 1$, $u(\phi) = v$, $u(\phi + \delta) = v + \delta$, $F(x) = x$ and $G(\phi) = \frac{\phi - \underline{\phi}}{\bar{\phi} - \underline{\phi}}$. As a result $\mathbb{E}(\phi) = \frac{1}{2}\left(\bar{\phi} + \underline{\phi}\right)$ and one can deduce that if $\phi \leq \mathbb{E}(\phi)$, then $I'_P(\phi) = \frac{\delta+\tau}{q}\left(\ln\left(\frac{\bar{\phi}-\underline{\phi}}{\phi-\underline{\phi}}\right) + \ln\left(\frac{\bar{\phi}-\underline{\phi}}{\bar{\phi}-\phi}\right) - 2\ln 2\right) \geq 0$ and if $\phi \geq \mathbb{E}(\phi)$, then $I'_P(\phi) = \frac{\delta+\tau}{q}\left(2\ln 2 - \ln\left(\frac{\bar{\phi}-\underline{\phi}}{\phi-\underline{\phi}}\right) - \ln\left(\frac{\bar{\phi}-\underline{\phi}}{\bar{\phi}-\phi}\right)\right) \leq 0$. In some sense, bell shapes for $I_P(\phi)$ indicates that the incentive to install DPU increases with the load profile for small consumers but decreases for big consumers. Indeed, an increase in the load factor (i.e., $d\phi$) modifies the consumption profile in the sense that, for a given DPU size q, the agent becomes a buyer at the margin. So for small consumers, i.e., $\phi \leq \mathbb{E}(\phi)$, the *selling argument* dominates to participate and install DPU, but that is becoming less and less so when the load factor increases. For big consumers i.e., $\phi \leq \mathbb{E}(\phi)$, the *consumption argument* dominates more and more when the load factor increases. In this setting, only prosumers with medium load factor will install a DPU when there is a platform.

Policy tools Finally we look at the effects of policy tools that are usually implemented and how they can help in promoting or deterring the development of energy P2P platforms.

For instance, one first can imagine that some subsidization schemes are implemented by governments in order to promote P2P platforms for environmental or innovative concerns. A simple lump sum subsidy for each DPU installed will have the effect of reducing the installation cost k and of course will directly increase the incentive for prosuming. However, this effect is not amplified by the existence of a P2P platform.

Price subsidization schemes could be more effective. Indeed, a unit rebate ρ on the purchasing price so that the price paid would be $p - \rho$, or a premium for the selling price so that the price paid would be $r + \rho$, would enhance demand and/or supply on the platform.[18] These premia and rebates have direct effects on the incentives for prosuming $I_P(\phi)$ as they influence positively the relative price spreads. However, they are bounded instruments, since, depending on the state of nature for the availability of DPU, a flat rebate or flat subsidy may be ineffective at some point. For instance, in unfavourable conditions of availability, i.e. x low, Lemma 2 indicates that the selling price is set to the upper bound $\bar{r}$ for which all energy in excess is supplied within the platform. In this case, adding a premium would not change the supply and so the selling price remains unchanged. The same applies for the purchasing price in favourable conditions, x high.

Finally, another way is to increase the grid price through directed taxation. This policy may have positive effects as it increases the total expected cost savings for a prosumer who had installed a DPU, i.e. $q\mathbb{E}[ax]$, and it decreases the purchase price spread. However, this also deflates the selling price spread which is a driver for prosuming, in favourable conditions.

[18] These rebates or premia call for compensation to the platform.

6 Extensions

Some extensions of the basic framework will now be developed in order to challenge our main result in Proposition 1. First, we consider zero-pricing within the platform. Second, we discuss the effect of a for-profit platform. Third, we look at a more sophisticated way to realize trades for prosumers, considering a matching platform. Fourth, we drop the assumption that the size of the DPU is fixed: it can now vary with the load profile in order to be adapted to the basic consumption profile of each agent. Last, we alter the analysis to take into account the impacts of individual shocks on prosumers.

Zero-pricing

An argument sometimes put forward to justify the emergence of these platforms is that to some extent participants could exchange energy for free because first the short run marginal cost of generation for DPU based on DRES is near zero and also they could benefit from a certain reciprocity within the community. Of course, one could argue that zero pricing is detrimental to investment in local generation capacities.

First of all, a permanent zero pricing scheme is not generally possible, except in one (potential) state of nature for which $D(0) = \bar{d} = S(0) = s(0)$, which implies that it cannot be supported as an equilibrium for each state. Second, a unilateral zero pricing scheme (i.e. $p = 0$ or $r = 0$, $\forall x$) is not feasible, as, for instance, when $x \leq \hat{x}, D(0) = \bar{d} > \bar{s} > s(r)$, there are not enough sellers on the platform to serve the high demand. However, a zero pricing scheme can be achieved. Indeed, if $p = 0$ for all $x \geq \hat{x}$, it is equivalent in terms of demand of a minimal pricing $p^* = \underline{p}$, and also in terms of local welfare.[19] So the platform can propose an optimal selling price $r^* : S(r^*) = D(0) = \bar{d}$. However, the same *does not* apply if $r = 0$ for $x \leq \hat{x}$. Indeed a market equilibrium is achievable by posting a price $p_z : D(p_z) = S(0)$ as $\bar{d} > \bar{s} > S(0)$, but it is no longer optimal, and $p_z > p^*$. Then the incentives to install DPU are now

$$\begin{aligned} I_Z(\phi) &= \mathbb{E}_{\mathcal{B}_z^P}\left[\Delta(\phi) - (p_z - a)(\phi - qx)\right] + \mathbb{E}_{\mathcal{B}_0^P}\left[\Delta(\phi) + a(\phi - qx)\right] \\ &\quad + \mathbb{E}_{\mathcal{S}_0^P}\left[\Delta(\phi) - a(qx - \phi)\right] + \mathbb{E}_{\mathcal{S}_*^P}\left[\Delta(\phi) + (r^* - a)(qx - \phi)\right] \end{aligned}$$

where $\mathcal{B}^P$ and $\mathcal{S}^P$ are subdivided into $\mathcal{B}_z^P = [\xi_b^z(\phi), \hat{x}]$, $\mathcal{B}_0^P = [\hat{x}, \phi/q]$; $\mathcal{S}_0^P = [\phi/q, \hat{x}]$ and $\mathcal{S}_*^P = [\hat{x}, \xi_s(\phi)]$ where $\xi_b^z(\phi)$ is higher than $\xi_b(\phi)$ in the optimal case, so $\mathcal{B}_z^P \subset \mathcal{B}_*^P$. First of all, we see that $I_Z(\phi)$ is positive for all ϕ as the trade sets are empty all together. Second, compared to the optimal case, zero pricing reduces these incentives in selling periods (the shortfall is not smaller) but increases them during buying periods only when the DPU availability is high. For low availability, a zero selling price implies huge purchase

[19] Of course the platform will not break even.

price increases, that drive consumers to turn to the grid. As a result, it is clear that $I_Z(\phi) < I_P(\phi)$.

Proposition 2 *Zero pricing creates less than optimal incentives but more than without a platform.*

To sum up, zero pricing is not detrimental to investment in local generation capacities, but creates low powered incentives. As a result, zero pricing cannot be the clincher in the creation and growth of energy platforms.

For-profit platform

In the main analysis, we assumed a welfare maximizing dealing platform. We have seen in Section 2 that some P2P energy trading platforms have been developed by private investors or start-ups which are for-profit organizations. Let us now suppose that the platform has a profit objective that can be written

$$\pi(x) = p(x)\, D(p(x)) - r(x)\, S(r(x)).$$

One can see that the platform as a dealer is a local node acting as an upstream monopsony and a downstream monopoly. The for-profit platform problem in x is then

$$\max_{p,r} \pi(x) \quad \text{s.t.} \quad D(p) = S(r)$$

which leads to an integrated monopsony-monopoly (interior) equilibrium[20]

$$\frac{p^d - r^d}{p^d} > \frac{1}{\eta_D} \text{ and } S\left(r^d\right) = D\left(p^d\right)$$

where η_D is the price elasticity of demand.

