Peer-to-Peer Energy Platforms: Incentives for Prosuming

Thomas Cortade, Jean-Christophe Poudou

Peer-to-Peer Energy Platforms: Incentives for Prosuming*

Thomas CORTADE† and Jean-Christophe POUDOU‡

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Abstract

In this paper, we analyze how new models of peer-to-peer exchanges in the electricity sector may be effective and could yield incentives to invest in decentralized domestic production units based on renewable energy sources. We model a local exchange system for electricity, designed as a dealing platform, which determines purchase and selling prices on a continuous time basis. This allows us to question the participation of prosumers in peer-to-peer energy exchanges and their willingness to invest in local energy production. Compared to the no-platform configuration, we show that a pure dealing welfare maximizing platform creates at least as much incentives to install domestic production units. Then we challenge this main result considering several relevant features for peer-to-peer energy exchanges.

JEL classification: L14, L81, L94, Q4
Keywords: Peer-to-peer, electricity, trading platform, renewables

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1 Introduction

Europe's 2050 targets for reducing CO2 emissions, promoting renewable energies and reducing energy consumption are drastic and require the implementation of strong public policies.[1] As part of the energy transition, the development of smart grids represents a major challenge: thanks to new technologies and smart grids, it will be possible to increase the share of renewable energies and reduce energy consumption. Moreover, the European Parliament adopted at first reading on 13 November 2018 with a view to the adoption of a new Directive of the European Parliament and of the Council in order to promote of the use of energy from renewable sources. This legal process should favor the development of new trading arrangements and new technological improvements in energy systems.[2] By 30 June 2021, national governments will need to transpose the laws (and the community energy rights) into their legal system.

A first technological step in this process has been the development of smart grids that focused initially on the reliability and security of energy networks. The smart meters roll-out, the development of IoT-based energy devices and the use of individual data on energy consumption facilitate the demand-side management. The economic analysis on smart grids focuses on costs and prices, in particular the tariffs design as a tool for reducing electricity demand during peak periods (peak-load pricing, capacity trading), thus allowing to reduce CO2 and GHG emissions.

Smart grids open up new perspectives and a revolution in the energy field. The raise of peer-to-peer (P2P) electricity trading, using exchange platforms, like Airbnb or Uber platforms, is the basis for significant societal changes that will make it possible to achieve the objectives of the energy transition. According to Rifkin (2011), these changes could arise "using Internet technology to transform the power grid of every continent into an energy internet that can help households sell surplus energy back to the grid and share it with neighbours". The development of self-consumption has now modified traditional economic models based on a clear distinction between consumer and energy producer. A new type of agent has appeared, the prosumer which is an "active" consumer that both consumes and produces electricity based on distributed renewable energy sources (DRES).

As it will be seen in the next section, these P2P trading systems are sometimes still at the stage of R&D projects, and their relevance may be questioned. P2P trading systems may allow consumers and prosumers to trade energy in real time within a local group of agent, i.e. a community. DRES are small and decentralized electricity generation with

[1]See, for example, on April 2021, the European Climate Law enshrines the EU's commitment to reaching climate neutrality by 2050 and the intermediate target of reducing net greenhouse gas emissions by at least 55% by 2030, compared to 1990 levels.

[2]Thus, the article 21-2a indicates that Member States shall ensure that renewables self-consumers, individually or through aggregators, are entitled: "*to generate renewable energy, including for their own consumption, store and sell their excess production of renewable electricity, including through renewables power purchase agreements, electricity suppliers and peer-to-peer trading arrangements*".

storage systems. They are mainly based on hybrid or combined technologies such as solar power and energy storage technologies, but can also be made up of a single technology like a diesel or gas genset.[3] As a result, they may be seen as a real opportunity for integrating these technologies into the electric system. As quoted by Mengelkamp et al. (2018) "this empowers small-scale energy consumers and prosumers, incentivizes investments in local generation, and helps to develop self-sustainable microgrid communities". In practice, P2P electricity trading systems rely on physical and virtual layers which embodies an energy management system, so P2P trading is facilitated by the existence of digital platforms connecting a large number of peers. According to IRENA (2020), "smart meters, broadband communication infrastructure, network remote control and automation systems (network digitalisation) are thus fundamental enablers of platform-based business models, such as the P2P electricity trading model".

The objective of this paper is to study a P2P energy trading system connected to the national grid which acts as market-dealer. We provide a simple model by considering prosumers who purchase or sell energy among themselves or to the grid. We aim to take into account the heterogeneity of agents within the community with respect to their energy needs (or load profiles) and also the inner intermittency of DRES used to produce their local electricity. More precisely, we analyze the impact of exchange platforms design on the effectiveness and the relevance of energy exchanges within the community. For that, we consider several configurations, dealing or matching platform, zero pricing, non-profit or for-profit platform. In this context, we discuss about price levels on the platform compared with prices on the grid, in order to assess the incentives to invest in a domestic production unit (hereafter DPU) based on DRES. We show that the existence of dealing platform can boost the installation process of DPU's. This comes from the fact that these platforms are able to generate economic intrinsic or monetary values for prosumers that are the fundamentals of trade. A consequence is that energy is purchased at a higher price and sold at a lower price than the grid reference. However, the expected net gains to be a trader within the platform is always greater than the expected average price on the grid. Investments in DPU's are able to increase these expected net gains, which provides incentives for prosumers to participate in the local trading.

The present paper is related to three strands of the literature. First, it touches upon the literature on energy communities and decentralised energy systems. There exists a growing economic literature on those topics which is particularly well exposed in Abada *et al.* (2020a,b). To sum-up, this literature is mainly oriented in an engineering or optimization perspective, depicting the optimal technical performances of the decentralized generation on energy communities and micro-grids. However, some papers adopt an economic point of view. Abada *et al.* (2020a) study the viability of the community by using cooperative

[3]Between 2017 and 2026, the annual average growth rate of these technologies is commonly estimated to be 7.7%

game approach and find that inadequate gain sharing may jeopardize the stability of a community but if aggregation benefits can compensate coordination costs, the community may be stable. The same authors (2020b) find also that the development of such energy communities are dependent on the grid tariff structure that can lead to over-investments in decentralised energy systems (mainly rooftop PVs). Instead, in the present paper we build a non-cooperative framework to analyze the relevance of energy communities based on local exchanges.

Second, this paper is closely related to the literature on P2P economics. The recent economic literature applied to digital platforms has mainly developed on the basis of questions raised by the emergence of service platforms such as eBay, Uber or Airbnb. The main objective of these platforms is to facilitate exchanges between a large number of heterogeneous buyers and sellers exchanging commodities, services and cultural goods. New economic issues arise about the economic and business model of actors, their pricing strategies, and how these activities could be regulated. Krishnan *et al.*(2003) argue that P2P networks could be perceived either as public goods or as club goods. They provide an overview of P2P networks, focusing on the agent behavior such as free-ridding, that is when users consume network resources without providing these resources to the network. Basically, with such behaviors a P2P network could collapse. This risk can be mitigated if the users' participation is conditioned by altruism, or if the viability of P2P networks is based on trust and reputation. In our paper, we rely also on some intrinsic preferences to be engaged in P2P energy trading. Einav *et al.* (2016) consider common elements to all these P2P platforms such as intermediation role for the platform owner, monitoring agents via technology, sophisticated pricing mechanisms and so on. They highlight the issue of matching heterogeneous buyers and sellers and determine the condition for which P2P markets are arising and efficient. Among these conditions, the choice of pricing mechanism or market design is essential. For example, by using data of Ebay, Einav *et al.* (2018) provide an empirical and theoretical analysis about trade-off between online auctions and posted prices. On our analysis, we will take stock of these ideas by comparing different platform design.

Last, a new branch of literature deals with energy digital platforms that use blockchain and distributed ledger technologies. Sousa *et al.* (2019) and Soto *et al.* (2021) provide wide overviews of P2P energy trading markets, focusing on technical, optimization and engineering. The development of such technologies had made possible decentralized exchanges with automated management systems that are essential for the balancing of supply and demand within microgrids, without intermediary (or aggregator), as noted by Mengelkamp *et al.* (2018). From an economic point of view, Gautier and Salem (2021) show that the social efficiency of P2P trading may depend on the strength of negative externalities created by too generous feed-in tariffs. In our paper, rather we consider that no feed-in tariffs support the prosumer investments and we analyse the platform design performance in a local efficiency perspective.

The rest of the paper is organized as follows. In a first section, we depict some P2P elexperiences : first we propose a benchmark, i.e without platform, in which we focus on the incentives for installing a fixed size DPU capacity. We go on with a simple dealing platform and we establish and compare those incentives. Finally we consider several extensions in order to challenge our basic framework and results. We study zero pricing schemes, market power for the dealing platform, a matching process fir platform and we extend our basic framework allowing for variable DPU size and individual shocks. Details and proofs are given in the Appendix.

2 P2P electricity trading: some experiences

Several experiences of P2P energy trading systems based on platforms has been achieved in the word, and we give a brief presentation of the most significant experiences on microgrids and smart grids. A lot of papers and reports describe the technological details of these P2P trading systems (Zhang et *al.* (2017), Sousa *et al.* (2019), IRENA (2020) and Soto *et al.* (2021))

Mainly, projects and experiences are located in Europe and United States and are are supported by public research programs.[4] However, it exists a lot of independent projects worldwide. For instance P2P microgrids can constitute home electrification solutions in developing countries such as Bangladesh, Malaysia or Colombia, with the Transactive Energy Colombia project implemented in Medellin. The key point of these projects is to connect low-income prosumers, equipped with photovoltaic roofs, and unequipped richer consumers. In Bangladesh, the Solshare company has developed similar technological solutions based on connected objects as smart phones.

Zhang et *al.* (2017), propose a comparison of several projects or start-ups based on the network size or their scope and on the information and communication technologies (hereafter ICT). Hence, Piclo (UK), Vandebron (Netherland), SonnenCommunity (Germany) and Litchtblick Swarm Energy (Germany) have national scope, whereas Smart Watts (Germany), Yeloha Mosaic[5] (US) are regional. The smallest size of platforms such as TransActive Grid/LO3 Energy (US) and Electron (UK) correspond to a local P2P market in which blockchain technology is used in order to simplify metering and billing system. Some studies have shown that, trading power with peers, prosumers can achieve overall savings on

[4]See also Gangale *et al.* (2017) for an overview of European smart grids projects.

[5]This project aimed to create a solar-like Airbnb, without success at the end. In Spain, Sotysolar aims to do the same.

their billing costs (up to 20% on average), such as for the german project Lition tested in 2018 (GJETC, 2020).

P2P energy trading systems are also based on three levels. The first level represents a P2P energy trading within a eco-neighborhood like for example the iconic Brooklyn microgrid (TransActive Grid/LO3 Energy) or Lyon Confluence in France. The second level is characterized by tradings between several microgrids (Multi-Microgrids, P2P within CELL). This is the case of two connected microgrids Walqa and Atenea located in Spain, distant from 150 km.[6] Energy trading is also possible between them, organized around industrial laboratories of small tertiary companies. Finally, the third level corresponds to P2P among CELLs (Multi-CELLs), which is mostly hypothetical at that time. The two last levels raise the question of conducive regulatory framework allowing such interconnection, but also of the structure of pricing scheme.

Sousa *et al.*(2019) classify R&D projects for P2P energy tradings considering 1) the market design and business models and 2) the implementation of local control and ICT platforms for prosumers. Projects as Enerchain, NRGcoin, Energy Collective are most advanced on the first dimension, whereas Empower, P2P-SmartTest focus on the second. Some, as Lumenaza (Germany), ambition to cover both dimensions.

Therefore it appears from these various experiments, that as well research organizations, entrepreneurs, or consumers consider as relevant the implementation P2P energy trading systems based on platforms. We now develop a model to proceed to their economic analysis.

3 Model

We develop a simple stylized model where heterogeneous agents aim to exchange excess energy flows they produce using renewable decentralized production unit. Our main goal is to see how such P2P trading arrangements can be viable for all participants. In our model prosumers, i.e. consumers and producers of energy goods can offer them in competition with professionals producers (i.e. companies or local communities) and interact with possible pure consumers on a dedicated platform. In a first step, the platform is just considered as a dealer that purchase energy in excess from some prosumers and resells it to consumers or through the grid.

Suppose that exists a mass n agents have a load factor (state of demand) of $\phi \in [\underline{\phi}, \overline{\phi}]$ distributed according to a cumulative $G(\phi)$ where $G'(\phi) = g(\phi)$ and $G(\bar{\phi}) = n$. This state describes the level of consumption they desire to achieve in all periods. This corresponds to their standard energy needs in relation to the size of the agent's households (i.e. dwelling

[6] They are included in the P2P-SmartTest R&D project, see the report available online: https://cordis.europa.eu/project/id/646469/reporting

area, number of people, installed power). We assume that with the surplus derived from this baseline level of consumption is $u(\phi)$, where $u(\cdot)$ is an increasing concave function.

