Peer-to-Peer Energy Platforms: Incentives for Prosuming

Thomas Cortade, Jean-Christophe Poudou

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Thomas CORTADE† and Jean-Christophe POUDOU‡

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Abstract

In this paper, we analyze how new models of exchanges in the electricity sector may be viable and yield incentives to invest in decentralized domestic production units based on renewable energy sources. We try to identify the factors and the elements in the platform design that influence participation of prosumers in peer-to-peer energy exchanges in local microgrids. Compared to the no-platform configuration, we find that a pure dealing platform exhibits no less incentives to install domestic production units. However, this main result is challenged by considering several relevant features for peer-to-peer energy exchanges.

JEL classification: L14, L81, L94, Q4
Keywords: Peer-to-peer, electricity, trading platform, renewables

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†MRE, Univ Montpellier, Montpellier, France thomas.cortade@umontpellier.fr

‡Correspoding author: jean-christophe.poudou@umontpellier.fr, MRE, Univ Montpellier, Espace Richter, avenue Raymond Dugrand - CS 79606 34960, Montpellier Cedex 2, France

1 Introduction

Europe's 2050 targets for reducing CO2 emissions, promoting renewable energies and reducing energy consumption are drastic and require the implementation of strong public policies.[1] As part of the energy transition, the development of smart grids represents a major challenge: thanks to new technologies and smart grids, it will be possible to increase the share of renewable energies and reduce energy consumption. Moreover, in 2016 the European Parliament adopted at first reading on 13 November 2018 with a view to the adoption of a new Directive of the European Parliament and of the Council in order to promote of the use of energy from renewable sources. This legal process should favor the development of new trading arrangements and new technological improvements in energy systems.[2] By 30 June 2021, national governments will need to transpose the laws (and the community energy rights) into their legal system.

The development of smart grids in the field of energy focused initially on the reliability and security of networks: the deployment of smart meters, development of energy oriented IoT (Internet of Things) and the fine use of data on energy consumption can facilitate balancing these networks through better demand-side management and increased opportunities for interruptibility during peak periods. Beyond the technical issues, the organization of smart grids has mobilized economists around the analysis of costs and prices, in particular the analysis of tariffs as a means of reducing electricity demand during peak periods (peak-load pricing, NEBEF rules, capacity trading), thus helping to reduce energy consumption and CO2 emissions.

Smart grids opens up new perspectives and a revolution in the energy field. The deployment of peer-to-peer (P2P) electricity exchange platforms, like Airbnb or Uber platforms, is the basis for significant societal changes that will make it possible to achieve the objectives of the energy transition.

P2P: related literature The recent economic literature applied to platforms has mainly developed on the basis of questions raised by the emergence of service platforms such as eBay, Uber and Airbnb. The main objective of these platforms is to facilitate exchanges between a large number of heterogeneous buyers and sellers and fragmented. It was initially to exchange current consumer goods or services (or intermediate), then cultural goods.

[1] See, for example, the European Parliament resolution of 14 March 2013 on the 2050 energy guidelines, which sets a target of reducing greenhouse gas emissions by 80 to 95% compared to 1990.

[2] Thus, the article 21-2a indicates that Member States shall ensure that renewables self-consumers, individually or through aggregators, are entitled: "*to generate renewable energy, including for their own consumption, store and sell their excess production of renewable electricity, including through renewables power purchase agreements, electricity suppliers and peer-to-peer trading arrangements*".

The development of these exchange platforms has naturally brought out new economic issues concerning the economic model of the players and their pricing strategies, the economic regulation of these activities, market efficiency measures and their societal impact.

Krishnan *et al.*(2003) argue that P2P networks could be perceived either as public goods (non rival in demand and non excludable in supply), or as club goods (excludable in supply but non rival in demand). Their article provides an overview of P2P networks. They focus on the user behavior, such that free-ridding, that means if a user provides any resource on the network, but consumes network resources. This is typically the case for music or video sharing network. Basically, with such behaviors a P2P network could collapse. This effect could be mitigated if the users' participation is conditioned by altruism, and if the viability of P2P networks is based on trust and reputation.