One can write the following proposition:

Proposition 3 *With a for-profit platform, prices denoted* $\left(p^d, r^d\right)$ *are such that*

$$p^d > p^m > p^* \geq \underline{p} > a > \bar{r} \geq r^* > r^d \geq 0$$

where p^m *would be the monopoly-side price and* $r = 0$ *the monopsony-side price (free purchase). The incentive to adopt a DPU is reduced compared to the non-profit platform, and then some prosumers do not invest any more, whereas they would do so without a platform.*

[20] This standard analysis of price setting by an intermediary can be found in Spulber (1999) for instance.

As a (non exclusive) dealer, the platform has both upstream and downstream market power which implies more market power than a stand-alone monopoly or monopsony. Hence compared to the grid price and the optimal prices, it both increases the energy price paid by consumers that are served through the platform and decreases the energy price received by prosumers that sell their energy in excess. These markups are possible as they incorporate partially the value of participating in the platform. In this case, the incentives to invest in DPU for an agent ϕ are $I_P^d(\phi) = \max\{\mathbb{E}[U] - k - (u(\phi) - \mathbb{E}[a\phi]), 0\}$ and comparing with the non-profit platform, that is the expression (6), when $q\mathbb{E}[ax] = k$, this leads to

$$\begin{aligned} I_P^*(\phi) - I_P^d(\phi) &= \mathbb{E}_{\mathcal{B}_*^P}[\Delta(\phi) - (p^* - a)(\phi - qx)] + \mathbb{E}_{\mathcal{S}_*^P}[\Delta(\phi) + (r^* - a)(qx - \phi)] \\ &\quad -\mathbb{E}_{\mathcal{B}_d^P}\left[\Delta(\phi) - \left(p^d - a\right)(\phi - qx)\right] - \mathbb{E}_{\mathcal{S}_d^P}\left[\Delta(\phi) + (r^d - a)(qx - \phi)\right] \geq 0 \end{aligned}$$

where here the subscripts d and $*$ refer to the for-profit platform and welfare maximizing cases, respectively. Therefore $\mathcal{B}_d^P \subset \mathcal{B}_*^P$ and $\mathcal{S}_d^P \subset \mathcal{S}_*^P$ as $a < p^* < p^d$ and $r^d < r^* < a$. Indeed, prices are set less "often" at their limit values (ceiling selling price and floor purchase price): the local energy costs more to purchase and gets less revenue from sales. As a result, prosumers trade less "often"[21] within the platform; so a for-profit platform generates less incentive to install DPU among prosumers. In the end, the platform cost–benefit trade-off is now less "often" positive for all agents. An important consequence is that if all agents would be prosumers even if there were no platform, i.e. $I_0 = 0$, then one may have $I_P^*(\phi) \geq I_0 = 0 > I_P^d(\phi)$: some prosumers do not invest any more when the platform is for-profit.

Matching platform

We look at a different way prosumers can find electricity through the platform, namely, if the dealer is now also matchmaker. The justification of this alternative assumption is twofold. On the one hand, as P2P energy trading platforms are supposed to mimic superstar digital platforms (such as Uber, Blablacar, and so on), matching can become a central issue of their business models. On the other hand, it comes from the standard literature on P2P markets where the matching process is at the core of the analysis. Indeed, P2P trading issues are now mainly analyzed using two-sided models which are more sophisticated than the simple dealing one.[22] In such environments, participants derive a utility from the basic trades but also from the origin or the destination of energy flows, this is akin to cross-side externalities in two-sided settings. They also value the characteristics of peers to whom they are connected, such as location and so on. As a result, even if there is sufficient local energy in excess at a moment of time, a transient form of mismatch can occur, implying

[21] That is to say, they are active on the platform in a narrower set of states of nature.

[22] For istance Khorasany *et al.* (2021) design a non-mediated negotiation algorithm which allows prosumers to select their trading partners, and negotiate directly with them through the platform.

that trades are completed only with a certain probability. What are the impacts of such a platform design on the incentives to participate, i.e., to install a DPU, for prosumers?

Following Goss *et al.* (2014), we assume that the platform is a closed environment in which the participants must declare themselves and install a DPU. In line with our main framework, we assume that the platform is non-profit, in the sense that it is welfare maximizing. Doing so they can be technically connected to the local micro-grid and at that time the matching's technology will make it possible to carry out exchanges between the participants (peer-to-peer exchanges) or if there is no match made between the participants and the central grid. The problem is to know which agents will participate in this platform, depending on the purchase and selling prices that the platform designer may choose, possibly one for all the states of nature.

The matching technology depends on the relative size of the potential supplies and demands to be matched in state x with a counterpart within the platform. Hence if there is a (endogenous) mass of buyers participating on the platform that corresponds to a mass D of energy to be consumed and a mass of sellers that corresponds to a mass S of energy to be supplied, then we assume that the total number of matches is given by the well-known matching function[23] $M = M(S, D)$.

As is standard in the matching literature, the matching function $M(S, D)$ is assumed to be twice continuously differentiable, weakly increasing and concave such that $M(S, 0) = M(0, D) = 0$ and $M \leq \min\{S, D\}$. The platform is a random matchmaker such that all participants on the same side have the same probability of being matched

$$m_B = \frac{M(S, D)}{D} \text{ and } m_S = \frac{M(S, D)}{S} \tag{7}$$

Under these weak regularity conditions, it has been shown that the match probability of buyers m_B is weakly decreasing in own-side participation D, which captures a negative own-side externality, and weakly increasing in cross-side participation S, which captures a positive cross-side externality. The same applies to m_S. A common example is $M(S, D) = S\,(1 - \exp(-D/S))$. Here the presence of the grid provides a non-zero outside option. It is useful to also define the matching elasticities for buyers and sellers, respectively, which can be written

$$\psi^B = \frac{M'_D(S, D)D}{M(S, D)} \text{ and } \psi^S = \frac{M'_S(S, D)S}{M(S, D)}$$

These numbers are lying in the interval $[0, 1]$ and are defined as the percentage increase in the mass of seller (respectively buyers) that match for a percentage increase in the mass agents active in buyer (respectively seller) side. Each type of elasticity reflects technological

[23]This matching process is clearly exogenous in this context. There is a growing literature grounding one-to-one and one to many matching procedures (see Chade *et al.*, 2017). However, the microfoundations of our setting, that is, many-to-many multidimensional matching with heterogeneous agents, have not yet been established (see however Gomes and Pavan (2016) for a primer). This is left for future research.

performances for a platform to create desirable matches according to the side. For instance, this may be related to the algorithmic efficiency or the performance of smart contracts.

On the dealer side, the necessity to maintain an overall grid balance implies that the matched demands and supplies must be equalized by the platform,[24] and the non-matched trade on the platform being ensured by the central grid. Hence, the platform proposes *ex ante* a menu of prices $(p(x), r(x))_{x\in[01,]}$ that balances energy exchanges within inner participants in each state x, that is[25]:

$$m_B D = m_S S \tag{8}$$

where here D is the potential energy demanded by participants from the platform in state x when price p is observed and S is the potential energy to be supplied when price r is observed in state x. As demand and supplies are in real time scale, potential demands and supplies can be viewed *ex post* as described by (3) and (4). Indeed, at each state of nature, the prosumer will prefer to trade with peers or with the grid, depending upon price conditions (a, p, r), so that they may demand or supply energy as in market conditions. For example, a smart contract can be signed with the matchmaker which states purchases and selling conditions for the prosumer.

In a matching process, the economic value arises through the fact of being matched to a peer only within the platform rather than being served through the grid. As a result, now the intrinsic value is affected by the probability of being served within the platform. To lighten the notation and the analysis, we simplify the model by assuming $u(\phi) = v + \delta$ within the platform and $u(\phi) = v$ outside. So for an agent with a load factor ϕ the expected utility from trading through the platform in state x is

$$\mathbb{U}(\phi, x, q) = \begin{cases} v + m_B\delta - (m_B p + (1 - m_B)a)(\phi - qx) & \text{if } \phi \geq qx \\ v + m_S\delta + (m_S r + (1 - m_S)a)(qx - \phi) & \phantom{\text{if }} \phi < qx \end{cases}$$

Then *ex post,* an agent whose expected utility is greater than the surplus of trading with the grid will only trade within the platform if $\mathbb{U}(\phi, x, q) \geq v - a(\phi - qx) - \tau$ so as explained above, we find again the same demand and supply as described by (3) and (4). So we can state $D = D(p)$ and $S = S(r)$. As a result we see from (7) that probabilities m_B and m_S depend now on both (p, r). These probabilities of being matched for buyers are increasing functions of p and r, whereas they are decreasing functions for sellers.