To satisfy her needs, an agent has the choice to install or not a domestic production unit of energy, here represent by a maximal production capacity of $q > 0$ kWp at a capacity up-front cost $k > 0$. We assume that q is fixed and relax this assumption in the Section 6. For example, it can be the case if the agent acquire a dwelling in a connected residential area where areas are normalized and so is the DPU.

Had this capacity installed, an agent can be a prosumer in the sense she can use it as she wants, to self-consume it or to sell it if it is possible according to the excess capacity she observes at each time $\phi - qx$. Here the variable qx represents the available amount of the renewable capacity q that is actually dispatchable in state $x \in [0, 1]$, they are distributed according to a cumulative $F(x)$, with $F'(x) = f(x)$. The state of nature x represents weather conditions or occurrence of failures, that is all external conditions that drives the intermittency feature of DPU's. Then in a given state of nature x, a prosumer (i.e. an agent that has installed a capacity q) may be either a pure consumer if $\phi - qx \geq 0$ or a potential seller if $\phi - qx < 0$. For the sake of simplicity let us assume that $\bar{\phi} \geq q > \underline{\phi}$, which means that in favourable conditions ($x = 1$), there are always some buyers (those with load factors near the upper bound $\bar{\phi}$) and sellers (those with small load factors near the lower bound $\underline{\phi}$).

Figure 1 depicts the heterogeneous consumption model. The sloping dotted lines represent the net consumption/production for the extremal agents, the sloping thick line is the one of a given agent with a load factor ϕ.

Now let us describe the supply side. First, we assume that a centralized professional supplier always exists and may provide unlimited energy volumes to all agents that demand them at a given price $a(x)$. This price may include the energy wholesale prices and volumetric parts of grid access tariffs. We also consider that a fixed periodic (non-volumetric) tariff τ is charged to pure consumers that are served fro the grid. However, as we focus on the P2P exchanges we refer to the external supply as of the (centralized) grid. In some, sense the grid supply is the outside option for all agents being or not prosumers. Second, we analyze the viability of a dealing platform through which all prosumers may want to trade their excess/lack energy volumes in any state of nature.

The basic business model to this platform is to resell the excess energy volumes to consumers that are connected to it or to the central grid if no deals are found. We assume that the platform cannot change the price $a(x)$ decided by the global market on the centralized price, so it cannot make profits on this external side. We denote by $p(x) \geq 0$ the platform purchase price and $r(x) \geq 0$ the platform selling price within the platform.[7] Then if a

[7]In such a model with vertical differentiation for participating to the platform, negative prices would be possible. However, we assume that in front of a negative price, a seller do not trade.

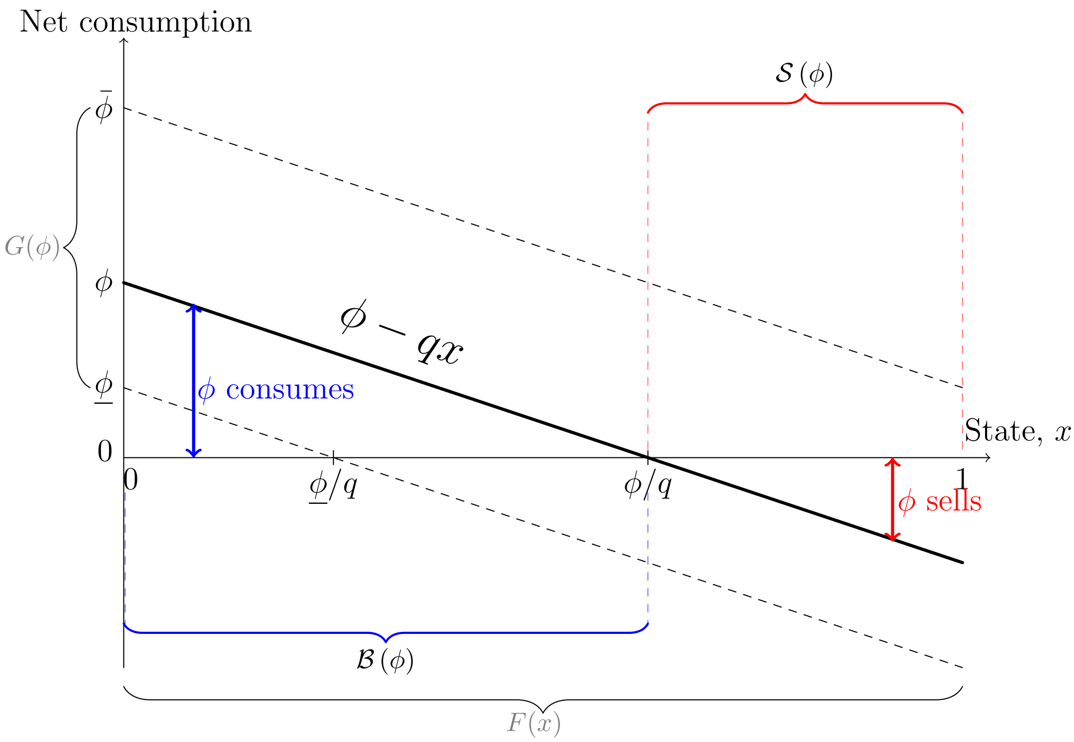

Figure 1: Net consumptions

agent ϕ is a consumer in state x, she will have to pay an amount $p(x)(\phi - qx) \geq 0$ if she purchases their energy needed through the platform. On the contrary, if she is a seller in state x, she will receive a profit $p(x)(qx - \phi) \geq 0$ if she sells their excess energy through the platform. We also consider that agents participate to the platform have an intrinsic preference when they are served through this channel, which is represented by a parameter $\delta \geq 0$. For instance it represents the surplus of being in sharing relationships with identified agents (neighbours, flatmates, members of an dedicated association). It can also represent a part of the surplus for avoiding power cuts when distribution grids are failing, or the reduction of transaction costs with the professional suppliers, or the gain from having the possibility of trading, or from some ancillary local services provided by the platform, or finally the environmental preference for by themselves a potential producer with residential renewable sources (i.e. "fossil fuel freedom"). This preference is also a way to represent the ability the platform has to provide specific services that are valuable to the connected consumers. As a result, an agent that fulfils her needs through the local channel or the platform derives an utility level of $u(\phi + \delta)$.

So for an agent with a load factor ϕ the utility from trading through the platform in state x is

$$U(\phi, x, q) = \begin{cases} u(\phi + \delta) - p(x)(\phi - qx) & \text{if } \phi \geq qx \\ u(\phi + \delta) + r(x)(qx - \phi) & \phantom{\text{if }} \phi < qx \end{cases}$$

The utility from trading through using the grid

$$\underline{U}(\phi, x, q) = \begin{cases} u(\phi) - a(x)(\phi - qx) - \tau & \phi \geq qx \\ u(\phi) + a(x)(qx - \phi) - \tau & \text{if } \phi < qx \\ u(\phi) - a(x)\phi - \tau & q = 0 \end{cases}$$

So for each x, it may exist $\hat{\phi}_x = qx$ such that the agent is a pure self-consumer (if $x > 0$).

4 No platform

Consider first the common situation in which the platform does not exist. The central grid is viewed an aggregator that purchases or sells energy at a given price $a(x)$. The only decision for all agents is to install or not the DPU capacity q at cost k. A prosumer ϕ installs the DPU if (expectation are taken over x):

$$\mathbb{E}[U_0] - k \geq \mathbb{E}[\underline{U}|q = 0] = u(\phi) - \mathbb{E}[a(x)\phi] - \tau$$

where

$$\mathbb{E}[U_0] = u(\phi) - \mathbb{E}_{\mathcal{B}(\phi)}[a(x)(\phi - qx)] + \mathbb{E}_{\mathcal{S}(\phi)}[a(x)(qx - \phi)] - \tau \tag{1}$$

and

$$\begin{aligned} \mathcal{B}(\phi) &= \{x \in [0,1] : 0 \leq x \leq \phi/q\} \\ \mathcal{S}(\phi) &= \{x \in [0,1] : 1 \geq x \geq \phi/q\} \end{aligned}$$

which are respectively the set of s.o.n in which the prosumer ϕ is a buyer, resp. a seller. Note that $\mathcal{S}(\phi)$ may be eventually *empty* as for instance when $\phi = \bar{\phi}$, $x \leq 1 < \bar{\phi}/q$. In Figure 1, both sets are depicted.

Looking for the indifferent prosumer ϕ_0 such $\mathbb{E}[U_0] = u(\phi_0) - \mathbb{E}[a(x)\phi_0]$, we have

$$\phi_0 : q\mathbb{E}[a(x)x] - k = 0$$

which does not depend on the value of ϕ. As a result with no platform, the incentives to invest in DPU for an agent ϕ amount $I_0 = \max\{q\mathbb{E}[a(x)x] - k, 0\}$, so $\phi_0 = \bar{\phi}$ if $q\mathbb{E}[a(x)x] < k$ and $\phi_0 = \underline{\phi}$ if $q\mathbb{E}[a(x)x] > k$.

Lemma 1 *With no platform, all agents are prosumers and install capacity $q > 0$, iff $q\mathbb{E}[a(x)x] > k$, and there are no prosumers otherwise.*

The result in this Lemma is just a cost-benefit trade-off for any prosumer. On one hand, the amount $q\mathbb{E}[a(x)x]$ represents the opportunity benefits of the total purchase cost savings expected for a prosumer ϕ that had installed a amount capacity of q. On the other hand, k is the fixed expenditure to have access to this capacity. As a result, a prosumer actually invests in this capacity if this benefit overcomes the cost. Moreover as these cost savings are independent of the load factor ϕ, then either all agents are prosumers either their are all pure consumers.

5 Simple dealing platform

Let us suppose that a dealing technical (and eventually commercial platform) has the ability to identify prosumers supplies and demands and ensures their equilibrium. In the sense of an electricity system, the platform is an also an aggregator that dispatch the power within the local grid and towards the central grid. It can purchase prosumers supplies if any at a price $r(x) \geq 0$ in state x and resell this electricity flows to connected consumers at a price $p(x) \geq 0$.

The objective platform can be profit-oriented or welfare maximizing. To start with, let us suppose that the platform has a local welfare objective. Indeed a first step, we could imagine that due to a technicality of the microgrids technology involved in local areas to connected prosumers, of the blockchain processes and the implementation of smart contracts needed to ensure the real-time equilibrium within the platform and outside with the grid, the dealing platform would be a for profit organization. However, one could also imagine that in the future "turnkey digital technologies" and microgrids may be installed by some energy communities. In that sense, up the installation cost, the trading platform could be socially managed and and even zero-pricing could be desired by users.

So if in state x, the total supply to the platform in order to be resold within is $S(r(x))$, it must match the total demand $D(p(x))$ from prosumers that are in lack of power with regard to their domestic production at that state. However, some agents may prefer not to purchase or resell to the platform but to the grid. The platform cannot make money from them.

Demand and supply to the dealing platform The platform will implement choices that are individually preferable for each participants so an agent ϕ will be a consumer within the platform if she prefers to purchase the energy needed or to sell the energy in excess in some state x, to the platform whereas to the grid.

Concerning purchases, that is for agents such that $\phi \geq qx$, this writes (omitting the argument x)

$$u(\phi + \delta) - p(\phi - qx) \geq u(\phi) - a(\phi - qx) - \tau \tag{2}$$

put differently:

$$\Delta(\phi) = u(\phi + \delta) - u(\phi) + \tau \geq (p - a)(\phi - qx)$$

Here $\Delta(\phi)$ represents the direct periodic gains for a prosumer both from being connected to the community through the platform and also due to saving from the fixed access costs. Note that $\Delta'(\phi) < 0$ by concavity of u. This implies that[8] :

$$\begin{array}{rclr} qx & \leq & \phi \leq \beta(p) & \text{if } p > a \\ \phi & \geq & qx > \beta(p) & \text{if } p \leq a \end{array}$$

where $\beta(p)$ is the highest load value for which the platform demand peaks at a given price p. If $p > a$, the demand is price-sensitive and there exists a minimal price level $\underline{p}$ such that $\beta(\underline{p}) = \bar{\phi}$. So the demand at state x is such that

$$D(p) = \left\{ \begin{array}{c} \bar{d} \\ d(p) \end{array} \text{ if } \begin{array}{c} p \leq \underline{p} \\ p > \underline{p} \end{array} \right. \tag{3}$$

with $\bar{d} = \int_{qx}^{\bar{\phi}} (\phi - qx)\, dG$ and $d(p) = \int_{qx}^{\beta(p)} (\phi - qx)\, dG$. In the same spirit an agent ϕ will be a (extra) supplier within the platform if, when $r < a$ that is if $qx \geq \phi \geq \sigma(r)$ and $\sigma(\bar{r}) = \underline{\phi}$ with $\bar{r} = a - \frac{\delta}{qx - \underline{\phi}}$. The aggregate supply at state x is such that

$$S(r) = \left\{ \begin{array}{c} \overline{s} \\ s(r) \end{array} \text{ if } \begin{array}{c} r \geq \bar{r} \\ r < \bar{r} \end{array} \right. \tag{4}$$

where $\overline{s} = \int_{\underline{\phi}}^{qx} (qx - \phi)\, dG$ and $s(r) = \int_{\sigma(r)}^{qx} (qx - \phi)\, dG$. Here $\sigma(r)$ is the lowest load value for which the platform supply peaks at a given price r.