From the point of view of the economic analysis, Einav *et al.* (2016) provide an interesting analysis based on common elements to all these P2P platforms : low entry costs for sellers or suppliers, an intermediation role for the platform owner, better monitoring buyers and sellers via technology, the introduction of flexible or auction-based prices and other sophisticated pricing mechanisms. A first key question according Einav *et al.* (2016) is the problem of matching buyers and sellers. Both important informational problems exist. Given the heterogeneity between buyers and differentiated sellers, the information is dispersed, so it is necessary to use information efficiently. The second problem is to keep low the transaction costs. They propose two matching process. A process in which the platform centralize the demands such as Uber which allows to keep low transactions costs. Another process based on decentralization, using by Airbnb for example, allows to take account into personal choices for the buyers. Then a second key question is related to the pricing mechanism. They consider trade-off between auctions and posted prices. On this point, Einav *et al.* (2018) provide a complete analysis about both pricing mechanism by using data of Ebay. A result well-known in the literature shows that auction mechanisms preferable if buyers owns private information about their willingness to pay,[3] and this mechanism provides an efficient allocation. Einav *et al.* (2018) show that the use of auction mechanisms decline for at least one decade, not only for Ebay but it is a general trend. On this point Einav *et al.* (2016) argue that *"in practice,using auctions can be cumbersome to identify potential buyers and sellers and to elicit information from them"*. By contrast, the posted prices are preferable if there exist few buyers, if the buyers are impatient or if there exist queuing for them.[4] We can think that is the case when we consider P2P energy platforms/ smarts grids. But an important constraint is imposed when we consider the energy trading since supplies and demands have to balanced.

The development of self-consumption modifies traditional economic models based on a clear distinction between consumer and energy producer. The consumer is led to internalize

[3]On this point see Harris and Townsend (1981); Myerson (1981); Riley and Samuelson (1981)
[4]See Wang 1993; Ziegler and Lazear 2003 for example

the production constraint of renewable energies and to become his own producer. The P2P electricity platforms make it possible to better organize groups of actors that can be extracted from traditional commercial circuits to exchange energy with each other. These platforms are primarily developed at the territorial level, while remaining connected to the national grid. We present below the different experiences Considering these elements, the economic literature was first of all concerned with the design of markets (market platforms) based on the experiences of the digital markets (see cf. Hagiu, Jullien (2010)), then on favorable conditions for production and prices adapted to irregular or intermittent demands, but also to certain emerging issues of economic regulation. Several experiences of energy platforms has been realized in the word, and we give presentation of the most significant experiences on microgrids and smart grids.[5]

Microgrids and Smart grids : related experiences Obviously, there exist several projects in Europe and United States, and we can rank them according to their geographic scope. Hence, the network size of the following projects, Piclo (UK), Vandebron (Netherland), SonnenCommunity (Germany) and Litchtblick Swarm Energy (Germany) is national, whereas the network size of Smart Watts (Germany), Yeloha, Mosaic (US) is regional. As Zhang et *al.* (2017) said that all these projects platforms support P2P energy trading among their participants. The smallest size of platforms (grids) such as TransActive Grid (US) and Electron are interessed by a local P2P market. The authors underline that but blockchain technology is used in order to simplify metering and billing system. Finally they provide a very interesting comparison between the different grid projects and Sousa *et al.*(2019) do the same for R& D projects with a relation to P2P markets.

According to Zhang et *al.* (2017) the future P2P energy trading will be based on three levels. The first level represents a P2P energy trading within a microgrid that means within a eco-neighborhood for example like Lyon Confluence or Nice Meridia in France. The second level is characterized by tradings between between several microgrids (Multi-Microgrids, P2P within CELL). This is the case of the smart-grid called Walqa & Atenea is located in Spain. Walqa & Atenea are two connected microgrids, that are 150 km away. Basically, in this case each microgrid own one's own distribution grid, but energy trading is also possible between them. Finally, the third level corresponds to P2P among CELLs (Multi-CELLs). The two last levels raise the question of interconnection and its regulation, but also of the scheme pricing. But for each level it is interesting to ask how the exchanges, among the participants to P2P energy trading, of self-produced energy can be realized.