Therefore the platform pricing is now affected by the matching process, as the expected local welfare[26] is based only on the agents that are matched with peers prosumers.

[24] On this point we rely on the analysis of Benjaafar *et al.* (2018) concerning P2P car sharing.

[25] At the "rational-expectations" equilibrium, as suggested by Caillaud and Jullien (2003), this is always true.

[26] We go on with the convention that no markups are possible when the platform trades with the grid.

Compared to the pure dealing platform, the matching process implies two-sided effects of pricing schemes that create countervailing forces that may operate. Resolving the matching platform problem,[27] which is to maximize the welfare for each state x, a Pigovian pricing structure appears. Matching prices (p^{μ}, r^{μ}) are driven by the underlying matching technology and can be written as an average of the matching elasticities for a buyer or a seller. Weights in this average are the weighted net match valuation of buyers or sellers, that means the ratio between values and quantities traded for buyers or sellers.

For the matching platform, increasing the purchase price or selling price helps attract buyers but it repels sellers. Decreasing the prices does the reverse. Hence, depending on the relative strength of the matching elasticities, the platform will prefer to push up one price rather than another. So it can be the case that for some states of nature (mainly for intermediate values of x), both prices admit mark-ups in the sense that $p^{\mu} > \underline{p} > \bar{r} > r^{\mu}$. This well-known balancing mechanism is only possible if the matching technology exhibits decreasing and limited returns to scale, that is, when $\psi^B + \psi^S < 1$. If not, the pricing scheme will be bounded by the price limits $\underline{p}$ or $\bar{r}$, as demand and supply are also bounded in the platform.

On top of this Pigovian pricing structure, one can define matching elasticity thresholds ψ_*^B and ψ_*^S such that, for elasticities above these thresholds, matching prices can be more profitable for prosumers than the dealing ones, that is $p^{\mu} < p^*$ or $r^{\mu} > r^*$. This is at the heart of the following result.

Proposition 4 *The more elastic the matching technology is, the more the incentives to adopt DPU may increase, compared to the non-profit dealing platform.*

The proposition is quite intuitive. When the matching technology is rigid (i.e. $\psi^B \leq \psi_*^B$ and/or $\psi^S \leq \psi_*^S$), negative own-side externalities have a greater impact than positive cross-side externalities. As a result, this calls for increasing the purchase price towards the weighted net match valuation of buyers or decreasing to the one of sellers, respectively. When the matching technology is sufficiently elastic (i.e. $\psi^B \geq \psi_*^B$ and/or $\psi^S \geq \psi_*^S$), positive cross-side externalities are more effective, so this calls for decreasing the purchase price towards the floor price or increasing the selling price towards the price ceiling. If we finally turn to the incentives to install DPU created by the existence of the matching platform, these price effects are beneficial only if matching technology is sufficiently elastic. For an agent ϕ, these incentives (if positive) are defined again by $I_M(\phi) = \mathbb{E}[\mathbb{U}] - k - (v - \mathbb{E}[a\phi])$ but now it could be written

$$\begin{aligned} I_M(\phi) &= q\mathbb{E}[ax] - k \\ &\quad + \mathbb{E}_{\mathcal{B}}\left[m_B^{\mu}\left\{\delta + \tau - (p^{\mu} - a)(\phi - qx)\right\}\right] \\ &\quad + \mathbb{E}_{\mathcal{S}}\left[m_S^{\mu}\left\{\delta + \tau + (r^{\mu} - a)(qx - \phi)\right\}\right] \end{aligned}$$

[27] See appendix for details.

where for $j = B, S, m_j^\mu$ are the matching probabilities evaluated at (p^μ, r^μ). Again, compared to the no platform benchmark, the prosumers are not worse off. However, it is not clear if prosumers are more or less well off than with a dealing (welfare maximizing) platform, that is, if $I_M(\phi) \geq (\leq) I_P(\phi)$. Indeed, first if *ex ante* in all states of nature a potential match is possible, this has a positive effect on the incentives to install the unit. Second, if the matching technology is sufficiently elastic (i.e. $\psi^B \geq \psi_*^B$ and $\psi^S \geq \psi_*^S$), then prices tend to their respective bounds, which also may boost prosumer's investments. Of course, the reverse holds if the matching technology is rigid. Finally, the matching itself as an uncertain process creates a depressive effect on the incentives to invest. As a result we cannot directly assess which effect will dominate.

Variable capacities

To extend our main setting analysed in Sections 3 and 5, we now assume that agents can calibrate their DPU with respect to their load factor, that is, now q is a variable depending on ϕ. We will seek a continuous differentiable equilibrium path $q(\phi)$ where for each x. As q is a choice of an agent with profile ϕ, we now assume that the capacity up-front cost $k(q)$ is increasing and convex for a production capacity of q kWp.

First, when there is no platform, Lemma 1 still holds. Now the gross expected gain for a prosumer with profile ϕ is $\mathbb{E}[U_0] = u(\phi) - \mathbb{E}[a(\phi - q(\phi)x)]$, so the incentive to adopt becomes $I_0(\phi) = \mathbb{E}[U_0] - k(q)$, where the marginal opportunity benefit can be written as $\frac{\partial \mathbb{E}[U_0]}{\partial q(\phi)} = \mathbb{E}[ax]$ is constant in q. This imply that all agents will install the same capacity $q_0(\phi) = q_0$ such that $\mathbb{E}[ax] = k'(q_0)$ for all ϕ. Second, when a dealing platform is active, following a similar line of reasoning as in Section 5, one can again derive the dealer prices that are now driven by *each* capacity installed $q(\phi)$ for all profiles ϕ. The aggregate demand and supply are defined accordingly, taking into account that the switching load profile is now $\hat{\phi}_x : \phi = q(\phi)x$. The result in Lemma 2 still holds with the main change that $q = q(\phi)$ for each ϕ. This implies that the switching state $\hat{x}$ is now defined as $\hat{x} = \frac{\mathbb{E}(\phi)}{n\mathbb{E}(q(\phi))}$. Therefore the optimal capacity $q^*(\phi)$ maximizes $I(\phi) = \mathbb{E}[U] - k(q)$, the net expected surplus of an agent with a load factor ϕ, where $\mathbb{E}[U]$ is still defined by (5), replacing q by $q(\phi)$. Then this solution is driven by their marginal net gains from increasing the capacity, taking as given those of the other agents on the platform, that is: $\frac{\partial I(\phi)}{\partial q} = \frac{\partial \mathbb{E}[U]}{\partial q} - k'(q)$. Now in general, agents with different load factors connected to a platform will install different levels of capacity as $q^*(\phi)$ is such that

$$\begin{aligned}
\frac{\partial I(\phi)}{\partial q} &= \frac{\partial I_0(\phi)}{\partial q} + \mathbb{E}_{\mathcal{B}^P}[(p^* - a)x] - \mathbb{E}_{\mathcal{B}^P}\left[\frac{\partial p^*}{\partial q}(\phi - q^*(\phi)x)\right] \\
&+ \mathbb{E}_{\mathcal{S}^P}[(r^* - a)x] + \mathbb{E}_{\mathcal{S}^P}\left[\frac{\partial r^*}{\partial q}(q^*(\phi)x - \phi)\right] = 0
\end{aligned}$$

where $\mathcal{B}^G$ and $\mathcal{S}^P$ are subdivided into subsets depending on the platform pricing (as depicted in Lemma 2). Hence, one can see for each load profile ϕ, the marginal incentive to install DPU, i.e., $\frac{\partial I(\phi)}{\partial q}$, is in general different from its counterpart without a platform that is $\frac{\partial I_0(\phi)}{\partial q}$ as $q^*(\phi) \neq q_0$. We also see that the local impacts on dealing prices are $\frac{\partial p^*}{\partial q}$ and $\frac{\partial r^*}{\partial q}$ are key variables to promote or to dampen the installation of DPU by prosumers. One can prove that:

Proposition 5 *With variable capacities,*
(i) dealing price are (weakly) reduced when more capacity is installed for any agent.
(ii) The ceiling selling (resp. floor purchase) price increases only when more capacity is installed by the agent $\underline{\phi}$ (resp. $\bar{\phi}$).
(iii) DPU capacities are increasing with the load profile and any agent connected to a dealing platform might not have strictly superior marginal incentives to install DPU than without being connected.