Market clearing and platform pricing In some state, $x > \underline{\phi}/q$, it may exist platform exchanges in the sense that the above demand and supply may meet. The market clearing price is then a couple of prices that equals demand and supply on the platform:

$$(p, r) : D(p) = S(r)$$

As the grid is a default option, the non served demands and supplies through the platform are served by the central grid. As a result, in any time, all energy flows are balanced. Let us now consider that at each state the platform chooses the prices (p, r) that maximize the

[8]More details are provided in the Appendix.

total welfare of the participant in the platform that is the sum of prosumer' surpluses and the profit of the platform:

$$W(x) = \int_{\underline{\phi}}^{\bar{\phi}} U(\phi, x, q)\, dG + \pi(x) = \int_{\sigma(r)}^{\beta(p)} u(\phi + \delta)\, dG$$

subject to $D(p) = S(r)$ and where $\pi(x) = pD(p) - rS(r)$ is the platform's profit. This leads to corner solutions[9] as depicted in the following Lemma, where $\hat{x} = \frac{\mathbb{E}[\phi]}{nq}$.

Lemma 2 *Optimal prices (p^*, r^*) are such that*

1. $r^* = \bar{r}$ *and* $p^* > \underline{p}$ *whenever* $\overline{s} < \bar{d}$ *that is for* $x < \hat{x}$*,*
2. $p^* = \underline{p}$ *and* $r^* \leq \bar{r}$ *whenever* $\overline{s} \geq \bar{d}$ *that is for* $x \geq \hat{x}$*.*
3. *and* $p^* > a > r^*$ *for all* x

In unfavourable availability conditions, i.e. x low, the aggregate demand to the platform is structurally high and the supply low, so the selling price is stated at least to its maximum value[10] in order to attract all sellers to the platform. As a result, the demand price is the one that just clears the market. In favourable availability conditions, i.e. x high, the aggregate supply to the platform is structurally high and the demand low, so the demand price is stated to its minimum value to push possible local buyers to be active on the platform. As a result, the selling price just clears the market.[11] The optimal market clearing is depicted in Figure 2. These optimal prices equilibrium put the agent in a trade set representing the states of nature in which the prosumer ϕ is a buyer to the grid ($\mathcal{B}^G$), to the platform ($\mathcal{B}^P$), a seller to the platform ($\mathcal{S}^P$) and finally a seller to the grid ($\mathcal{S}^G$). They write[12]

$$\begin{aligned}
\mathcal{B}^G &= \{x \in [0,1] : x \leq \xi_b(\phi)\} \\
\mathcal{B}^P &= \{x \in [0,1] : \xi_b(\phi) \leq x \leq \phi/q\} \\
\mathcal{S}^P &= \{x \in [0,1] : \xi_s(\phi) \geq x \geq \phi/q\} \\
\mathcal{S}^G &= \{x \in [0,1] : x \geq \xi_s(\phi)\}
\end{aligned}$$

[9]Indeed, there are multiple solutions as they are depicted in the proof in the Appendix. We pick down the less favourable for prosumers in order to give as little chance as possible to the platform to dominate and not to favor the platform situation in an artificial way. However, if we chose others prices further results are unchanged.

[10]This is also equivalent is terms of demands or supplies to set alternatively the price equal to $a(x)$ or lower. But it is not in terms of net welfare as the platform generates a additional utility.

[11]One might expect a cost associated with managing the platform, and this cost would be increasing and convex in the number of suppliers and consumers on the platform (need for huge servers). However, if platforms are ICT based, at least for microgrids those costs may be negligible. Nevertheless, if we introduce such a cost, the optimal pricing is now changed such that price are now interior: $p^* > \underline{p} > a > \bar{r} > r^*$. At the end, this does not deeply change the Proposition 1 as this create the same effect on prices than a for-profit platform described in section 6.

[12]These sets could further subdivided to take into account the pricing structure of the platform, as it is shown is Figure 3.

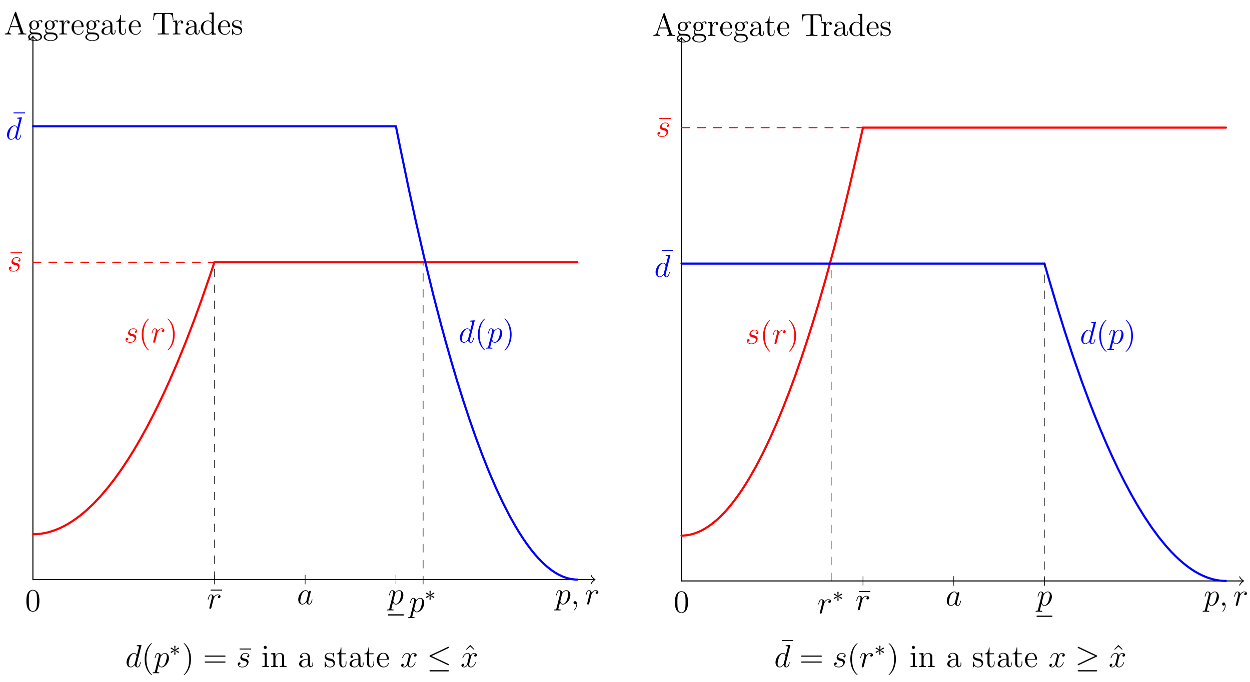

Figure 2: Market clearing

and when[13] $x = \xi_b(\phi) : \beta(p^*) = \phi$ and $x = \xi_s(\phi) : \sigma(r^*) = \phi$. Note by definition that $\xi_b(\bar{\phi}) = \xi_s(\underline{\phi}) = \frac{\mathbb{E}[\phi]}{nq}$ as when $\beta(p^*) = \bar{\phi}$ and $\sigma(p^*) = \underline{\phi}$ then $\bar{d} = \bar{s}$, which occurs in $\hat{x} = \frac{\mathbb{E}[\phi]}{nq}$.

The Figure 3 represents the equilibrium trade sets in the (ϕ, x) plane where red/blue areas are such that agents buy/sell on the platform. We see that for a given state of intermittency (a given x-axis), depending on her load profile ϕ a prosumer may be a seller on the platform (red hatched area) or to the grid (gray hatched area on the right); or she may be a buyer on the platform (blue hatched area) or to the grid (gray hatched area on the left).

Incentives to install DPU Now, we analyze the incentives to install DPU created by the existence of the exchange platform. For an agent ϕ, the expected surplus for participating to the platform is then

$$\begin{aligned} \mathbb{E}[U] &= u(\phi) - \mathbb{E}_{\mathcal{B}^G}[a(\phi - qx)] + \mathbb{E}_{\mathcal{B}^P}[\Delta(\phi) - p^*(\phi - qx)] \\ &\quad + \mathbb{E}_{\mathcal{S}^P}[\Delta(\phi) + r^*(qx - \phi)] + \mathbb{E}_{\mathcal{S}^G}[a(qx - \phi)] \end{aligned} \tag{5}$$

[13]Indeed we always have $\xi_b(\phi) \leq \phi/q$ as

$$\xi_b(\phi) = \frac{\phi}{q} - \frac{\Delta(\phi)}{q(p^* - a)} < \phi/q$$

Identically for $\xi_s(\phi) \geq \phi/q$. Moreover they are both increasing in ϕ.

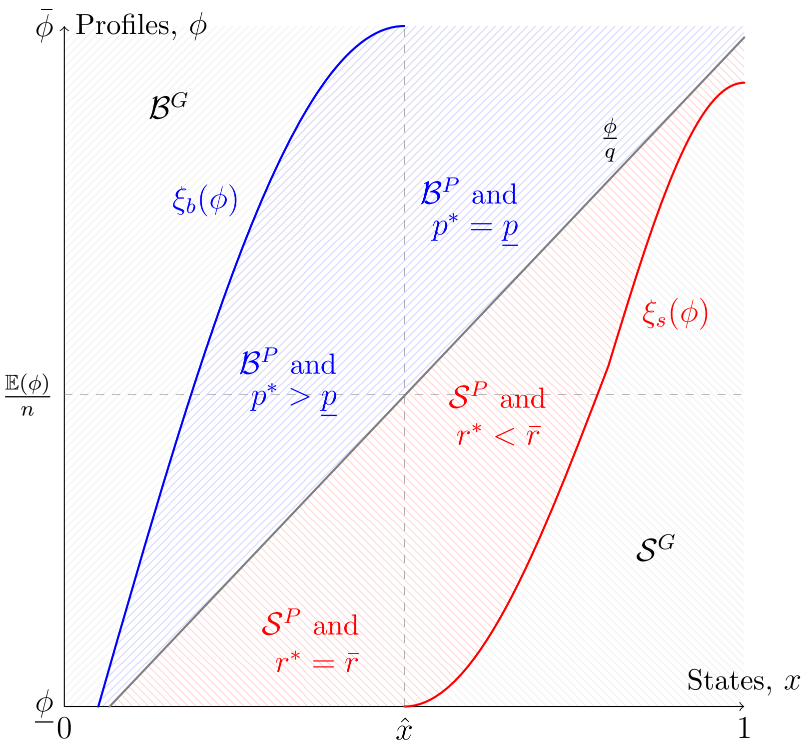

Figure 3: Trade regions

Actually she installs the capacity q when $\mathbb{E}[U]-k \geq [\underline{U}|q=0]$ and looking for the indifferent prosumer ϕ^* such $\mathbb{E}[U]-k = u(\phi) - \mathbb{E}[a\phi]$. Rearranging the terms, this leads to the equality:

$$\begin{aligned}\mathbb{E}[U] - k - (u(\phi^*) - \mathbb{E}[a\phi^*]) &= q\mathbb{E}[ax] - k \\ &+\mathbb{E}_{\mathcal{B}^P}[\Delta(\phi^*) - (p^* - a)(\phi^* - qx)] \\ &+\mathbb{E}_{\mathcal{S}^P}[\Delta(\phi^*) + (r^* - a)(qx - \phi^*)] = 0\end{aligned}$$

First, we now see that *in general* not all consumers are willing to participate to the platform and installing DPU. Indeed, we see that the load factor now is involved in the decision. Here the incentives to invest in DPU for an agent ϕ are $I_P(\phi) = \max\{\mathbb{E}[U] - k - (u(\phi) - \mathbb{E}[a\phi]), 0\}$.