On this point, Mengelkamp *et al.* (2018) deals with the fact that consumers and prosumers can trade by using a P2P operating condition in microgrid energy markets without central intermediaries like a aggregator. More precisely, the authors investigate the incentives, both for consumers and prosumers, to participate and to invest in the P2P platform,

[5]See also Gangale *et al.* (2017) for an overview of European smart grids projects. doi:10.2760/701587

Project Name	Country	Start Year	Objectives	Network Size	P2P? Layers	Outcomes	Shortcoming
Piclo	UK	2014	P2P energy trading platform from suppliers perspective	National	Business	A P2P energy trading platform	No discussion on local markets
Vandebron	Netherland	2014	P2P energy trading platform from suppliers perspective	National	Business	A P2P energy trading platform	No discussion on local markets
PeerEnergyCloud	Germany	2012	Cloud-based P2P Energy Trading Platform, Smart Home	Microgrids	Energy Network, ICT	Cloud-based platform for smart homes	No discussion on control system
Smart Watts	Germany	2011	Optimizing energy supply via ICT	Regional	Energy Network, ICT	A smart meter gateway as interface to Internet of energy	No discussion on control system
Yeloha, Mosaic	US	2015	Solar sharing network for lower energy bills.	Regional	Business	Terminated due to funding issues	No discussion on local markets
SonnenCommunity	Germany	2015	P2P energy trading with storage system	National	Energy Network, Business	A P2P energy trading platform (online)	No discussion on local markets
Lichtblick Swarm Energy	Germany	2010	IT platform for energy markets and customers	National	Energy Network, ICT	Plenty of services provided by the energy supplier	No discussion on local markets
Community First! Village	US	2015	Energy sharing from donations	Community	Business	Saving energy bills for poor people	No discussion on ICT and control system
TransActive Grid	US	2015	P2P energy trading within Microgrids using Blockchain	Grid-connected Microgrids	Energy Network, Control, ICT, Business	Automatic energy trading platform within Microgrids	Communication before exchange was ignored
Electron	UK	2016	Energy metering and billing platform using Blockchain	Unknown	Energy Network, ICT, Business	Not started yet	Not started yet

Figure 1: Comparison of different projects, from Zhang *et al.* *(2017)*

and the incentives to balance locally demand and supply. Moreover, they focus on the microgrid management by the market participants and so their cooperative behaviors. For that, it is necessary to use an innovative system without central intermediaries. Thus the authors present blockchain system as a solution, by analyzing the case of Brooklyn microgrid (TransActive Grid).[6]

They show that blockchain-based microgrid could be an efficient technology, according to seven components.

1. Microgrid setup in order to answer to the following question : who are the market members and what is the type of energy traded ? It is an essential question since heterogeneous participants could have conflicting interests. Moreover, what is the optimal size for the market ? This setup has implication in particular on the pricing schemes

2. Grid Connection : important in order to be able to balance demand and supply

3. Information system : an efficient information system is essential to connect the participants, and to ensure the absence of discrimination to the platform access.

4. Market mechanism : market's allocation and pricing schemes

[6]The project called Electron, in UK, is also blockchain-based microgrid.

Project name	Country/Region	Starting year	Focus level	Outcomes	Classification
P2P-SmartTest	Europe (Finland, United Kingdom, Spain, Belgium)	2015 (ongoing)	Distribution grid level	Advanced control and ICT for P2P energy market	Local control and ICT; Market design
EMPOWER	Europe (Norway; Switzerland, Spain, Malta, Germany)	2015 (ongoing)	Distribution grid level	Architecture and ICT solutions for provider in local market	Local control and ICT
NRGcoin	Europe (Belgium, Spain)	2013 (finish)	Consumer/ prosumer	P2P wholesale trading platform	Market design
Enerchain	Europe	2017 (ongoing)	Wholesale market	P2P wholesale trading platform	Market design
Community First! Village	USA	2015 (ongoing)	Consumer/ prosumer	Build self-sustained community for homeless	Local control and ICT
PeerEnergy Cloud	Germany	2012 (finish)	Consumer/ prosumer	Cloud-based energy trading for excessive production	Local control and ICT
Smart Watts	Germany	2011 (finish)	Consumer/ prosumer	ICT to control consumption in a secure manner	Local control and ICT
NOBEL	Europe (Germany, Spain, Greece, Sweden, Spain)	2012 (finish)	Consumer/ prosumer	ICT for energy brokerage system with consumers	Local control and ICT
Energy Collective	Denmark	2016 (ongoing)	Consumer/ prosumer	Deployment of local P2P markets in Denmark	Market design
P2P3M	Europe (United kingdom), Asia (South Korea)	2016 (ongoing)	Consumer/ prosumer	Prototype P2P energy trading/sharing platform	Market design