Point *(i)* of Proposition 5 tells us that on one hand, installing more DPUs reduces the demand on the buyer-side as self-consumption is more likely, but on the other hand, it increases the supply on the seller-side. As a result, dealing prices are reduced, driven by changes in the supply and demand fundamentals. What is more surprising is, as stated by point *(ii)*, the corner prices $\bar{r}$ and $\underline{p}$ are positively affected by investments for the extreme agents in terms of the load. When the smallest consumer $\underline{\phi}$ invests, that agent increases the maximal supply achievable $\bar{s}$ at a given state and then also pushes up the maximum price. For the biggest consumer $\bar{\phi}$, investing reduces the maximal demand achievable $\bar{d}$ at a given state and so pushes down the minimum price. The consequences of these price effects is that a prosumer with a higher load profile will install a higher DPU capacity. The intuition is that a prosumer with higher electricity needs is more often a buyer than a seller and prefers to see a fall in the purchase price than a rise in the selling price, and so will prefer increasing own-DPU capacities to achieve this goal.

The last point of Proposition 5 depicts how, for a given agent, the marginal incentives to install DPU $\frac{\partial I(\phi)}{\partial q}$ depend on that agent's load profile. For all agents (except $\underline{\phi}$ and $\bar{\phi}$), the marginal incentives to install DPU are boosted by the positive marginal effects on the purchase expenditures, as[28] $\mathbb{E}_{\mathcal{B}^P}\left[(p^* - a)\,x\right] - \mathbb{E}_{\mathcal{B}^P}\left[\frac{\partial p^*}{\partial q}(\phi - qx)\right] > 0$. But they are dampened by the negative marginal effects they produce on revenues from sales, as $\mathbb{E}_{\mathcal{S}^P}\left[(r^* - a)\,x\right] - \mathbb{E}_{\mathcal{S}^P}\left[\frac{\partial r^*}{\partial q}(qx - \phi)\right] < 0$. For the smallest consumer $\underline{\phi}$, this is increased by $\mathbb{E}_{\mathcal{S}^P}\left[\frac{\partial \bar{r}}{\partial q}\left(qx - \underline{\phi}\right)\right] > 0$ and for the biggest consumer $\bar{\phi}$, it is reduced by $-\mathbb{E}_{\mathcal{B}^P}\left[\frac{\partial \underline{p}}{\partial q}\left(\bar{\phi} - qx\right)\right] < 0$. In the end, we see that $\frac{\partial I(\phi)}{\partial q} - \frac{\partial I_0(\phi)}{\partial q}$ is not always positive. For affiliated agents, the marginal incentives to increase domestic capacities are not always greater.

[28] We drop the argument ϕ.

Of course on average, the distribution of loads matters for identifying whether more or less total capacity will be installed. As for the main analysis with fixed capacities (see Corollary 1), without further precise information on the state of of intermittency, one cannot assess which prosumer will be better off.

Individual shocks

In the main model, we assume that the state of nature (i.e. intermittency) affects all prosumers in the same way, that is, they are not affected by individual shocks. However, one could argue that this assumption is no longer justified if P2P platforms are connecting people that are located in different places.[29] What would some heterogeneity in DPU ability do to the effectiveness of the platform? To analyse this, we will now assume that prosumers are affected by the state of nature x in different ways, i.e. the availability of the renewable capacity is distributed according to a cdf which is also dependent on the load profile, that is, $F(x, \phi)$. As a result, prior each prosumer is facing a different external conditions. However, one can see that the result in Proposition 1 is not deeply affected by this setting and one can state the result.

Proposition 6 *When agents are affected by individual shocks, Proposition 1 still holds.*

Indeed, with no platform, the incentives to invest in DPU for an agent ϕ are now $I_0(\phi) = \max\{q\mathbb{E}_\phi[a(x)\,x] - k, 0\}$, where $\mathbb{E}_\phi$ are expectations over x for an agent with profile ϕ. As a result, Lemma 1 is no longer valid in the sense that now some agents may prefer not become prosumer, depending on the DPU capacity q, the capacity up-front cost k and the grid pricing $a(x)$. A typical example is when $\mathbb{E}_\phi$ is a monotone increasing function of ϕ, that if small consumers face unfavourable DPU generation conditions and large consumers face favourable ones. In this case, only large consumers will prefer becoming prosumers and install the DPU capacity, i.e. $\phi \geq \phi_0 : I_0(\phi_0) = 0$. In contrast, when $\mathbb{E}_\phi$ decreases with ϕ, small consumers will prefer to install the DPU capacity. However, one cannot easily generalize such examples, and indeed $\mathbb{E}_\phi$ may be very non monotonic with respect to ϕ.

Nevertheless, considering the gains from joining a dealing platform for an agent who would install DPU capacity without it, a weak version of Proposition 1 holds: if an agent would install a DPU when there is no platform, then that agent will do so and will not be worse off when the dealing platform is active. Indeed, now $I_P(\phi) = I_0(\phi) + \mathbb{E}_\phi[U] - (u(\phi) - \mathbb{E}_\phi[a\phi])$ so if $I_0(\phi) > 0$, $I_P(\phi)$ does, as (6) still holds, replacing $\mathbb{E}[.]$ by $\mathbb{E}_\phi[.]$. In some sense this could justify the creation of communities of prosumers from an individual point of view.

[29]This is the idea of Internet Energy proposed by Rifkin, mentioned in the Introduction, and also illustrated by the Walqa–Altenea experiment, see Section 2.

7 Conclusion

In this paper, we provided the first economic analysis of how new models of peer-to-peer exchanges in the electricity sector may be effective and may yield sufficient incentives to invest in DPU based on renewable energy sources. We analysed how a P2P energy trading system could lead to a desirable economic outcome for a community of prosumers. We provided a simple model by first considering heterogeneous prosumers with respect to their energy needs and, second, intermittency in the production of electricity based on distributed renewable energy sources. In this context, we determined the equilibrium price levels for a welfare maximizing dealing platform, showing that the purchase prices are always greater than the central grid price, and selling prices are always lower. However, the expected net gains to be a trader within the platform are always greater than the expected average price on the grid. On top of this, we determined the optimal incentives for participants to adopt DPUs in P2P energy trading platforms, showing that they are always greater than the incentives when there is no platform. The intuition is that investing in DPUs allows prosumers to increase their expected net gains to participate in local trading. As a result, a welfare maximizing platform will always be profitable for a community of prosumers.

This strong result is challenged when we consider some extensions of our analysis. First, zero pricing is considered. There is a common reciprocity argument put forward to justify the emergence of these platforms, by remarking that the short run marginal cost of DRES generation is near zero. We show that zero pricing creates less incentives than the optimal ones, but more than those without a platform. Indeed, as both selling and purchase prices cannot be zero at any time, this depends on the conditions of availability of the DPU. If they are favourable, a zero purchase price has no effect compared to the dealing platform. By contrast, if they are unfavourable, a zero selling price implies huge purchase price increases, that drive consumers to turn to the grid. Second, as a number of P2P energy trading platforms have been developed by private investors or start-ups, one could think about for-profit platforms rather non-profit ones. With market power on both sides, such a platform will further increase purchase prices and lower selling prices, reducing the incentives to adopt DPU compared to the non-profit case. Consequently, the platform cost-benefit trade-off is now less “often” positive for all agents, and some prosumers do not invest any more whereas they would have done so without a platform. Third, since P2P energy trading platforms learn their business models from digital platforms, matching will become a central issue. Considering a matching platform rather than a dealer affects the market outcomes and incentives to adopt DPU. Mainly, we have found that the more the matching technology is efficient to match sellers and buyers, the more the incentives to adopt DPU may increase, compared to a pure dealer. With a more efficient technology, more matches are realized, so purchase prices tend to be reduced and selling prices to be increased which may boost prosumer’s investments.