However, assume that $q\mathbb{E}[ax] = k - \varepsilon$, so that no agent would be a prosumer in the benchmark case (without platform). Then in that case we see that being prosumers connected to the platform all agents are not worse off, as

$$\mathbb{E}[U] - (u(\phi) - \mathbb{E}[a\phi]) = \mathbb{E}_{\mathcal{B}^P}[\Delta(\phi) - (p^* - a)(\phi - qx)] + \mathbb{E}_{\mathcal{S}^P}[\Delta(\phi) + (r^* - a)(qx - \phi)] \geq 0 \tag{6}$$

Indeed, depending on price levels, mainly if the spread $p^* - r^*$ is large, the sets $\mathcal{B}^P$ and $\mathcal{S}^P$ may be empty and agents are in the same conditions as in the no platform case. But when the sets $\mathcal{B}^P$ and $\mathcal{S}^p$ are not empty, for an agent with a load factor ϕ, both terms in the RHS of (6) are not negative. So for these states of nature, an agent with a load factor ϕ has a greater surplus trading with peers on the platform than with the grid so (6) holds.

Assume now that $q\mathbb{E}[ax] > k$, such that all agents install a DPU without a platform, as their incentives to invest are $I_0 = q\mathbb{E}[ax] - k > 0$. However connected to the platform, their incentives to invest $I_P(\phi)$ are never less than I_0 as $I_P(\phi) = I_0 + \mathbb{E}[U] - (u(\phi) - \mathbb{E}[a\phi])$ and (6) holds. The following proposition sums up the previous discussion.

Proposition 1 *If all agents install a DPU when there is no platform, they do and are not worse off when the dealing platform is active.*

Even if the energy prices are less favourable, the intrinsic and differentiated services provided by the platform (safer distribution, local trades, traceability or just sharing renewables sources) as well as the grid cost savings, lead some prosumers to use the platform to trade their domestic production. The intuition that drives the Proposition 1 is that on top of the cost-benefit trade-off for any prosumer to install the DPU (being a trader on the platform or not), there are now further gains and costs to participate to P2P trading for some agents. This gains come from intrinsic values of participation and grid cost savings. The costs are market based: electricity purchase or selling platform prices are less favourable than those from the grid. However, installing a DPU for trading with peers allows to trigger these gains and to avoid those costs at least for some state of nature. At the end, the platform cost-benefit trade-off is positive for all agents.

Indeed, if no agent would be a prosumer without a platform, i.e. if $q\mathbb{E}[ax] < k$, with the dealing platform, there is a room for some agents to install the DPU, that is for which the platform cost-benefit trade-off is positive. So it exist a set of agents $\Phi^* \subset [\underline{\phi}, \bar{\phi}]$, for which $I_P(\phi) > 0 > I_0$. However, one cannot state generally what kind of agents will be concerned (low or high load profile), so we have the result.

Corollary 1 *If no agent install a DPU when there is no platform, there are some agents that do and are not worse off when the dealing platform is active. However without further information on the distribution of intermittency state of nature, one cannot assess which set of load profiles will be better off.*

To understand this result, let us analyze the shape of such incentives to install capacity with respect to the load profile of agents. Indeed, the variations of those incentives are a non monotonic function of ϕ:

$$I'_P(\phi) = \Delta'(\phi) - \mathbb{E}_{\mathcal{B}^P}[p^* - a] + \mathbb{E}_{\mathcal{S}^p}[a - r^*]$$

It depends first on the marginal utility from being "more" served within the platform Δ' which is negative (as u is concave). Second, it relies on the relative price spreads $p^* - a$

and $a - r^*$ at each state and also on the skewness of the distribution of states of nature. On one hand, agents with higher load profiles will be buyers more often (at the margin) and accordingly on the platform, then will have to pay the premium $p^* - a$ as a cost of sourcing, this reduces their incentives to invest i.e. $-\mathbb{E}_{\mathcal{B}^P}\left[p^* - a\right] < 0$. On the other, agents with higher load profiles will be sellers on the platform less often, then they will not have to bear shortfalls resulting from selling to the platform, this increases their incentives to invest i.e. $\mathbb{E}_{\mathcal{S}^P}\left[a - r^*\right] > 0$ at the margin.

When $I_P'(\phi) < 0$, for all ϕ then $\Phi^* = [\underline{\phi}, \phi^*]$, prosumers connected to the platform are those who have low load profiles (i.e. small consumers), and they are motivated by a selling argument to participate and install DPU: the shortfall $a - r^*$ is not so important for them. Big consumers are not interested by participating the premium is $p^* - a$ is too costly for them. When $I_P'(\phi) > 0$, then $\Phi^* = [\phi^*, \bar{\phi}]$, prosumers connected to the platform are those who have high load profiles (i.e. big consumers), they are motivated by a consuming argument to participate.

To finish with, we can show as an example, that some kind of bell shapes can be found for $I_P(\phi)$ using uniform distributions for ϕ and x, which can be viewed as neutral configurations with respect to variability. Indeed, let us consider the following specifications: $n = 1$, $u(\phi) = v$, $u(\phi + \delta) = v + \delta$, $F(x) = x$ and $G(\phi) = \frac{\phi - \underline{\phi}}{\bar{\phi} - \underline{\phi}}$. As a result $\mathbb{E}(\phi) = \frac{1}{2}\left(\bar{\phi} + \underline{\phi}\right)$ and one can derive that if $\phi \leq \mathbb{E}(\phi)$, then $I_P'(\phi) = \frac{\delta + \tau}{q}\left(\ln\left(\frac{\bar{\phi} - \underline{\phi}}{\phi - \underline{\phi}}\right) + \ln\left(\frac{\bar{\phi} - \underline{\phi}}{\bar{\phi} - \phi}\right) - 2\ln 2\right) \geq 0$ and if $\phi \leq \mathbb{E}(\phi)$, then $I_P'(\phi) = \frac{\delta + \tau}{q}\left(2\ln 2 - \ln\left(\frac{\bar{\phi} - \underline{\phi}}{\phi - \underline{\phi}}\right) - \ln\left(\frac{\bar{\phi} - \underline{\phi}}{\bar{\phi} - \phi}\right)\right) \leq 0$. In some sense, the incentives to install increase with the load profile when the prosumer is a small consumer but decreases if it is a big consumer.

Policy tools Finally we look at the effects of policy tools that are usually implemented and how they can help promoting or deterring the development of energy P2P platforms.

For instance, one first can imagine that some subsidization schemes are implemented by governments in order to promote P2P platforms for environmental or innovative concerns. A simple lumpsum subsidy for each DPU installed will have the effect of reducing the installation cost k and of course will directly increase the incentives for prosuming. However, this effect is not amplified by the existence of a P2P platform.

Price subsidization schemes could be more effective. Indeed, a unit rebate ρ allowed for the purchasing price so that the paid price would be $p - \rho$ or a premium for the selling price so that the paid price would be $r + \rho$, would enhance demand and/or supply on the platform.[14] This premia and rebates have direct effects on the incentives for prosuming $I_P(\phi)$ as they influence positively the relative price spreads. However, they are bounded instruments as depending of the state of nature for DPU availability, a flat rebate or flat subsidy may be ineffective at some point. For instance, in unfavourable availability conditions, i.e. x low,

[14] These rebate or premium call for compensations for the platform

Lemma 2 indicates that the selling price is set to the upper bound $\bar{r}$ for which all energy in excess is supplied within the platform. In this case adding a premium would not change the supply and then the selling price remains unchanged. The same applies for the purchasing price in favourable availability conditions, x high.

Finally, another way is to increase the grid price through directed taxation. This policy may have positive effects as it increases the total expected cost savings for a prosumer that had installed a DPU, i.e. $q\mathbb{E}[ax]$ and it decreases the purchase price spread. However, this also deflates the selling price spread which is a driver for prosuming, in favourable availability conditions.

6 Extensions

Some extensions of the basic framework are developed in order to challenge our main result in Proposition 1. First, we consider zero-pricing within the platform. Second, we discuss about the effect of a for-profit platform. Third, we look at a more sophisticated way to realize trades for prosumers considering a matching platform. Fourth, we drop the assumption that the DPU size is fixed, that now can vary with the load profile in order to be adapted to the basic consumption profile of each agent. Last, we alter the analysis to take into account the impacts of individual shocks for prosumers.

Zero-pricing

An argument sometimes put forward to justify the emergence of these platforms is that to some extent participants could exchange energy for free because first the short run marginal cost of generation for DPU based on DRES is near zero and also they could benefit from a certain reciprocity within the community. Of course, one could argue that zero pricing is detrimental for investments in local generation capacities.

First of all, a permanent zero pricing scheme is not generally possible, except in one (potential) state of nature for which $D(0) = \bar{d} = S(0) = s(0)$ which implies that it cannot be supported as an equilibrium for each states. Second, a unilateral zero pricing scheme (i.e. $p = 0$ or $r = 0$, $\forall x$) is not feasible as, for instance when $x \leq \hat{x}, D(0) = \bar{d} > \bar{s} > s(r)$, there are not enough sellers on the platform to serve the high demand. However, a zero pricing scheme can be achieved. Indeed, if $p = 0$ for all $x \geq \hat{x}$, it is equivalent in term of demand of a minimal pricing $p^* = \underline{p}$, and also in term of local welfare.[15] So the platform can propose an optimal selling price $r^* : S(r^*) = D(0) = \bar{d}$. However, the same *does not* apply if $r = 0$ for $x \leq \hat{x}$. Indeed a market equilibrium is achievable by posting a price

[15] Of course the platform will not break even.

$p_z : D(p_z) = S(0)$ as $\bar{d} > \bar{s} > S(0)$, but it is no more optimal and $p_z > p^*$. Then incentives to install DPU are now:

$$\begin{aligned} I_Z(\phi) \;\; = \;\; & \mathbb{E}_{\mathcal{B}_z^P}\left[\Delta(\phi) - (p_z - a)(\phi - qx)\right] + \mathbb{E}_{\mathcal{B}_0^P}\left[\Delta(\phi) + a(\phi - qx)\right] \\ & + \mathbb{E}_{\mathcal{S}_0^P}\left[\Delta(\phi) - a(qx - \phi)\right] + \mathbb{E}_{\mathcal{S}_*^P}\left[\Delta(\phi) + (r^* - a)(qx - \phi)\right] \end{aligned}$$

where $\mathcal{B}^P$ and $\mathcal{S}^P$ are subdivided into $\mathcal{B}_z^P = [\xi_b^z(\phi), \hat{x}]$, $\mathcal{B}_0^P = [\hat{x}, \phi/q]$; $\mathcal{S}_0^P = [\phi/q, \hat{x}]$ and $\mathcal{S}_*^P = [\hat{x}, \xi_s(\phi)]$ where $\xi_b^z(\phi)$ is higher than $\xi_b(\phi)$ in the optimal case so $\mathcal{B}_z^P \subset \mathcal{B}_*^P$. First of all, we see that $I_Z(\phi)$ is positive for all ϕ as the trade sets are empty all together. Second, compared to the optimal case, zero pricing reduces these incentives in selling periods (the shortfall is not smaller) but increases them during buying periods only when the DPU availability is high. For low availability, a zero selling price implies a huge purchase price increases that drives consumers to turn to the grid. As a result, it is clear that $I_Z(\phi) < I_P(\phi)$.

Proposition 2 *Zero pricing creates less than optimal incentives but more than without platform.*

To sum up, zero pricing is not detrimental for investments in local generation capacities, but creates low powered incentives. As a result, zero pricing cannot be the clincher in the creation and growth of energy platforms.