Figure 2: Comparison of different R&D projects, from Sousa *et al.* *(2018)*

5. Pricing mechanism : it is issued from the market mechanism. The objective of this pricing mechanism (auction or posted prices) is to reach the most efficient allocation between supply and demand.

6. Energy Management Trading System : in order to secure the energy supply. It is also necessary to have an access to the data in real time (both for demand and supply)

7. Regulation : it refers to a Cost–benefit analysis since on the one hand the government may encourage the development of microgrids based on renewable energy sources and so which provide a solution for environment. But on the other hand it may discourage this development since microgrids would negative impact on the centralized system.

Generally it seems necessary to focus on these components from an economic perspective, with special insights on the microgrid setup, market mechanism, pricing mechanism and their implications for the regulation by taking account into the characteristic of energy network.

More precisely, we can note that it exist a need for a balance between injection and extraction is not without consequences on the organization of the network. Thus it is necessary to study intermediation carried out by an aggregator or "traders" to facilitate commercial operations within the microgrids. Then the economic model of such platforms could be based on the implementation of autonomous contracts through the development of innovative technology like the "blockchain". In these conditions, questions about the way in which micrograds operate (*efficiently*) raise.

We provide a simple analysis based on P2P energy trading in a community. We retain the definition proposed by Abada *et al.* (2020a) : *"households of a common building or close geographical area may decide to combine their effort and jointly build solar panels on their roofs (or windmills in a nearby field)".* They study the viability of the community by using cooperative game approach and find that inadequate gain sharing may jeopardize the stability of a community but if aggregation benefits can compensate coordination costs, the community may be stable. In our paper, we study the non-cooperative viability of these communities.

In this paper, a P2P energy trading community connected to the national grid acts as market-dealer. We provide a simple model by considering heterogeneity among prosumers who offer variables quantities. In this context, we discuss about the price levels on the platform and we offer a comparison with the price on the central (national) grid, but also about incentives to invest in a domestic production unit (hereafter DPU) in the P2P energy trading platform. We show that the existence of dealing platform can boost the installation process of DPU's. This comes from the fact that these platforms are able to generate economic intrinsic values for participant that are the fundamentals of trade. A consequence is that energy is purchased at a higher price and sold at a lower price than the grid reference. However, these spreads become the drivers of the incentives to install DPU.

The rest of the paper is organized as follows : first we propose a benchmark, i.e without platform, in which we focus on the incentives for installing a fixed size DPU capacity. We go on with a simple dealing platform and we establish and compare those incentives. Finally we consider extensions in order to challenge the basic framework, allowing for variable DPU size, market power for the dealing platform and considering a matching platform. Details and proofs are in the Appendix.

2 Model

We develop a simple stylized model where heterogeneous agents aim to exchange excess energy flows they produce using renewable decentralized production unit. Our main goal is to see how such P2P trading arrangements can be viable for all participants. In our model prosumers, i.e. consumers and producers of energy goods can offer them in competition with professionals producers (i.e. companies or local communities) and interact with possible pure consumers on a dedicated platform. In a first step, the platform is just a dealer that accept to purchase energy from some prosumers in excess and resell it to consumers or through the grid.