Finally, we also challenged our main result by assuming that a variable DPU size was possible and that intermittency shocks differ between prosumers. We showed that with variable capacities, DPU capacities are increasing with the load profile, but the main result does not always hold. There are two effects on platform prices of a marginal capacity increase: a positive one on the purchase expenditures price (as the purchase price decreases) and a negative one on the revenues from sales (as the selling price also decreases). Then the marginal incentive to install DPU is boosted by the former and dampened by the latter. When individual shocks are considered, the main result still holds (in a weak version): if an agent would have installed a DPU if there were no platform, then that same agent will do so and will not be worse off when a dealing platform is active. This comes from the fact that for a given agent, individual shocks are intended to be the same with or without a platform, so expected gains are specific of each prosumer and then the main result is not affected.

Our results have some economic implications for the regulator and for economic policy. First, as we have shown that P2P energy trading platforms have some economic importance and local efficiency, regulators should ensure a level playing field for platform-based businesses and governments should support their emergence, in order to reap the benefits of P2P electricity trading. To some extent, this has been initiated in Europe, as the European Commission has defined P2P trading of renewable energy in EU Directive 2018/2001. However, this is not applied worldwide, for instance in the United States, where only microgrids are eligible for P2P trading. This has limited the implementation of the LO3 Energy (the Brooklyn experiment) in the public distribution network. Second, we have discussed the role of the external grid price in the design of the platform and as a determinant of prosumer' participation and investments in DRES. Usually, at least one component of this external grid price is subject to regulation by the energy authorities. As a result, the integration of P2P trading platforms in the overall electric system could be subject to disproportionate or non-discriminatory (network) charges or procedures. This is a real challenge for regulators to ensure such a grid neutrality and transparency for the external grid price as in the future, P2P trading platforms could complement the wholesale electricity market.

Some issues have been left aside and our analysis could be extended along at least three lines. First, we have supposed the external grid pricing as totally exogenous. Grid pricing issues when prosumers are active have been analysed by Gautier *et al.* (2018) but without considering a P2P trading system. The interdependence between the platform price equilibrium and grid pricing (regulated or market-based) may have strong impacts on both investments by prosumers and the network profitability of the grid. Following Abada *et al.* (2020b), one may anticipate the existence of a snowball effect between the expansion of platforms and the external grid price. The main idea is that depending on whether the grid pricing is based on the average cost or on the marginal cost, the contraction of grid exchanges due to the existence of the trading platform may respectively increase or decrease the supply price to the grid. In return, this modifies the incentives for potential prosumers to install DPU.

Second, we have seen that P2P trading platforms are often located within microgrids. As a result, an issue is how the platform or the connected agents may provide electricity backups (batteries and storage capacities) instead of withdrawing/injecting electricity from/to the grid. As the first step, for each prosumer there will appear a trade-off between the storage costs for withdrawal/injection and the external grid price viewed as an opportunity cost.

In our analysis we have considered the information ICT devices and blockchain technologies needed for the effectiveness P2P trading platform merely as black boxes. What are the improvements expected with ICT and blockchain technologies with smart contracting? This is left for further research.

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Appendix

Demand and Supply

Demands. From (2), one can derive

$$\begin{array}{rcll} qx & \leq & \phi \leq \beta(p) & \text{if } p > a \\ \phi & \geq & qx > \beta(p) & \text{if } p \leq a \end{array}$$

If $p \leq a$, then all the agents connected to the platform will demand energy within it and then the demand is rigid i.e. $\bar{d} = \int_{qx}^{\bar{\phi}} (\phi - qx)\, dG$. If $p > a$, the demand is price-sensitive and we denote $\beta(p)$, the value of ϕ such that

$$\phi = qx + \frac{\Delta(\phi)}{p-a}$$

with $\beta'(p) = -\frac{\Delta/(p-a)^2}{1-(\Delta')/(p-a)} < 0$ and $\beta''(p) > 0$. Moreover, it may exist a maximum value of $\underline{p} > a$ such that $\beta(\underline{p}) = \bar{\phi}$ with $\frac{dp}{da} = 1$, more precisely

$$\underline{p} = a + \frac{\Delta(\bar{\phi})}{\bar{\phi} - qx} \tag{9}$$

Using the notation $\dot{y} = \frac{dy}{dx}$, note that $\dot{\underline{p}} - \dot{a} = \frac{\left[u(\bar{\phi}+\delta)-u(\bar{\phi})\right]}{(\bar{\phi}-qx)^2} q > 0$. So if $p \geq \underline{p}$, the demand to the platform is

$$d(p) = \int_{qx}^{\beta(p)} (\phi - qx)\, dG = \int_{qx}^{\beta(p)} \left[G(\beta(p)) - G(\phi)\right] d\phi$$

with $d'(p) = \beta'(p)(\beta(p) - qx)\, g(\beta(p)) < 0$. As expected, the demand is normal (downward sloping). Note that this demand is always positive as we assumed that $\bar{\phi}/q \geq 1$.Note that with this vertical differentiation framework, the demand is never choked off. So the demand at state x is such that

$$D(p) = \begin{cases} \bar{d} & \text{if } p \leq \underline{p} \\ d(p) & \phantom{\text{if }} p > \underline{p} \end{cases}$$

Note that for all p, $\dot{\bar{d}} = -q(n - G(qx)) < 0$ with $\bar{d} = \mathbb{E}(\phi)$ if $x \leq \underline{\phi}/q$.

Supplies. For agents such that $\phi \leq qx$, an agent ϕ will be a (extra) supplier within the platform if $U(\phi, x, q) \geq \underline{U}(\phi, x, q)$. This implies

$$\Delta(\phi) + r(qx - \phi) \geq a(qx - \phi)$$

we denote $\sigma(r)$, the value of ϕ such that

$$\sigma(r) = qx - \frac{\Delta(\phi)}{a - r} \geq 0 \text{ if } r < a$$

with $\sigma'(r) < 0$. If $r > a$ then $\sigma(r) = \underline{\phi}$ and all potential sellers are suppliers to the platform. Moreover, it may exist a maximum value of $\bar{r} < a$ such that $\sigma(\bar{r}) = \underline{\phi}$ with $\frac{d\bar{r}}{da} = 1$. Namely

$$\bar{r} = a - \frac{\Delta(\underline{\phi})}{qx - \underline{\phi}} \tag{10}$$

note that $\dot{\bar{r}} - \dot{a} = \frac{\delta}{qx - \underline{\phi}} > 0$. Then if $r \geq \bar{r}$, the supply is rigid and equal to $\bar{s} = \int_{\underline{\phi}}^{qx} (qx - \phi)\, dG$. When $0 \leq r \leq \bar{r}$, the supply to the platform is

$$s(r) = \int_{\sigma(r)}^{qx} (qx - \phi)\, dG = \int_{\sigma(r)}^{qx} [G(\phi) - G(\sigma(r))]\, d\phi$$

Note that $s'(r) = -\sigma'(r)(qx - \sigma(r))\, g(\sigma(r)) > 0$, the supply is upward sloping. Of course, this supply is zero if $x \leq \underline{\phi}/q$. Note that as $\sigma(0) < qx$ then $S(0) > 0$: there are always sellers willing to sell electricity for free. Finally the supply at state x is such that

$$S(r) = \begin{cases} \bar{s} & \text{if } r \geq \bar{r} \\ s(r) & \phantom{\text{if }} r < \bar{r} \end{cases}$$

Note that for all r, $\dot{\bar{s}} = qG(qx) > 0$ and $\bar{s} = 0$ if $x \leq \underline{\phi}/q$.