For-profit platform

In the main analysis, we consider a welfare maximizing dealing platform. We have seen in section 2 that some P2P energy trading platforms have been developed by private investors or start-ups which are for-profit organizations. Let us now suppose that the platform has a profit objective that writes

$$\pi(x) = p(x)\, D(p(x)) - r(x)\, S(r(x))$$

One can see that the platform as a dealer is a local node acting as an upstream monopsony and a downstream monopoly. The for-profit platform problem in x is then

$$\max_{p,r} \pi(x) \quad \text{s.t.} \quad D(p) = S(r)$$

which leads to an integrated monopsony-monopoly (interior) equilibrium[16]

$$\frac{p^d - r^d}{p^d} > \frac{1}{\eta_D} \text{ and } S\left(r^d\right) = D\left(p^d\right)$$

[16]This standard analysis of price setting by an intermediary can be found in Spulber (1999) for instance.

where η_D is the price elasticity of demand. As a (non exclusive) dealer, the platform has upstream and downstream market power which implies more market power than stand-alone monopoly or monopsony. Hence compared to the grid price and the optimal prices, it both increases the energy price paid by consumers that are served through the platform and decreases the energy price received by prosumers that sell their energy in excess. These markups are possible as they incorporate partially the value of participating to the platform. In this case, the incentives to invest in DPU for an agent ϕ are $I_P^d(\phi) = \max\{\mathbb{E}[U] - k - (u(\phi) - \mathbb{E}[a\phi]), 0\}$ and compare with the non-profit platform (i.e. 6), when $q\mathbb{E}[ax] = k$, this leads to:

$$\begin{aligned} I_P^*(\phi) - I_P^d(\phi) &= \mathbb{E}_{\mathcal{B}_*^P}[\Delta(\phi) - (p^* - a)(\phi - qx)] + \mathbb{E}_{\mathcal{S}_*^P}[\Delta(\phi) + (r^* - a)(qx - \phi)] \\ &\quad - \mathbb{E}_{\mathcal{B}_d^P}\left[\Delta(\phi) - \left(p^d - a\right)(\phi - qx)\right] - \mathbb{E}_{\mathcal{S}_d^P}\left[\Delta(\phi) + \left(r^d - a\right)(qx - \phi)\right] \geq 0 \end{aligned}$$

where here lower indexes d and $*$ refer to the for-profit platform and welfare maximizing cases, respectively. Therefore $\mathcal{B}_d^P \subset \mathcal{B}_*^P$ and $\mathcal{S}_d^P \subset \mathcal{S}_*^P$ as $a < p^* < p^d$ and $r^d < r^* < a$. So a for-profit platform generates less incentives to install DPU among prosumers that trade less "often"[17] within the platform as price are at their limit values (maximum selling price and minimum purchase price). A important consequence is that, if all agents are prosumers without platform, i.e. $I_0 = 0$, then one may have $I_P^*(\phi) \geq I_0 = 0 > I_P^d(\phi)$: some prosumers do not invest anymore when the platform is for-profit.

To sum-up the discussion above, one can state the following Proposition:

Proposition 3 *With a for-profit platform, prices denoted* $\left(p^d, r^d\right)$ *are such that*

$$p^d > p^m > p^* \geq \underline{p} > a > \bar{r} \geq r^* > r^d \geq 0$$

where p^m *would be the monopoly-side price and* $r = 0$ *,the monopsony-side price (free purchase).*
The incentives to adopt DPU are reduced compared to the non-profit platform, and then some prosumers do not invest anymore whereas they will do without platform.

Matching platform

We look at a different way prosumers can find electricity through the platform, that is the dealer is now also matchmaker. The justification of alternative assumption is twofold. On one hand, as P2P energy trading platforms are supposed to mimic superstar digital platforms (as Uber, Blablacar and so on), matching can become a central issue of their business models. On the other hand, it comes from the standard literature on P2P markets where the matching process is at the core of the analysis. Following Goss *et al.* (2014), we

[17] That is to say the are active on the platform in a narrower set of states of nature.

assume that the platform is a closed environment in which the participants must declare themselves and install a DPU. In line with our main framework, we consider that the platform is non-profit, in the sense that it is welfare maximizing. Doing so they can be technically connected to the local micro-grid and at that time the matching's technology will make it possible to carry out exchanges between the participants (peer-to-peer exchanges) or if there is no match made between the participants and the central grid. The problem is to know which agents will participate in this platform, depending on purchase and selling prices that the platform designer may choose, possibly one for all the states of nature.

The matching technology depends on the relative size of potential supplies and demands to be matched in state x with a counterpart within the platform. Hence if their a (endogenous) mass of buyers participating on the platform that corresponds to a mass D of energy to be consumed and a mass of sellers that corresponds to a mass S of energy to be supplied, then we assume that the total number of matches is given by the well-known matching function[18]

$$M = M(S, D)$$

As is standard in the matching literature, the matching function $M(S, D)$ is assumed to be twice continuously differentiable, weakly increasing and concave such that $M(S, 0) = M(0, D) = 0$ and $M \leq \min\{S, D\}$. The platform is a random matchmaker such that all participants on the same side have the same probability of being matched

$$m_B = \frac{M(S, D)}{D} \text{ and } m_S = \frac{M(S, D)}{S}$$

Under these weak regularity conditions, it has been shown that the match probability of buyers m_B is weakly decreasing in own-side participation D which captures a negative own-side externality, and weakly increasing in cross-side participation S which captures a positive cross-side externality. The same applies to m_S. A common example $M(S, D) = S\,(1 - \exp\,(-D/S))$. Here the presence of the grid provides an non-zero outside option. It is useful to also define the matching elasticities for buyers and sellers respectively that write:

$$\psi^B = \frac{M'_D(S, D)D}{M(S, D)} \text{ and } \psi^S = \frac{M'_S(S, D)S}{M(S, D)}$$

There numbers lying in the interval $[0, 1]$, they represents the percentage increase in the total number of matches for a percentage increase in own-side participation.

On the dealer side, the necessity to maintain an overall grid balance implies that the matched demands and supplies must be equalized by the platform,[19] the non-matched trade

[18] This matching process is clearly exogenous in this context. A growing literature exists in order to ground one-to-one and one to many matching procedures (see Chade *et al.*, 2017). However the micro-foundations of our setting, that is many-to-many multidimensional matching with heterogeneous agents, are not yet established (see however Gomes and Pavan (2016) for a primer). It is left for future research.

[19] On this point we rely on the analysis of Benjaafar *et al.* (2018) concerning P2P car sharing.

on the platform being ensured by the central grid. Hence, the platform proposes *ex ante* a menu of prices $(p(x), r(x))_{x\in[01,]}$ that balances energy exchanges within inner participants in each state x, that is[20]:

$$m_B D = m_S S \tag{7}$$

where here D is the potential energy demanded by participants to the platform in state x when price p is observed and S is the potential energy to be supplied when price r is observed in state x. As demand and supplies are in real time scale, potential demands and supplies can be viewed *ex post* as described by (3) and (4). Indeed, at each state of nature, the prosumer will prefer to trade within among peers or with the grid, depending upon price conditions (a, p, r), so she may demand or supply energy as in market conditions. For example, a smart contract can be signed with the matchmaker which states purchases and selling conditions for the prosumer.

In a matching process, the economic value rises through the fact of being matched to a peer only within the platform rather than being served through the grid. As a result now the intrinsic value is affected by the probability of being served within the platform. To lighten notations and the analysis we simplify the model assuming $u(\phi) = v + \delta$ within the platform and $u(\phi) = v$ outside. So for an agent with a load factor ϕ the expected utility from trading through the platform in state x is

$$\mathbb{U}(\phi, x, q) = \begin{cases} v + m_B\delta - (m_B p + (1 - m_B)\, a)(\phi - qx) & \text{if } \phi \geq qx \\ v + m_S\delta + (m_S r + (1 - m_S)\, a)(qx - \phi) & \phantom{\text{if }} \phi < qx \end{cases}$$

Then *ex post* an agent will trade within the platform if her expected utility is greater then the surplus of trading with the grid only $\mathbb{U}(\phi, x, q) \geq v - a(\phi - qx) - \tau$ so as explained above, we find again the same demand and supply as described by (3) and (4). So we can state $D = D(p)$ and $S = S(r)$. As a result m_B and m_S depend on both (p, r) as

$$m_B(p, r) = \frac{M(S(r), D(p))}{D(p)} \text{ and } m_S(p, r) = \frac{M(S(r), D(p))}{S(r)}$$

Indeed, the probability of being matched for buyers is increasing function p and r and the probability of being matched for sellers is decreasing function r and p. Let us denote the *net* expected utility of trading within the platform

$$\mathbb{V}(\phi, x, q) = \begin{cases} m_B(\delta + \tau - (p - a)(\phi - qx)) & \text{if } \phi \geq qx \\ m_S(\delta + \tau + (r - a)(qx - \phi)) & \phantom{\text{if }} \phi < qx \end{cases}$$

[20] At the "rational-expectations" equilibrium, as suggested by Caillaud and Jullien (2003), this is always true.

Therefore the platform pricing is now impacted by the matching process as the expected local welfare when agents are matched with peers prosumers[21]

$$\mathbb{W}(x) = \int_{\underline{\phi}}^{\bar{\phi}} \mathbb{V}(\phi, x, q)\, dG + \pi(x)$$

where $\pi(x) = (p-a)\, m_B D(p) - (r-a)\, m_S S(r) = (p-r)\, M(S(r), D(p))$, o we can rewrite the platform welfare as:

$$\mathbb{W}(x) = m_B(p,r)\,\delta\{G(\beta(p)) - G(qx)\} + m_S(p,r)\,\delta\{G(qx) - G(\sigma(r))\}$$

Compared to the pure dealing platform, the matching process implies two-sided effects of pricing schemes that create countervailing forces that may operate. Resolving the matching platform problem which is to maximize $\mathbb{W}(x)$ for each state x, one can state the following lemma for pigovian pricing

Lemma 3 *Matching prices (p^{μ}, r^{μ}) are driven by the underlying matching technology and entail :*

$$\begin{aligned} p^{\mu} &= \max\{a + \left(1-\psi^B\right) A^B - \psi^B A^S, \bar{p}\} \\ r^{\mu} &= \min\{a + \psi^S A^B - \left(1-\psi^S\right) A^S, \bar{r}\} \end{aligned}$$

where ψ^B, ψ^S is the matching elasticities for a buyer and a seller respectively and A^B, A^S stands for the weighted net match valuation of buyers and sellers respectively.

For the matching platform increasing purchase price or selling price helps attracting buyers but it repels sellers. Decreasing prices do the reverse. Hence depending on the relative strength of the matching elasticities the matchmaker will prefer to push up a price than another. So it can be the case that for some state of nature (mainly for intermediate values of x) that both prices admit mark-ups in the sense that $p^{\mu} > \underline{p} > \underline{r} > r^{\mu}$. This well-known balancing mechanism is only possible if the matching technology exhibits decreasing and limited return to scale, that is when $\psi^B + \psi^S < 1$. If not, the pricing scheme will be bounded by the price limits $\bar{p}$ or $\bar{r}$, as demand and supply is also bounded in the platform.

[21] We go on with the convention that no markups are possible when the platform trades with the grid.

Proposition 4 *With a matching welfare maximizing platform, compared to optimal prices, there exists elasticities thresholds* $\psi_*^B \leq \bar{\psi}^B$ *and* $\psi_*^S \leq \bar{\psi}^S$ *such they are*

$$\begin{cases} p^\mu \geq p^* & \\ p^* \geq p^\mu \geq \bar{p} & \end{cases} for \begin{array}{c} \psi^B \leq \psi_*^B \leq \bar{\psi}^B \\ \psi_*^B \leq \psi^B \leq \bar{\psi}^B \end{array}$$

$$\begin{cases} r^\mu \leq r^* & \\ r^* \leq r^\mu \leq \bar{p} & \end{cases} for \begin{array}{c} \psi^S \leq \psi_*^S \leq \bar{\psi}^S \\ \psi_*^S \leq \psi^S \leq \bar{\psi}^S \end{array}$$

The more the matching technology is elastic, the more the incentives to adopt DPU are increased, compared to the non-profit dealing platform.

The last Proposition is quite intuitive. When the matching technology is rigid (i.e. $\psi^B \leq \psi_*^B$ and/or $\psi^S \leq \psi_*^S$) negative own-side externalities have a greater impact than positive cross-side externalities, as a result this calls for increasing the purchase price towards the weighted net match valuation of buyers or decreasing to the one of sellers respectively. When the matching technology is sufficiently elastic (i.e. $\psi^B \geq \psi_*^B$ and/or $\psi^S \geq \psi_*^S$) positive cross-side externalities are more effective so this calls for decreasing the purchase price towards the ceiling price or increasing the selling price to price cap. Finally, we turn to the incentives to install DPU created by the existence of the matching platform. For an agent ϕ, these incentives (if positive) are defined again by $I_M(\phi) = \mathbb{E}[\mathbb{U}] - k - (v - \mathbb{E}[a\phi])$ but now write

$$\begin{aligned} I_M(\phi) &= q\mathbb{E}[ax] - k \\ &\quad + \mathbb{E}_{\mathcal{B}}\left[m_B^\mu \left\{\delta + \tau - (p^\mu - a)(\phi - qx)\right\}\right] \\ &\quad + \mathbb{E}_{\mathcal{S}}\left[m_S^\mu \left\{\delta + \tau + (r^\mu - a)(qx - \phi)\right\}\right] \end{aligned}$$

where $m_j^\mu = m_j(p^\mu, r^\mu)$ for $j = B, S$. Again, compared to the no platform benchmark, the prosumers are not worse off. However, it is not clear if prosumers are more or less better off than with a dealing (welfare maximizing) platform, that is if $I_M(\phi) \geq (\leq) I_P(\phi)$. Indeed, first *ex ante* in all state of nature a possible match is possible, this has a positive effect on the incentives to install the unit. Second, if the matching technology is sufficiently elastic (i.e. $\psi^B \geq \psi_*^B$ and $\psi^S \geq \psi_*^S$) then prices tend to their respective bounds which also may boost prosumer's investments. Of course, the reverse holds if the matching technology is rigid. Finally, the matching itself as a uncertain process creates a depressive effect on the incentives to invest. As a result we cannot directly assess which effect will dominate.