Suppose that exists a mass n agents have a load factor (state of demand) of $\phi \in [\underline{\phi}, \overline{\phi}]$ distributed according to a cumulative $G(\phi)$ where $G'(\phi) = g(\phi)$ and $G(\bar{\phi}) = n$. This state describes the level of consumption they desire to achieve in all periods. This corresponds to their standard energy needs in relation to the size of the agent's households (i.e. dwelling area, number of people, installed power). We assume that with the surplus $u(\cdot)$ derived from this baseline level of consumption ϕ is v but decreases to 0 if it is less than needed that is

$$u(\phi) = v > u(z) = 0 \text{ when } z < \phi$$

As a result an agent with a load factor ϕ with shall always try to reach this level but she would not have to overtake it.

To satisfy her needs, an agent has the choice to install or not a domestic production unit of energy, here represent by a maximal production capacity of $q > 0$ kW at a capacity up-front cost $k > 0$. We assume that q is fixed and relax this assumption in the Section 5. For example, it can be the case if the agent acquire a dwelling in a connected residential area where areas are normalized and so is the DPU.

Had this capacity installed, an agent can be a prosumer in the sense she can use it as she wants, to self-consume it or to sell it if it is possible according to the excess capacity she observes at each time $\phi - qx$. Here the variable qx represents the available amount of the renewable capacity q that is actually dispatchable in state $x \in \mathcal{X} = [0, 1]$, they are distributed according to a cumulative $F(x)$, with $F'(x) = f(x)$.

The state of nature represents weather conditions or occurrence of failures, that is all external conditions that drives the intermittency feature of DPU's. Then in a given state of nature x, a prosumer (i.e. an agent that has installed a capacity q) may be either a pure consumer if $\phi - qx \geq 0$ or a potential seller if $\phi - qx < 0$. For the sake of simplicity let us assume that $\bar{\phi} \geq q > \underline{\phi}$, which means that in favorable conditions ($x = 1$), there are always some buyers (those with load factors near the upper bound $\bar{\phi}$) and sellers (those with small load factors near the lower bound $\underline{\phi}$).

Figure 3 depicts the heterogeneous consumption model. The sloping dotted lines represent the net consumption/production for the extremal agents, the sloping thick line is the one of a given agent with a load factor ϕ.

Now let us describe the supply side. First, we assume that a centralized professional supplier always exists and may provide unlimited energy volumes to all agents that demand them at a given price $a(x)$. This price may include grid access tariffs and energy wholesale prices. However, as we focus on the P2P exchanges we refer to the external supply as of the (centralized) grid. In some, sense the grid supply is the outside option for all agents being or not prosumers. Second, we analyze the viability of a dealing platform through which all prosumers may want to trade their excess/lack energy volumes in any state of nature.