Proof of Lemma 2

If $p < \underline{p}$ then $D(p) = \bar{d}$ and $r^* = S^{-1}(\bar{d})$ so using the mean theorem $W(x) = u(\bar{\nu} + \delta)\{n - G(\sigma(r^*))\}$, where $\bar{\nu} < \bar{\phi}$. If $r > \bar{r}$ then $S(r) = \bar{s}$ and $p^* = D^{-1}(\bar{s})$ so $W(x) = u(\underline{\nu} + \delta)\, G(\beta(p^*))$, where $\underline{\nu} > \underline{\phi}$, where $\bar{\nu} > qx > \underline{\nu}$. When $p \geq \underline{p}$ and $r \leq \bar{r}$, and $D(p) = S(r)$, let us denote the Lagrangean $L = W(x) + \lambda(d(p) - s(r)) + \lambda_p(p - \underline{p}) + \lambda_r(\bar{r} - r)$, with $\lambda \neq 0$, a Lagrange multiplier, others $(\lambda_p, \lambda_r) \geq 0$, Khun-Tucker multipliers, first-order conditions imply:

$$\begin{aligned}
\frac{\partial L}{\partial p} &= \{u(\beta(p) + \delta) + \lambda(\beta(p) - qx)\}\, g(\beta(p))\, \beta'(p) + \lambda_p = 0 \\
\frac{\partial L}{\partial r} &= \{-u(\sigma(r) + \delta) + \lambda(qx - \sigma(r))\}\, g(\sigma(r))\, \sigma'(r) - \lambda_r = 0 \\
\frac{\partial L}{\partial \lambda} &= d(p) - s(r) = 0 \\
\lambda_p \frac{\partial L}{\partial \lambda_p} &= \lambda_p(p - \underline{p}) = 0 \\
\lambda_r \frac{\partial L}{\partial \lambda_r} &= \lambda_r(\bar{r} - r) = 0
\end{aligned}$$

which gives

$$\begin{aligned}
\{u(\beta(p)+\delta)+\lambda(\beta(p)-qx)\}\, g(\beta(p))\, \beta'(p) &= -\lambda_p \le 0 \\
\{-u(\sigma(r)+\delta)+\lambda(qx-\sigma(r))\}\, g(\sigma(r))\, \sigma'(r) &= \lambda_r \ge 0
\end{aligned}$$

so if $p > \underline{p}$ and $r < \bar{r}$ then $(\lambda_p, \lambda_r) = (0,0)$ and we have the contradiction: $\lambda = u(\sigma(r)+\delta)/(qx-\sigma(r)) > 0$ and

$$0 < u(\beta(p)+\delta) + \lambda(\beta(p)-qx) = 0$$

If $p > \underline{p}$ and $r = \bar{r}$ then $\lambda_p = 0$ so $\lambda = -u(\beta(p)+\delta)/((\beta(p)-qx)) < 0$ and

$$\lambda_r = -\left\{u(\underline{\phi}+\delta) + u(\beta(p)+\delta)\frac{qx-\underline{\phi}}{\beta(p)-qx}\right\} g(\underline{\phi})\, \sigma'(\bar{r}) > 0$$

with $p^* = d^{-1}(\bar{s})$, this implies that $\bar{s} = s(\bar{r}) > \bar{d}$. When $p = \underline{p}$ and $r < \bar{r}$ then $\lambda_r = 0$ so $\lambda = u(\sigma(r)+\delta)/(qx-\sigma(r)) > 0$ and

$$\lambda_p = -\left\{u(\bar{\phi}+\delta) + u(\sigma(r)+\delta)\frac{\bar{\phi}-qx}{qx-\sigma(r)}\right\} g(\bar{\phi})\, \beta'(\underline{p}) > 0$$

with $r^* = s^{-1}(d)$ whenever $\bar{s} < \bar{d}$. Finally if $p^* = \underline{p}$ and $r^* = \bar{r}$ for which it may exist a level of $x = \hat{x} : \bar{s} = \bar{d}$, such that

$$\bar{s} - \bar{d} = \int_{\underline{\phi}}^{\bar{\phi}} (\phi - qx)\, dG = \mathbb{E}(\phi) - nqx = 0$$

here expectations are over ϕ, so

$$\hat{x} = \frac{\mathbb{E}(\phi)}{nq}$$

As a result there are a multiple of solutions. If $x \le \hat{x}$ then $p^* = d^{-1}(\bar{s}) > 0$ and $r^* \in [\bar{r}, +\infty]$ and if $x \ge \hat{x}$ then $p^* \in [0, \underline{p}]$ and $r^* = s^{-1}(\bar{d}) > 0$. Of course if we add a breakeven constraint for the platform account $\pi(x) = (p-r)\min\{\bar{d}, \bar{s}\} \ge 0$ then this restrict the set of optima to $p^* \ge r^*$. This restrict selling prices to $r^* \in [\bar{r}, p^*]$ when $x \le \hat{x}$ and purchase prices to $p^* \in [r^*, \underline{p}]$ when $x \ge \hat{x}$.

Proof of Proposition 3

From the market clearing condition $D(p) = S(r)$ one can define a locus $\hat{r}(p)$ such that

$$S(\hat{r}(p)) = D(p)$$

which entails $r^*(p)$ decreasing in $p \in [\underline{p}, \infty[$ whenever $S(0) < D(\underline{p})$ i.e.

$$D(\underline{p}) - S(0) = \int_{qx-\frac{\Delta}{a}}^{\bar{\phi}} (\phi - qx)\, dG > 0$$

So the dealer problem writes $\max_{p \ge \underline{p}} (p - \hat{r}(p))\, D(p)$ and the first order condition gives:

$$\begin{aligned}
\frac{p^d - \hat{r}(p^d)}{p^*} &= \frac{1-\hat{r}'(p^d)}{\eta_D} > \frac{1}{\eta_D} \\
S(\hat{r}(p^*)) &= D(p^*)
\end{aligned} \tag{11}$$

where

$$\eta_D = -\frac{D'(p)\,p}{D(p)} > 0$$

so $p^d \geq p^m$. As a result

$$p^d > p^* \geq \underline{p} \text{ and } \bar{r} \geq r^* > r^d$$

Proof of Proposition 4

In order to derive the result in the Proposition, we need to determine (p^μ, r^μ) the dealer equilibrium price when matching applies. Let us denote the *net* expected utility of trading within the platform by

$$\mathbb{V}(\phi, x, q) = \begin{cases} m_B\left(\delta + \tau - (p-a)(\phi - qx)\right) & \text{if } \phi \geq qx \\ m_S(\delta + \tau + (r-a)(qx - \phi)) & \phantom{\text{if }} \phi < qx \end{cases}$$

So the platform welfare writes

$$\mathbb{W}(x) = \int_{\underline{\phi}}^{\bar{\phi}} \mathbb{V}(\phi, x, q)\, dG + \pi(x)$$

where $\pi(x) = (p-a)\, m_B D(p) - (r-a)\, m_S S(r)$ that is $\pi(x) = (p-r)\, M(S(r), D(p))$, so we can rewrite the platform welfare as

$$\mathbb{W}(x) = (\delta + \tau)\left[m_B(p,r)\left\{G(\beta(p)) - G(qx)\right\} + m_S(p,r)\left\{G(qx) - G(\sigma(r))\right\}\right]$$

Assume that $p > \underline{p}$ and $r < \bar{r}$, then (interior) first order conditions write:

$$\frac{\partial \mathbb{W}(x)}{\partial p} = 0 \text{ and } \frac{\partial \mathbb{W}(x)}{\partial r} = 0$$

Derivatives write

$$\begin{aligned}
\frac{\partial \mathbb{W}(x)}{\partial p} &= m_B(p,r)(\delta+\tau)\, g(\beta(p))\, \beta'(p) \\
&\quad + \frac{\partial m_S(p,r)}{\partial p}(\delta+\tau)\left[G(qx) - G(\sigma(r))\right] + \frac{\partial m_B(p,r)}{\partial p}(\delta+\tau)\left[G(\beta(p)) - G(qx)\right] \\
\frac{\partial \mathbb{W}(x)}{\partial r} &= -m_S(p,r)(\delta+\tau)\, g(\sigma(r))\, \sigma'(r) + \frac{\partial m_B(p,r)}{\partial r}(\delta+\tau)\left[G(\beta(p)) - G(qx)\right] \\
&\quad + \frac{\partial m_S(p,r)}{\partial r}(\delta+\tau)\left[G(qx) - G(\sigma(r))\right]
\end{aligned}$$

with

$$\frac{\partial m_B(p,r)}{\partial p} = m_B(p,r)\left(\psi^B - 1\right)\frac{D'(p)}{D(p)} > 0 \;\; ; \;\; \frac{\partial m_B(p,r)}{\partial r} = m_B(p,r)\,\psi^S \frac{S'(r)}{S(r)} > 0 \tag{12}$$

$$\frac{\partial m_S(p,r)}{\partial r} = m_S(p,r)\left(\psi^S - 1\right)\frac{S'(r)}{S(r)} < 0 \text{ and } \frac{\partial m_S(p,r)}{\partial p} = m_S(p,r)\,\psi^B \frac{D'(p)}{D(p)} < 0 \tag{13}$$

where

$$\psi^B = \frac{M'_D(S,D)D}{M(S,D)} \;\; ; \; \psi^S = \frac{M'_S(S,D)S}{M(S,D)}$$