Variable capacities

We now consider that agents can calibrate their DPU with respect to their load factor, that is now $q(\phi)$ is a variable depending on ϕ, at the equilibrium these prices will be impacted by the DPU choices made by the agents. We will seek at an continuous differentiable equilibrium path $q = q(\phi)$ where for each x. As q is a choice of an agent with profile ϕ, we now assume that a capacity up-front cost $k(q)$ that is increasing and convex for a production capacity q kWp.

First, when there is no platform the Lemma 1 still holds. Now the gross expected gain for a prosumer with profile ϕ is $\mathbb{E}[U_0] = u(\phi) - \mathbb{E}[a(x)(\phi - q(\phi)x)]$, so the incentives to adopt becomes $I_0(\phi) = \mathbb{E}[U_0] - k(q)$, where her marginal opportunity benefit writes $\frac{\partial \mathbb{E}[U_0]}{\partial q(\phi)} = \mathbb{E}[ax]$ is constant in q. This imply that all agents will install the same capacity $q_0(\phi) = q_0$ such that $\mathbb{E}[ax] = k'(q_0)$ for all ϕ. Second, when a dealing platform is active, following similar developments as above, one can again derive the dealer prices that are now function of the entire path $\{q(\phi)\}_{\phi\in[\underline{\phi},\bar{\phi}]}$ where the aggregate demand is now $d(p) = \int_{\hat{\phi}_x}^{\beta(p)} (\phi - q(\phi)x)\, dG$ where the switching load profile is now $\hat{\phi}_x : \phi = q(\phi)x$, and the aggregate supply $s(r) = \int_{\sigma(r)}^{\hat{\phi}_x} (q(\phi)x - \phi)\, dG$. The result in Lemma 2 still holds with the main change that $q = q(\phi)$ for each ϕ. This implies that the switching state $\hat{x}$ is now defined as $\hat{x} = \frac{\mathbb{E}(\phi)}{n\mathbb{E}(q(\phi))}$. Therefore the optimal capacity $q^*(\phi)$ maximizes $I(\phi) = \mathbb{E}[U] - k(q)$, the net expected surplus of an agent with a load factor ϕ, where $\mathbb{E}[U]$ is still defined by (5), replacing q by $q(\phi)$. Then this solution is driven by her marginal net gains from increasing the capacity, taking as given those of others agents on the platform, that is: $\frac{\partial I(\phi)}{\partial q} = \frac{\partial \mathbb{E}[U]}{\partial q} - k'(q)$. Now in general, connected to a platform, agents with different load factor will install different levels of capacity as $q^*(\phi)$ is such that

$$\begin{aligned}\frac{\partial I(\phi)}{\partial q} &= \frac{\partial I_0(\phi)}{\partial q} + \mathbb{E}_{\mathcal{B}^P}[(p^* - a)x] - \mathbb{E}_{\mathcal{B}^P}\left[\frac{\partial p^*}{\partial q(\phi)}(\phi - q^*(\phi)x)\right] \\ &\quad + \mathbb{E}_{\mathcal{S}^P}[(r^* - a)x] + \mathbb{E}_{\mathcal{S}^P}\left[\frac{\partial r^*}{\partial q(\phi)}(q^*(\phi)x - \phi)\right] = 0\end{aligned}$$

where $\mathcal{B}^G$ and $\mathcal{S}^P$ are subdivided into subsets depending on the platform pricing (as depicted in Lemma 2). Hence, one can see for each ϕ, $\frac{\partial I(\phi)}{\partial q} \neq \frac{\partial I_0(\phi)}{\partial q}$ so $q^*(\phi) \neq q_0$ in general. We also see that the local impact on prices are $\frac{\partial p^*}{\partial q(\phi)}$ and $\frac{\partial r^*}{\partial q(\phi)}$ are key variables to promote or to dampen the installation of DPU by prosumers. One can prove that in general that (for $\phi \in \mathrm{int}\mathcal{B}^P(x)$ or $\phi \in \mathrm{int}\mathcal{S}^P(x)$).

Lemma 4 *For $x < \hat{x}$, then $r^* = \bar{r}$; $p^* > \underline{p}$ and*

$$\frac{\partial p^*}{\partial q(\phi)} \leq 0 \textit{ for all } \phi \textit{ while } \frac{\partial \bar{r}}{\partial q(\underline{\phi})} > 0 \textit{ and } \frac{\partial \bar{r}}{\partial q(\phi)} = 0 \textit{ for all } \phi > \underline{\phi}$$

For $x > \hat{x}$, then $p^* = \underline{p}$; $r^* < \underline{p}$ and

$$\frac{\partial r^*}{\partial q(\phi)} \leq 0 \textit{ for all } \phi \textit{ while } \frac{\partial \underline{p}}{\partial q(\bar{\phi})} > 0 \textit{ and } \frac{\partial \underline{p}}{\partial q(\phi)} = 0 \textit{ for all } \phi < \bar{\phi}$$

As a result $q^*(\phi)$ is strictly increasing with ϕ.

Lemma 4, tells us that on one hand installing more DPU's reduces the demand on the buyer-side as self-consumption is more likely, but on the other hand it increases the supply on the seller-side. As a result, adjusted prices (i.e p^* or r^*) are reduced driven changes in the supply and demand fundamentals. More surprising, the corner prices $\bar{r}$ and $\underline{p}$ are positively impacted by investments for the extreme agents in terms of load. When the smallest consumer $\underline{\phi}$ invests she increases the maximal supply achievable $\bar{s}$ at a given state and then she also pushes up the maximum price. For the biggest consumer $\bar{\phi}$, investing reduces the maximal demand achievable $\bar{d}$ at a given state and then she pushes down the minimum price. The consequences of those price effects is that a prosumer with a higher load profile will install a higher DPU capacity. The intuition is that a prosumer with higher electricity needs is more often a buyer than a seller and she prefers to see a fall in the purchase price than a rise in the selling price, so she will prefer increasing her DPU capacities to achieve this goal.

Now for a given agent, depending on her load profile the marginal incentives to install DPU $\frac{\partial I(\phi)}{\partial q}$ differ. For all agents (expect $\underline{\phi}$ and $\bar{\phi}$), the marginal incentives to install DPU are boosted by the positive marginal price effects it yields on the purchase price as[22] $\mathbb{E}_{\mathcal{B}^P}\left[(p^* - a)\,x\right] - \mathbb{E}_{\mathcal{B}^P}\left[\frac{\partial p^*}{\partial q}\left(\phi - qx\right)\right] > 0$. But they are dampened by the negative marginal price effects they produce on the selling price as $\mathbb{E}_{\mathcal{S}^P}\left[(r^* - a)\,x\right] - \mathbb{E}_{\mathcal{S}^P}\left[\frac{\partial r^*}{\partial q}\left(qx - \phi\right)\right] < 0$. For the smallest consumer $\underline{\phi}$, this is increased by $\mathbb{E}_{\mathcal{S}^P}\left[\frac{\partial \bar{r}}{\partial q}\left(qx - \underline{\phi}\right)\right] > 0$ and for the biggest consumer $\bar{\phi}$, it is reduced by $-\mathbb{E}_{\mathcal{B}^P}\left[\frac{\partial \underline{p}}{\partial q}\left(\bar{\phi} - qx\right)\right] < 0$. At the end, we see that $\frac{\partial I(\phi)}{\partial q} - \frac{\partial I_0(\phi)}{\partial q}$ is not always positive, the marginal incentives to increase capacities are not always greater agents connected to a dealing platform . To sum-up one can state the following result.

Proposition 5 *With variable capacities, DPU capacities are increasing with the load profile and any agent connected to a dealing platform have not strictly superior marginal incentives to install DPU than without being connected.*

Of course on average, the distribution of loads matters to identify if more or less total capacities will be installed. As for the main analysis with fixed capacities (see Corollary 1), without further precise information on the state of of intermittency, one cannot assess which prosumer will be better off.

Individual shocks

In the main model, we assume that the state of the nature (i.e. intermittency) affects all prosumers in the same way, that is they are not affected by individual shocks. However

[22] We drop the argument ϕ.

one could argue that this assumption is no more justified if P2P platforms are connecting people that are located in different places.[23] What would some heterogeneity in DPU ability yield for the platform effectiveness? To analyze this let us, consider now that prosumers are affected by any state of nature x in a different way, i.e. the availability of the renewable capacity is distributed according to a cumulative type dependent that is $F(x, \phi)$. As a result, prior each prosumer is facing a different external conditions. However, one can see that the result in Proposition 1 is not deeply affected by this setting and one can state the result.

Proposition 6 *When agents are affected by individual shocks, the Corollary 1 still holds.*

Indeed, with no platform, the incentives to invest in DPU for an agent ϕ amount now $I_0(\phi) = \max\{q\mathbb{E}_\phi[a(x)x] - k, 0\}$, where $\mathbb{E}_\phi$ are expectations over x for an agent with profile ϕ. As a result, Lemma 1 is no more valid in the sense that now some agents may prefer not to join the platform, depending on the DPU capacity q, the capacity up-front cost k and grid pricing $a(x)$. A typical example is when $\mathbb{E}_\phi$ is a monotone increasing function of ϕ, that if small consumers face unfavourable DPU generation conditions and large consumers face favourable ones, then only large consumers will prefer becoming prosumers and install the DPU capacity, i.e. $\phi \geq \phi_0 : I_0(\phi_0) = 0$. On the contrary, when $\mathbb{E}_\phi$ decreases with ϕ, small consumers will prefer to install the DPU capacity. However, one cannot easily generalize such examples, and indeed $\mathbb{E}_\phi$ may show strong non monotonicity with respect to ϕ.

Nevertheless, considering the gains from joining a dealing platform for an agent that would install the DPU capacity without it, a weak version of Proposition 1 holds: if an agent installs a DPU when there is no platform, then she will does and will not be worse off when the dealing platform is active. Indeed, now $I_P(\phi) = I_0(\phi) + \mathbb{E}_\phi[U] - (u(\phi) - \mathbb{E}_\phi[a\phi])$ so if $I_0(\phi) > 0$, $I_P(\phi)$ does, as (6) still holds, replacing $\mathbb{E}[.]$ by $\mathbb{E}_\phi[.]$. In some sense this could justify the creation of communities of prosumers from an individual point of view.

7 Conclusion

In this paper, we provide a first economic analysis of how new models of peer-to-peer exchanges in the electricity sector may be effective and may yield sufficient incentives to invest in DPU based on renewable energy sources. We analyze how a P2P energy trading system could lead to a desirable economic outcome for a community of prosumers. We provide a simple model by considering first heterogeneity among prosumers with respect to their energy needs and second intermittency in the electricity production based on DRES.

[23]This is the idea of Internet Energy proposed by Rifkin mentioned in the Introduction, and also illustrated by the Walqa-Altenea experiment, see section 2.

In this context, we determine equilibrium price levels for a welfare maximizing dealing platform, showing purchase price are always greater than the central grid price, and selling prices are always lower. However, the expected net gains to be a trader within the platform are always greater than the expected average price on the grid. On top of that, we determine the optimal incentives to adopt DPU's in P2P energy trading platforms for participants, showing that they are always greater than the incentives when there is no platform. The intuition is that investing in DPU's allows the prosumer to increase her expected net gains to participate in the local trading. As a result, a welfare maximizing platform will always be profitable for a community of prosumers.

This strong result is challenged when we consider some extensions of our analysis. First, zero pricing is considered. It is a common reciprocity argument put forward to justify the emergence of these platforms as the short run marginal cost of DRES generation is near zero. We show that zero pricing creates less incentives than optimal ones but more than without platform. Second, as a bunch of P2P energy trading platforms have been developed by private investors or start-ups, one could think about for-profit platforms rather non-profit ones. With market power on both sides, such a platform will further increase purchase prices and lower selling prices, reducing the incentives to adopt DPU compared to the non-profit case. Consequently, some prosumers do not invest anymore whereas they will do without platform. Third, since P2P energy trading platforms learn their business models from digital platforms, matching will become a central issue. Considering a matching platform rather than a dealer impacts the market outcomes and incentives to adopt DPU. Mainly, we find that the more the matching technology is elastic, the more the incentives to adopt DPU are increased, compared to a pure dealer. Finally, we also challenge our main result considering that variable DPU size are possible and that intermittency shocks differ between prosumers. We show that with variable capacities, DPU capacities are increasing with the load profile, but the main result does not always hold. Indeed, any agent connected to a dealing platform have not strictly superior marginal incentives to install DPU than without being connected. When individual shocks are considered, the main result still holds (in a weak version): if an agent installs a DPU when there is no platform, then she will does and will not be worse off when the dealing platform is active.