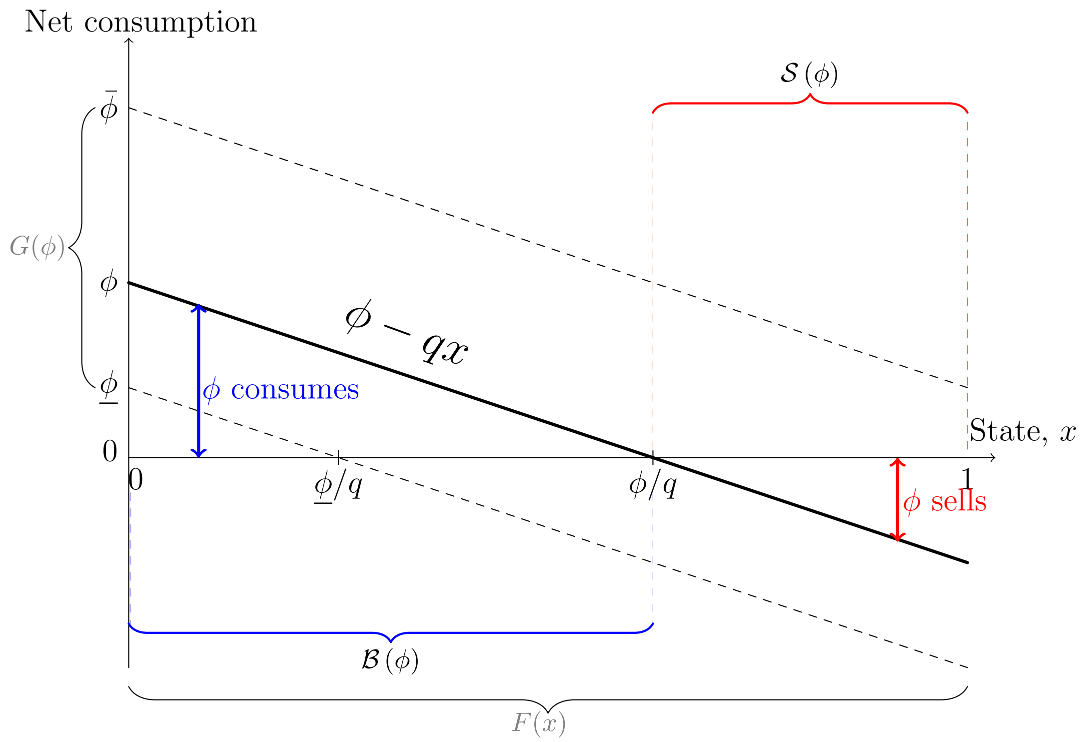


Figure 3: Net consumptions

The basic business model to this platform is to resell the excess energy volumes to consumers that are connected to it or to the central grid if no deals are found. We assume that the platform cannot change the price $a(x)$ decided by the global market on the centralized price, so it cannot make profits on this external side. We denote by $p(x) \geq 0$ the platform purchase price and $r(x) \geq 0$ the platform selling price withing the platform.[7] Then if a agent ϕ is a consumer in state x, she will have to pay an amount $p(x)(\phi - qx) \geq 0$ if she purchases their energy needed through the platform. On the contrary, if she is a seller in state x, she will receive a profit $p(x)(qx - \phi) \geq 0$ if she sells their excess energy through the platform. We also consider that agents participate to the platform have an intrinsic preference when they are served through this channel equals to $\delta \geq 0$, for instance it represents the surplus of being in sharing relationships with identified agents (neighbors, flatmates, members of an dedicated association). It can also represent a part of the surplus for avoiding power cuts when distribution grids are unadapted, or the reduction of transaction costs with the professional suppliers, or the gain from having the possibility of trading, or from some ancillary local services provided by the platform, or finally the environmental preference for by themselves a potential producer with residential renewable sources (i.e. "fossil fuel freedom"). This preference is also a way to represent the ability the platform has to provide specific services that are valuable to the connected consumers.

[7] In such a model with vertical differentiation for participating to the platform, negative prices would be possible. However, we assume that in front of a negative price, a seller do not trade.

So for an agent with a load factor ϕ the utility from trading through the platform in state x is

$$U(\phi, x, q) = \begin{cases} \delta + v - p(x)(\phi - qx) & \phi \geq qx \\ \delta + v + r(x)(qx - \phi) & \phi < qx \end{cases} \text{if}$$

The utility from trading through using the grid

$$\underline{U}(\phi, x, q) = \begin{cases} v - a(x)(\phi - qx) & \phi \geq qx \\ v + a(x)(qx - \phi) & \text{if } \phi < qx \\ v - a(x)\phi & q = 0 \end{cases}$$

So for each x, it may exist $\hat{\phi}_x = qx$ such that the agent is a pure self-consumer (if $x > 0$).

3 No platform

Consider first the common situation in which the platform does not exist. The central grid is viewed an aggregator that purchases or sells energy at a given price $a(x)$. The only decision for all agents is to install or not the DPU capacity q at cost k. A prosumer ϕ installs the DPU if (expectation are taken over x):

$$\mathbb{E}[U_0] - k \geq \mathbb{E}[\underline{U}|q = 0] = v - \mathbb{E}[a(x)\phi]$$

where

$$\mathbb{E}[U_0] = v - \mathbb{E}_{\mathcal{B}(\phi)}[a(x)(\phi - qx)] + \mathbb{E}_{\mathcal{S}(\phi)}[a(x)(qx - \phi)] \tag{1}$$

and

$$\begin{aligned} \mathcal{B}(\phi) &= \{x \in [0,1] : 0 \leq x \leq \phi/q\} \\ \mathcal{S}(\phi) &= \{x \in [0,1] : 1 \geq x \geq \phi/q\} \end{aligned}$$

which are respectively the set of s.o.n in which the prosumer ϕ is a buyer, resp. a seller. Note that $\mathcal{S}(\phi)$ may be eventually *empty* as for instance when $\phi = \bar{\phi}$, $x \leq 1 < \bar{\phi}/q$. In Figure 3, both sets are depicted.