are the matching elasticities for buyers and sellers respectively such that

$$0 \leq \psi^j \leq 1 \text{ for } j = B, S$$

So one can rewrite FOC using $D'(p) = \beta'(p)(\beta(p) - qx)\, g(\beta(p))$ and $S'(r) = -\sigma'(r)\,(qx - \sigma(r))\, g(\sigma(r))$:

$$\begin{aligned}\frac{\partial \mathbb{W}(x)}{\partial p} &= 0 = \frac{D'(p)}{D(p)}(\delta+\tau)\left\{ m_B(p,r)\frac{D(p)}{\beta(p)-qx}\right.\\ &\quad \left. + m_S(p,r)\,\psi^B\left[G(qx) - G(\sigma(r))\right] + m_B(p,r)\left(\psi^B - 1\right)\left[G(\beta(p)) - G(qx)\right]\right\}\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathbb{W}(x)}{\partial r} &= 0 = \frac{S'(r)}{S(r)}(\delta+\tau)\left\{ m_S(p,r)\frac{S(r)}{qx-\sigma(r)}\right.\\ &\quad \left. + m_B(p,r)\,\psi^S\left[G(\beta(p)) - G(qx)\right] + m_S(p,r)\left(\psi^S - 1\right)\left[G(qx) - G(\sigma(r))\right]\right\}\end{aligned}$$

Using definitions of $\beta(p)$ and $\sigma(r)$, this leads to define (p^μ, r^μ) as :

$$p^\mu = a + \left(1-\psi^B\right)A^B - \psi^B A^S \qquad (14)$$

$$r^\mu = a + \psi^S A^B - \left(1-\psi^S\right)A^S \qquad (15)$$

where

$$A_B = \frac{G(\beta(p^\mu)) - G(qx)}{D(p^\mu)}(\delta+\tau) > 0 \text{ and } A_S = \frac{G(qx) - G(\sigma(r^\mu))}{S(r^\mu)}(\delta+\tau) > 0$$

These expressions stand for the weighted net match valuation of buyers and sellers respectively. Note that this solution is not valid for matching technology characterized by constant or increasing returns to scale (Cobb Douglas technology for instance), indeed $\psi^B + \psi^S \geq 1$:

$$r^\mu \geq a + \left(\psi^S A^B - \psi^B A^S\right) \geq p^\mu$$

so one cannot verify $p^\mu > \underline{p} > \underline{r} > r^\mu$. Conditions (14) and (10) are reminiscent of Equation (17) in Goos *et al.* (2013) in a different context. Existence for the interior solution is ensured by the "rational-expectations" equilibrium we adopted as suggested by Caillaud and Jullien (2003). By the mean theorem we see that $d(p) = (\varphi - qx)(G(\beta(p)) - G(qx))$ where $\varphi < \beta(p) \leq \bar{\phi}$, so

$$a + A_B > \underline{p}$$

Identically, $s(r) = (qx - \xi)(G(qx) - G(\sigma(r)))$ where $\xi > \rho(r) \geq \underline{\phi}$, so

$$a - A^S < \bar{r}$$

So it exists value of the elasticicties (i.e. forms of the underlying matching technology) such that the interior solution is valid for some x

$$\psi^B \leq \bar{\psi}^B = \frac{(qx-\xi)(\bar{\phi}-\varphi)}{(\varphi-\xi)(\bar{\phi}-qx)} \geq 0$$

$$\psi^S \leq \bar{\psi}^S = \frac{(\varphi-qx)(\xi-\underline{\phi})}{(\varphi-\xi)(qx-\underline{\phi})} \geq 0$$

As $\bar{\psi}^B$ is monotonically increasing with respect to x and maps $[\frac{\xi}{q}, \min\{1, \bar{\phi}/q\}]$ into $[0, +\infty]$ so it exists a unique $x_b \in]\frac{\xi}{q}, \min\{1, \bar{\phi}/q\}[$: $\bar{\psi}^B = 1$. Identically, $\bar{\psi}^S$ is monotonically decreasing with respect to x and maps $[\underline{\phi}/q, \min\{1, \varphi/q\}]$ into $[0, +\infty]$ so it exists a unique $x_s \in]\underline{\phi}/q, \min\{1, \varphi/q\}[$: $\bar{\psi}^S = 1$. As a result it exists a unique $x_e \in]x_s, x_b[$ such that $\bar{\psi}^B = \bar{\psi}^S$ when $x = x_e$. As a result, the interior solution is valid for some underlying matching technologies (with decreasing return to scale) and some state of nature. Otherwise a corner solution applies which implies either $p = \underline{p}$ or $r = \bar{r}$.

To sump-up the matching prices (p^μ, r^μ) are driven by the underlying matching technology and imply

$$\begin{aligned} p^\mu &= \max\{a + \left(1 - \psi^B\right) A^B - \psi^B A^S, \underline{p}\} \\ r^\mu &= \min\{a + \psi^S A^B - \left(1 - \psi^S\right) A^S, \bar{r}\} \end{aligned}$$

where ψ^B, ψ^S are the matching elasticities for a buyer and a seller, respectively, and A^B, A^S stand for the weighted net match valuation of buyers and sellers, respectively.

Finally one can see that it exists levels of $\left(\psi^B_*, \psi^S_*\right)$ such that

$$\begin{aligned} p^\mu &= a + \left(1 - \psi^B_*\right) A^B - \psi^B_* A^S = p^* \\ r^\mu &= a + \psi^S_* A^B - \left(1 - \psi^S_*\right) A^S = r^* \end{aligned}$$

where (p^*, r^*) are defined in Lemma 2.

Indeed when at p^* such that $D(p^*) = \bar{s}$ we have $p^* = a + \frac{\delta}{\beta(p^*) - qx}$ and one can rewrite

$$p^\mu\left(\psi^B\right) = a + \left(1 - \psi^B\right)\frac{\delta}{\varphi - qx} - \psi^B \frac{\delta}{qx - \xi}$$

so $p^\mu(0) > p^*$. As $p^\mu\left(\psi^B\right)$ is linear decreasing in ψ^B it exists $\psi^B_* : p^\mu\left(\psi^B_*\right) = p^*$. Same reasoning applies for r^μ. Hence one can depicts the relative positions of these prices by

$$\begin{array}{lll} p^\mu \geq p^* & \text{for} & \psi^B \leq \psi^B_* \leq \bar{\psi}^B \\ p^* \geq p^\mu \geq \underline{p} & \text{for} & \psi^B_* \leq \psi^B \leq \bar{\psi}^B \\ r^\mu \leq r^* & \text{for} & \psi^S \leq \psi^S_* \leq \bar{\psi}^S \\ r^* \leq r^\mu \leq \bar{r} & \text{for} & \psi^S_* \leq \psi^S \leq \bar{\psi}^S \end{array}$$

Now the incentive to install DPU with a dealing platform writes

$$\begin{aligned} I_M(\phi) &= q\mathbb{E}[ax] - k \\ &\quad + \mathbb{E}_{\mathcal{B}}\left[m^\mu_B\left\{\delta + \tau - (p^\mu - a)(\phi - qx)\right\}\right] \\ &\quad + \mathbb{E}_{\mathcal{S}}\left[m^\mu_S\left\{\delta + \tau + (r^\mu - a)(qx - \phi)\right\}\right] \end{aligned}$$

So when $\left(\psi^B, \psi^S\right) = \left(\psi^B_*, \psi^S_*\right)$ we have $I_M(\phi) \leq I_P(\phi)$ as $m^\mu_j \leq 1$ for $j = B, S$. Then if $\left(\psi^B, \psi^S\right) \geq \left(\psi^B_*, \psi^S_*\right)$

$$\begin{aligned} \delta + \tau - (p^\mu - a)(\phi - qx) &\geq \delta + \tau - (p^* - a)(\phi - qx) \\ \delta + \tau + (r^\mu - a)(qx - \phi) &\geq \delta + \tau + (r^* - a)(qx - \phi) \end{aligned}$$

so this as an inflating effect for $I_M(\phi)$ in each state x. Moreover with a matching technology all states $x \in \mathcal{B}$ or $x \in \mathcal{S}$ are eligible to become a profitable match whereas only subsets of them (resp. $\mathcal{B}^P$ and $\mathcal{S}^P$) are with a dealing platform. So

$$\begin{aligned}
\mathbb{E}_{\mathcal{B}}\left[\delta+\tau-(p^{\mu}-a)(\phi-qx)\right] &> \mathbb{E}_{\mathcal{B}^P}\left[\delta+\tau-(p^{*}-a)(\phi-qx)\right] \\
\mathbb{E}_{\mathcal{S}}\left[\delta+\tau+(r^{\mu}-a)(qx-\phi)\right] &> \mathbb{E}_{\mathcal{S}^P}\left[\delta+\tau+(r^{*}-a)(qx-\phi)\right]
\end{aligned}$$

Hence, from (12) and (13), we see m_S^{μ} increases with (p,r) and m_S^{μ} decreases with (p,r) so if $\psi^B=\psi_*^B$ and $\psi^S>\psi_*^S$ then m_B^{μ} increases and matched agents are better off when buying. However, as m_S^{μ} decreases as r increases, sellers are matched with a lower probability. If $\psi^B>\psi_*^B$ and $\psi^S=\psi_*^S$ then m_S^{μ} increases as p decreases and matched agents are better off when selling. However, as m_B^{μ} decreases, buyers are matched with a lower probability.