Our results have some economic implications for the regulator and in terms of economic policy. First, as we have shown that P2P energy trading platforms have some economic relevance and local efficiency, regulators should ensure a level playing field for platform-based businesses and governments should support their emergence, in order to reap the benefits of P2P electricity trading. To some extent, this has been initiated in Europe, as the European Commission has defined P2P trading of renewable energy in EU Directive 2018/2001. However this is not applied worldwide, for instance in the United-States, where only microgrids are eligible to P2P trading. This has limited the implementation of the LO3 Energy (the Brooklyn experiment) in the public distribution network. Second, we have discuss about the role of the external grid price in the platform design and as a determinant of prosumer' participation and investments in DRES. Usually, at least one component of this external grid price is subject to regulation about energy authorities. As a result the integration of P2P trading platforms in the overall electric system could be subject to disproportionate or non-discriminatory (network) charges or procedures. This is a real challenge for regulators to ensure such a neutrality and transparency for the external grid price as in the future, P2P trading platforms could complement the wholesale electricity market.

Finally, some issues have been left aside and our analysis could be extended along at least three lines. First we have supposed the external grid pricing as totally exogenous. Grid pricing issues when prosumers are active have been analyzed by Gautier *et al.* (2018) but without considering a P2P trading system. The interdependence between the platform price equilibrium and grid pricing (regulated or market-based) may have strong impacts on both investments for prosumers and the network grid profitability. Following Abada *et al.* (2020b), one may anticipate the existence of a snowball effect between the expansion of platforms and the external grid price. The main idea is that depending on whether the grid pricing is average cost-based or marginal cost-based, the contraction of grid exchanges due to the existence of the trading platform may respectively increase or decrease the supply price to the grid. In return, this modifies the incentives to install DPU from potential prosumers.

Second, we have seen that P2P trading platforms are often located within microgrids. As a result, an issue is how the platform or the connected agents may provide electricity backups (batteries and storage capacities) instead of withdrawing/injecting electricity from/to the grid. At a first step, for each prosumer it will appear a trade-off between the storage costs for withdrawal/injection and the external grid price viewed as an opportunity cost.

Finally, in our analysis we have considered ITC devices and blockchain technologies needed for the effectiveness P2P trading platform, as black boxes. What are the improvements expected with ITC and blockchain technologies with smart contracting? This is left for further research.

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Appendix

Demand and Supply

Demands. If $p \leq a$, then all the agents connected to the platform will demand energy within it and then the demand is rigid i.e. $\bar{d} = \int_{qx}^{\bar{\phi}} (\phi - qx)\, dG$. If $p > a$, the demand is price-sensitive and we denote $\beta(p)$, the value of ϕ such that

$$\phi = qx + \frac{\Delta(\phi)}{p - a}$$

with $\beta'(p) = -\frac{\Delta/(p-a)^2}{1-(\Delta')/(p-a)} < 0$ and $\beta''(p) > 0$. Moreover, it may exist a maximum value of $\underline{p} > a$ such that $\beta(\underline{p}) = \bar{\phi}$ with $\frac{d\underline{p}}{da} = 1$, more precisely

$$\underline{p} = a + \frac{\Delta(\bar{\phi})}{\bar{\phi} - qx} \tag{8}$$

Using the notation $\dot{y} = \frac{dy}{dx}$, note that $\dot{\underline{p}} - \dot{a} = \frac{[u(\bar{\phi}+\delta) - u(\bar{\phi})]}{(\bar{\phi}-qx)^2} q > 0$. So if $p \geq \underline{p}$, the demand to the platform is

$$d(p) = \int_{qx}^{\beta(p)} (\phi - qx)\, dG = \int_{qx}^{\beta(p)} [G(\beta(p)) - G(\phi)]\, d\phi$$

with $d'(p) = \beta'(p)(\beta(p) - qx)\, g(\beta(p)) < 0$. As expected, the demand is normal (downward sloping). Note that this demand is always positive as we assumed that $\bar{\phi}/q \geq 1$.Note that with this vertical differentiation framework, the demand is never choked off. So the demand at state x is such that

$$D(p) = \begin{cases} \bar{d} & \text{if } p \leq \underline{p} \\ d(p) & \phantom{\text{if }} p > \underline{p} \end{cases}$$

Note that for all p, $\dot{\bar{d}} = -q(n - G(qx)) < 0$ with $\bar{d} = \mathbb{E}(\phi)$ if $x \leq \underline{\phi}/q$.

Supplies. An agent ϕ will be a (extra) supplier within the platform if

$$\Delta(\phi) + r(qx - \phi) \geq a(qx - \phi)$$

we denote $\sigma(r)$, the value of ϕ such that

$$\sigma(r) = qx - \frac{\Delta(\phi)}{a - r} \geq 0 \text{ if } r < a$$

with $\sigma'(r) < 0$. If $r > a$ then $\sigma(r) = \underline{\phi}$ and all potential suppliers to the platform. Moreover, it may exist a maximum value of $\bar{r} < a$ such that $\sigma(\bar{r}) = \underline{\phi}$ with $\frac{d\bar{r}}{da} = 1$. Namely

$$\bar{r} = a - \frac{\Delta(\underline{\phi})}{qx - \underline{\phi}} \tag{9}$$

note that $\dot{\bar{r}} - \dot{a} = \frac{\delta}{qx - \underline{\phi}} > 0$. Then if $r \geq \bar{r}$, the supply is rigid and equal to $\overline{s} = \int_{\underline{\phi}}^{qx} (qx - \phi)\, dG$. When $0 \leq r \leq \bar{r}$, the supply to the platform is

$$s(r) = \int_{\sigma(r)}^{qx} (qx - \phi)\, dG = \int_{\sigma(r)}^{qx} [G(\phi) - G(\sigma(r))]\, d\phi$$

Note that $s'(r) = -\sigma'(r)(qx - \sigma(r))\, g(\sigma(r)) > 0$, the supply is upward sloping. Of course, this supply is nil if $x \leq \underline{\phi}/q$. Note that as $\sigma(0) < qx$ then $S(0) > 0 :$ there are always sellers willing to sell electricity for free. Finally the supply at state x is such that

$$S(r) = \begin{cases} \overline{s} & \text{if } r \geq \bar{r} \\ s(r) & \phantom{\text{if }} r < \bar{r} \end{cases}$$

Note that for all r, $\dot{\overline{s}} = qG(qx) > 0$ and $\overline{s} = 0$ if $x \leq \underline{\phi}/q$.

Proof of Lemma 2

If $p < \underline{p}$ then $D(p) = \bar{d}$ and $r^* = S^{-1}(\bar{d})$ so using the mean theorem $W(x) = u(\bar{\nu} + \delta)\{n - G(\sigma(r^*))\}$, where $\bar{\nu} < \bar{\phi}$. If $r > \bar{r}$ then $S(r) = \overline{s}$ and $p^* = D^{-1}(\overline{s})$ so $W(x) = u(\underline{\nu} + \delta)\, G(\beta(p^*))$, where $\underline{\nu} > \underline{\phi}$, where $\bar{\nu} > qx > \underline{\nu}$. When $p \geq \underline{p}$ and $r \leq \bar{r}$, and $D(p) = S(r)$, let us enote the Lagrangean $L =$

$W(x) + \lambda(d(p) - s(r)) + \lambda_p(p - \underline{p}) + \lambda_r(\bar{r} - r)$, with $\lambda \neq 0$, a Lagrange multiplier, others $(\lambda_p, \lambda_r) \geq 0$, Khun-Tucker multipliers, first-order conditions imply:

$$\begin{aligned}
\frac{\partial L}{\partial p} &= \{u(\beta(p) + \delta) + \lambda(\beta(p) - qx)\} g(\beta(p)) \beta'(p) + \lambda_p = 0 \\
\frac{\partial L}{\partial r} &= \{-u(\sigma(r) + \delta) + \lambda(qx - \sigma(r))\} g(\sigma(r)) \sigma'(r) - \lambda_r = 0 \\
\frac{\partial L}{\partial \lambda} &= d(p) - s(r) = 0 \\
\lambda_p \frac{\partial L}{\partial \lambda_p} &= \lambda_p(p - \underline{p}) = 0 \\
\lambda_r \frac{\partial L}{\partial \lambda_r} &= \lambda_r(\bar{r} - r) = 0
\end{aligned}$$

which gives

$$\begin{aligned}
\{u(\beta(p) + \delta) + \lambda(\beta(p) - qx)\} g(\beta(p)) \beta'(p) &= -\lambda_p \leq 0 \\
\{-u(\sigma(r) + \delta) + \lambda(qx - \sigma(r))\} g(\sigma(r)) \sigma'(r) &= \lambda_r \geq 0
\end{aligned}$$

so if $p > \underline{p}$ and $r < \bar{r}$ then $(\lambda_p, \lambda_r) = (0, 0)$ and we have the contradiction: $\lambda = u(\sigma(r)+\delta)/(qx - \sigma(r)) > 0$ and

$$0 < u(\beta(p) + \delta) + \lambda(\beta(p) - qx) = 0$$

If $p > \underline{p}$ and $r = \bar{r}$ then $\lambda_p = 0$ so $\lambda = -u(\beta(p) + \delta)/((\beta(p) - qx)) < 0$ and

$$\lambda_r = -\left\{u(\underline{\phi} + \delta) + u(\beta(p) + \delta)\frac{qx - \underline{\phi}}{\beta(p) - qx}\right\} g(\underline{\phi}) \sigma'(\bar{r}) > 0$$

with $p^* = d^{-1}(\bar{s})$, this implies that $\bar{s} = s(\bar{r}) > \bar{d}$. When $p = \underline{p}$ and $r < \bar{r}$ then $\lambda_r = 0$ so $\lambda = u(\sigma(r) + \delta)/(qx - \sigma(r)) > 0$ and

$$\lambda_p = -\left\{u(\bar{\phi} + \delta) + u(\sigma(r) + \delta)\frac{\bar{\phi} - qx}{qx - \sigma(r)}\right\} g(\bar{\phi}) \beta'(\underline{p}) > 0$$

with $r^* = s^{-1}(d)$ wenever $\bar{s} < \bar{d}$. Finally if $p^* = \underline{p}$ and $r^* = \bar{r}$ for which it may exist a level of $x = \hat{x} : \bar{s} = \bar{d}$, such that

$$\bar{s} - \bar{d} = \int_{\underline{\phi}}^{\bar{\phi}} (\phi - qx)\, dG = \mathbb{E}(\phi) - nqx = 0$$

here expectations are over ϕ, so

$$\hat{x} = \frac{\mathbb{E}(\phi)}{nq}$$

As a result there are a multiple of solutions. If $x \leq \hat{x}$ then $p^* = d^{-1}(\bar{s}) > 0$ and $r^* \in [\bar{r}, +\infty]$ and if $x \geq \hat{x}$ then $p^* \in [0, \underline{p}]$ and $r^* = s^{-1}(\bar{d}) > 0$. Of course if we add a breakeaven constraint for the plateform account $\pi(x) = (p - r)\min\{\bar{d}, \bar{s}\} \geq 0$ then this restrict the set of optima to $p^* \geq r^*$. This restrict selling prices to $r^* \in [\bar{r}, p^*]$ when $x \leq \hat{x}$ and purchase prices to $p^* \in [r^*, \underline{p}]$ when $x \geq \hat{x}$.

Proof of Lemma 4

From (8) and (9) in the Proof of Lemma 2 one can derive that

$$\begin{aligned} \frac{\partial \underline{p}}{\partial q(\bar{\phi})} &= \frac{\Delta(\phi)\, x}{\left(\bar{\phi} - q(\bar{\phi})\, x\right)^2} > 0 \text{ and } \frac{\partial \underline{p}}{\partial q(\phi)} = 0 \text{ for all } \phi \neq \bar{\phi} \\ \frac{\partial \bar{r}}{\partial q(\underline{\phi})} &= \frac{\Delta(\phi)\, x}{\left(q(\underline{\phi})\, x - \underline{\phi}\right)^2} \geq 0 \text{ and } \frac{\partial \bar{r}}{\partial q(\phi)} = 0 \text{ for all } \phi \neq \underline{\phi} \end{aligned}$$

Moreover if $0 \leq x \leq \hat{x}$ then $d(p^*) = \bar{s}$ then for a given ϕ such that

$$d'(p) \frac{\partial p^*}{\partial q(\phi)} = \frac{\partial \bar{s}}{\partial q(\phi)} - \frac{\partial d(p)}{\partial q(\phi)}$$

- $\phi \in [\underline{\phi}, \sigma(r^*)[$ or $\phi \in [\beta(p^*), \bar{\phi}[$ then $\frac{\partial p^*}{\partial q(\phi)} = 0$
- $\phi \in [\sigma(r^*), \hat{\phi}_x]$ then $d'(p) \frac{\partial p^*}{\partial q(\phi)} = \frac{\partial \bar{s}}{\partial q(\phi)} - 0 = xg(\phi) \geq 0 \Rightarrow \frac{\partial p^*}{\partial q(\phi)} \leq 0$
- $\phi \in [\hat{\phi}_x, \beta(p^*)]$ then $d'(p) \frac{\partial p^*}{\partial q(\phi)} = 0 - \frac{\partial d(p)}{\partial q(\phi)} = xg(\phi) \geq 0 \Rightarrow \frac{\partial p^*}{\partial q(\phi)} \leq 0$.

If $1 \geq x \geq \hat{x}$ then $s(r^*) = \bar{d}$ then for a given ϕ such that $s'(r) \frac{\partial r^*}{\partial q(\phi)} = \frac{\partial \bar{d}}{\partial q(\phi)} - \frac{\partial s(r)}{\partial q(\phi)}$

- $\phi \in [\underline{\phi}, \sigma(r^*)[$ or $\phi \in [\beta(p^*), \bar{\phi}[$ then $\frac{\partial r^*}{\partial q(\phi)} = 0$
- $\phi \in [\sigma(r^*), \hat{\phi}_x]$ then $s'(r) \frac{\partial r^*}{\partial q(\phi)} = 0 - \frac{\partial s(r)}{\partial q(\phi)} = -xg(\phi) \leq 0 \Rightarrow \frac{\partial r^*}{\partial q(\phi)} \leq 0$
- $\phi \in [\hat{\phi}_x, \beta(p^*)]$ then $s'(r) \frac{\partial r^*}{\partial q(\phi)} = \frac{\partial \bar{d}}{\partial q(\phi)} - 0 = -xg(\phi) \leq 0 \Rightarrow \frac{\partial r^*}{\partial q(\phi)} \leq 0$

Finally, for all $\phi \neq \left(\underline{\phi}, \bar{\phi}\right)$, one can derive

$$\left[\frac{\partial^2 \mathbb{E}[U]}{\partial q^2(\phi)} - k''(q^*(\phi))\right] \frac{dq^*(\phi)}{d\phi} = \mathbb{E}_{\mathcal{B}^P}\left[\frac{\partial p^*}{\partial q(\phi)}\right] + \mathbb{E}_{\mathcal{S}^P}\left[\frac{\partial r^*}{\partial q(\phi)}\right] \leq 0$$

so if $\frac{\partial^2 \mathbb{E}[U]}{\partial q^2} - k''(q) \leq 0$ then $q^*(\phi)$ is unique solution for the prosumer's problem $\max_q \mathbb{E}[U] - k(q)$ and then we have $\frac{dq^*(\phi)}{d\phi} \geq 0$.

Proof of Proposition 3

From the market clearing condition $D(p) = S(r)$ one can define a locus $\hat{r}(p)$ such that

$$S(\hat{r}(p)) = D(p)$$

which entails $r^*(p)$ decreasing in $p \in [\underline{p}, \infty[$ whenever $S(0) < D\left(\underline{p}\right)$ i.e.

$$D\left(\underline{p}\right) - S(0) = \int_{qx - \frac{\Delta}{a}}^{\bar{\phi}} (\phi - qx)\, dG > 0$$

So the dealer problem writes $\max_{p \geq \underline{p}} (p - \hat{r}(p))\, D(p)$ and the first order condition gives:

$$\begin{aligned} \frac{p^d - \hat{r}\left(p^d\right)}{p^*} &= \frac{1 - \hat{r}'\left(p^d\right)}{\eta_D} > \frac{1}{\eta_D} \\ S(\hat{r}(p^*)) &= D(p^*) \end{aligned} \tag{10}$$

where

$$\eta_D = -\frac{D'(p)\,p}{D(p)} > 0$$

so $p^d \geq p^m$. As a result

$$p^d > p^* \geq \underline{p} \text{ and } \bar{r} \geq r^* > r^d$$

Proof of Lemma 3

Assume that $p > \underline{p}$ and $r < \bar{r}$, then (interior) first order conditions write:

$$\frac{\partial \mathbb{W}(x)}{\partial p} = 0 \quad \text{and} \quad \frac{\partial \mathbb{W}(x)}{\partial r} = 0$$

Derivatives write

$$\begin{aligned}
\frac{\partial \mathbb{W}(x)}{\partial p} &= m_B(p,r)\,\delta g(\beta(p))\,\beta'(p) \\
&\quad + \frac{\partial m_S(p,r)}{\partial p}\delta\left[G(qx) - G(\sigma(r))\right] + \frac{\partial m_B(p,r)}{\partial p}\delta\left[G(\beta(p)) - G(qx)\right] \\
\frac{\partial \mathbb{W}(x)}{\partial r} &= -m_S(p,r)\,\delta g(\sigma(r))\,\sigma'(r) + \frac{\partial m_B(p,r)}{\partial r}\delta\left[G(\beta(p)) - G(qx)\right] \\
&\quad + \frac{\partial m_S(p,r)}{\partial r}\delta\left[G(qx) - G(\sigma(r))\right]
\end{aligned}$$

with

$$\begin{aligned}
\frac{\partial m_B(p,r)}{\partial p} &= m_B(p,r)\left(\psi^B - 1\right)\frac{D'(p)}{D(p)} \;\;;\;\; \frac{\partial m_B(p,r)}{\partial r} = m_B(p,r)\,\psi^S\frac{S'(r)}{S(r)} \\
\frac{\partial m_S(p,r)}{\partial r} &= m_S(p,r)\left(\psi^S - 1\right)\frac{S'(r)}{S(r)} \;\;\text{and}\;\; \frac{\partial m_S(p,r)}{\partial p} = m_S(p,r)\,\psi^B\frac{D'(p)}{D(p)}
\end{aligned}$$

where

$$\psi^B = \frac{M'_D(S,D)D}{M(S,D)} \;\;;\;\; \psi^S = \frac{M'_S(S,D)S}{M(S,D)}$$

are the matching elasticities for buyers and sellers respectively such that

$$0 \leq \psi^j \leq 1 \text{ for } j = B, S$$

So one can rewrite FOC using $D'(p) = \beta'(p)\,(\beta(p) - qx)\,g(\beta(p))$ and $S'(r) = -\sigma'(r)\,(qx - \sigma(r))\,g(\sigma(r))$:

$$\begin{aligned}
\frac{\partial \mathbb{W}(x)}{\partial p} &= 0 = \frac{D'(p)}{D(p)}\left\{m_B(p,r)\,\delta\frac{D(p)}{\beta(p) - qx}\right. \\
&\quad \left. + m_S(p,r)\,\psi^B\delta\left[G(qx) - G(\sigma(r))\right] + m_B(p,r)\left(\psi^B - 1\right)\delta\left[G(\beta(p)) - G(qx)\right]\right\}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mathbb{W}(x)}{\partial r} &= 0 = \frac{S'(r)}{S(r)}\left\{m_S(p,r)\,\delta\frac{S(r)}{qx - \sigma(r)}\right. \\
&\quad \left. + m_B(p,r)\,\psi^S\delta\left[G(\beta(p)) - G(qx)\right] + m_S(p,r)\left(\psi^S - 1\right)\delta\left[G(qx) - G(\sigma(r))\right]\right\}
\end{aligned}$$

Using definitions of $\beta(p)$ and $\sigma(r)$, this leads to define (p^μ, r^μ) as :

$$\begin{aligned} p^\mu &= a + \left(1-\psi^B\right) A^B - \psi^B A^S \qquad (11) \\ r^\mu &= a + \psi^S A^B - \left(1-\psi^S\right) A^S \qquad (12) \end{aligned}$$

where

$$A_B = \frac{G(\beta(p^\mu)) - G(qx)}{D(p^\mu)}\delta > 0 \text{ and } A_S = \frac{G(qx) - G(\sigma(r^\mu))}{S(r^\mu)}\delta > 0$$

These expressions stand for the weighted net match valuation of buyers and sellers respectively. Note that this solution is not valid for matching technology characterized by constant or increasing returns to scale (Cobb Douglas technology for instance), indeed $\psi^B + \psi^S \geq 1$:

$$r^\mu \geq a + \left(\psi^S A^B - \psi^B A^S\right) \geq p^\mu$$

so one cannot verify $p^\mu > \underline{p} > \underline{r} > r^\mu$.

Conditions (11) and (9) are reminiscent of (17) in Goss *et al.* (2014) in a different context. Existence for the interior solution is ensured by the "rational-expectations" equilibrium we adopted as suggested by Caillaud and Jullien (2003).

By the mean theorem we see that $d(p) = (\varphi - qx)(G(\beta(p)) - G(qx))$ where $\varphi < \beta(p) \leq \bar{\phi}$, so

$$a + A_B > \bar{p}$$

Identically, $s(r) = (qx - \xi)(G(qx) - G(\sigma(r)))$ where $\xi > \rho(r) \geq \underline{\phi}$, so

$$a - A^S < \bar{r}$$

So it exists value of the elasticies (i.e. forms of the underlying matching technology) such that the interior solution is valid for some x

$$\begin{aligned} \psi^B &\leq \bar{\psi}^B = \frac{(qx-\xi)(\bar{\phi}-\varphi)}{(\varphi-\xi)(\bar{\phi}-qx)} \geq 0 \\ \psi^S &\leq \bar{\psi}^S = \frac{(\varphi-qx)(\xi-\underline{\phi})}{(\varphi-\xi)(qx-\underline{\phi})} \geq 0 \end{aligned}$$

As $\bar{\psi}^B$ is monotonically increasing with respect to x and maps $[\frac{\xi}{q}, \min\{1, \frac{\bar{\phi}}{q}\}]$ into $[0, +\infty]$ so it exists aunique $x_b \in]\frac{\xi}{q}, \min\{1, \frac{\bar{\phi}}{q}\}[$: $\bar{\psi}^B = 1$. Identically, $\bar{\psi}^S$ is monotonically decreasing with respect to x and maps $[\frac{\underline{\phi}}{q}, \min\{1, \frac{\varphi}{q}\}]$ into $[0, +\infty]$ so it exists a unique $x_s \in]\frac{\underline{\phi}}{q}, \min\{1, \frac{\varphi}{q}\}[$: $\bar{\psi}^S = 1$. As a result it existe a unique $x_e \in]x_s, x_b[$ such that $\bar{\psi}^B = \bar{\psi}^S$ when $x = x_e$.

As a result, the interior solution is valid for some underlying matching technologies (with decreasing return to scale) and some state of nature. Otherwise a corner solution applies which implies either $p = \underline{p}$ or $r = \bar{r}$.

Proof of Proposition 4

One can see that it exists levels of $\left(\psi_*^B, \psi_*^S\right)$ such that

$$\begin{aligned} p^\mu &= a + \left(1 - \psi_*^B\right) A^B - \psi_*^B A^S = p^* \\ r^\mu &= a + \psi_*^S A^B - \left(1 - \psi_*^S\right) A^S = r^* \end{aligned}$$

Indeed when at p^* such that $D\left(p^*\right) = \bar{s}$ we have $p^* = a + \frac{\delta}{\beta(p^*) - qx}$ and one can rewrite

$$p^\mu\left(\psi^B\right) = a + \left(1 - \psi^B\right) \frac{\delta}{\varphi - qx} - \psi^B \frac{\delta}{qx - \xi}$$

so $p^\mu(0) > p^*$. As $p^\mu\left(\psi^B\right)$ is linear decreasing in ψ^B it exists $\psi_*^B : p^\mu\left(\psi_*^B\right) = p^*$. Same reasoning applies for r^μ.

Hence one can depicts prices as

$$\begin{array}{ll} p^\mu \geq p^* & \text{for} \quad \psi^B \leq \psi_*^B \leq \bar{\psi}^B \\ p^* \geq p^\mu \geq \bar{p} & \text{for} \quad \psi_*^B \leq \psi^B \leq \bar{\psi}^B \\ r^\mu \leq r^* & \text{for} \quad \psi^S \leq \psi_*^S \leq \bar{\psi}^S \\ r^* \leq r^\mu \leq \bar{p} & \text{for} \quad \psi_*^S \leq \psi^S \leq \bar{\psi}^S \end{array}$$