So it *may exist* values of probabilities (m_B^{μ}, m_S^{μ}) such that

$$\begin{aligned}
\mathbb{E}_{\mathcal{B}}\left[m_B^{\mu}\left\{\delta+\tau-(p^{\mu}-a)(\phi-qx)\right\}\right] &> \mathbb{E}_{\mathcal{B}^P}\left[\delta+\tau-(p^{*}-a)(\phi-qx)\right] \\
\mathbb{E}_{\mathcal{S}}\left[m_S^{\mu}\left\{\delta+\tau+(r^{\mu}-a)(qx-\phi)\right\}\right] &> \mathbb{E}_{\mathcal{S}^P}\left[\delta+\tau+(r^{*}-a)(qx-\phi)\right]
\end{aligned}$$

and

$$I_M(\phi) > I_P(\phi)$$

Proof of Proposition 5

In order to derive the result in the Proposition, we need to determine once again the dealer equilibrium price. These prices are now functions of the entire path $\{q(\phi)\}_{\phi\in[\underline{\phi},\bar{\phi}]}$ where the aggregate demand is now $d(p)=\int_{\hat{\phi}_x}^{\beta(p)}(\phi-q(\phi)x)\,dG$ and the aggregate supply $s(r)=\int_{\sigma(r)}^{\hat{\phi}_x}(q(\phi)x-\phi)\,dG$. The result in Lemma 2 still holds with the main change that $q=q(\phi)$ for each ϕ. This implies that the switching state $\hat{x}$ is now defined as $\hat{x}=\frac{\mathbb{E}(\phi)}{n\mathbb{E}(q(\phi))}$. We first show that from (9) and (10) in the Proof of Lemma 2 one can derive that

$$\begin{aligned}
\frac{\partial \underline{p}}{\partial q(\bar{\phi})} &= \frac{\Delta(\phi)\,x}{\left(\bar{\phi}-q(\bar{\phi})\,x\right)^2} > 0 \text{ and } \frac{\partial \underline{p}}{\partial q(\phi)} = 0 \text{ for all } \phi \neq \bar{\phi} \\
\frac{\partial \bar{r}}{\partial q(\underline{\phi})} &= \frac{\Delta(\phi)\,x}{\left(q(\underline{\phi})\,x-\underline{\phi}\right)^2} \geq 0 \text{ and } \frac{\partial \bar{r}}{\partial q(\phi)} = 0 \text{ for all } \phi \neq \underline{\phi}
\end{aligned}$$

Moreover if $0 \le x \le \hat{x}$ then $d(p^*)=\bar{s}$ then for a given ϕ such that

$$d'(p)\frac{\partial p^*}{\partial q(\phi)} = \frac{\partial \bar{s}}{\partial q(\phi)} - \frac{\partial d(p)}{\partial q(\phi)}$$

- $\phi \in [\underline{\phi}, \sigma(r^*)[$ or $\phi \in [\beta(p^*), \bar{\phi}[$ then $\frac{\partial p^*}{\partial q(\phi)}=0$
- $\phi \in [\sigma(r^*), \hat{\phi}_x]$ then $d'(p)\frac{\partial p^*}{\partial q(\phi)} = \frac{\partial \bar{s}}{\partial q(\phi)} - 0 = xg(\phi) \ge 0 \Rightarrow \frac{\partial p^*}{\partial q(\phi)} \le 0$
- $\phi \in [\hat{\phi}_x, \beta(p^*)]$ then $d'(p)\frac{\partial p^*}{\partial q(\phi)} = 0 - \frac{\partial d(p)}{\partial q(\phi)} = xg(\phi) \ge 0 \Rightarrow \frac{\partial p^*}{\partial q(\phi)} \le 0.$

If $1 \ge x \ge \hat{x}$ then $s(r^*)=\bar{d}$ then for a given ϕ such that $s'(r)\frac{\partial r^*}{\partial q(\phi)} = \frac{\partial \bar{d}}{\partial q(\phi)} - \frac{\partial s(r)}{\partial q(\phi)}$

- $\phi \in [\underline{\phi}, \sigma(r^*)[$ or $\phi \in [\beta(p^*), \bar{\phi}[$ then $\frac{\partial r^*}{\partial q(\phi)}=0$

- $\phi \in \left[\sigma\left(r^*\right), \hat{\phi}_x\right]$ then $s'\left(r\right) \frac{\partial r^*}{\partial q(\phi)} = 0 - \frac{\partial s(r)}{\partial q(\phi)} = -xg\left(\phi\right) \leq 0 \Rightarrow \frac{\partial r^*}{\partial q(\phi)} \leq 0$
- $\phi \in \left[\hat{\phi}_x, \beta\left(p^*\right)\right]$ then $s'\left(r\right) \frac{\partial r^*}{\partial q(\phi)} = \frac{\partial \bar{d}}{\partial q(\phi)} - 0 = -xg\left(\phi\right) \leq 0 \Rightarrow \frac{\partial r^*}{\partial q(\phi)} \leq 0$

So we have proved that for $x < \hat{x}$, then $r^* = \bar{r}$; $p^* > \underline{p}$ and

$$\frac{\partial p^*}{\partial q\left(\phi\right)} \leq 0 \text{ for all } \phi \text{ while } \frac{\partial \bar{r}}{\partial q(\underline{\phi})} > 0 \text{ and } \frac{\partial \bar{r}}{\partial q\left(\phi\right)} = 0 \text{ for all } \phi > \underline{\phi}$$

For $x > \hat{x}$, then $p^* = \underline{p}$; $r^* < \underline{p}$ and

$$\frac{\partial r^*}{\partial q\left(\phi\right)} \leq 0 \text{ for all } \phi \text{ while } \frac{\partial \underline{p}}{\partial q\left(\bar{\phi}\right)} > 0 \text{ and } \frac{\partial \underline{p}}{\partial q\left(\phi\right)} = 0 \text{ for all } \phi < \bar{\phi}$$

These are points *(i)* and *(ii)* of the Proposition.

Finally, for all $\phi \neq \left(\underline{\phi}, \bar{\phi}\right)$, one can derive

$$\left[\frac{\partial^2 \mathbb{E}\left[U\right]}{\partial q^2\left(\phi\right)} - k''\left(q^*\left(\phi\right)\right)\right] \frac{dq^*\left(\phi\right)}{d\phi} = \mathbb{E}_{\mathcal{B}^P}\left[\frac{\partial p^*}{\partial q\left(\phi\right)}\right] + \mathbb{E}_{\mathcal{S}^P}\left[\frac{\partial r^*}{\partial q\left(\phi\right)}\right] \leq 0$$

so if $\frac{\partial^2 \mathbb{E}[U]}{\partial q^2} - k''\left(q\right) \leq 0$ then $q^*\left(\phi\right)$ is unique solution for the prosumer's problem $\max_q \mathbb{E}\left[U\right] - k\left(q\right)$ and then we have

$$\frac{dq^*\left(\phi\right)}{d\phi} \geq 0$$

As a result $q^*(\phi)$ is strictly increasing with ϕ, that is the first part of point *(iii)* of the Proposition is proved. The last part is done in the text as a discussion.