Looking for the indifferent prosumer ϕ_0 such $\mathbb{E}[U_0] = v - \mathbb{E}[a(x)\phi_0]$, we have

$$\phi_0 : q\mathbb{E}[a(x)x] - k = 0$$

which does not depend on the value of ϕ. As a result with no platform, the incentives to invest in DPU for an agent ϕ amount $I_0(\phi) = \max\{q\mathbb{E}[a(x)x] - k, 0\}$.

Lemma 1 *With no platform, all agents are prosumers and install capacity* $q > 0$*, iff* $q\mathbb{E}[a(x)x] > k$ *, and there are no prosumers otherwise.*

As the total expected cost savings for a prosumer ϕ that had installed a amount capacity of q are $q\mathbb{E}[a(x)x]$, she actually invests in this capacity if it overcomes the fixed expenditure $k > 0$. Moreover these cost savings are independent of the load factor ϕ, then either all agents are prosumers either their are all pure consumers.

4 Simple dealing platform

Let us suppose that a dealing technical (and eventually commercial platform) has the ability to identify prosumers supplies and demands and ensures their equilibrium. In the sense of an electricity system, the platform is an also an aggregator that dispatch the power within the local grid and towards the central grid. It can purchase prosumers supplies if any at a price $r(x) \geq 0$ in state x and resell this electricity flows to connected consumers at a price $p(x) \geq 0$.

The objective platform can be profit-oriented or welfare maximizing. To start with, let us suppose that the platform has a local welfare objective. Indeed a first step, we could imagine that due to a technicality of the microgrids technology involved in local areas to connected prosumers, of the blockchain processes and the implementation of smart contracts needed to ensure the real-time equilibrium within the platform and outside with the grid, the dealing platform would be a for profit organization. However, one could also imagine that in the future "turnkey digital technologies" and microgrids may be installed by some energy communities. In that sense, up the installation cost, the trading platform could be socially managed and and even zero-pricing could be desired by users.

So if in state x, the total supply to the platform in order to be resold within is $S(r(x))$, it must match the total demand $D(p(x))$ from prosumers that are in lack of power with regard to their domestic production at that state. However, some agents may prefer not to purchase or resell to the platform but to the grid. The platform cannot make money from them.

Demand and supply to the dealing platform The platform will implement choices that are individually preferable for each participants so an agent ϕ will be a consumer within the platform if she prefers to purchase the energy needed or to sell the energy in excess in some state x, to the platform whereas to the grid.

Concerning purchases, that is for agents such that $\phi \geq qx$, this writes (omitting the argument x)

$$\delta + v - p(\phi - qx) \geq v - a(\phi - qx) \tag{2}$$

which implies that[8] :

$$
\begin{array}{rcl}
qx & \leq & \phi \leq \beta\left(p\right) = qx + \dfrac{\delta}{p-a} \qquad \text{if } p > a \\
\phi & \geq & qx > \beta\left(p\right) \qquad \text{if } p \leq a
\end{array}
$$

If $p > a$, the demand is price-sensitive and $\beta(\underline{p}) = \bar{\phi}$ with $\underline{p} = a + \frac{\delta}{\bar{\phi} - qx}$. So the demand at state x is such that

$$
D\left(p\right) = \begin{cases} \bar{d} & \text{if } p \leq \underline{p} \\ d\left(p\right) & \quad\;\, p > \underline{p} \end{cases} \tag{3}
$$

with $\bar{d} = \int_{qx}^{\bar{\phi}} \left(\phi - qx\right) dG$ and $d\left(p\right) = \int_{qx}^{\beta(p)} \left(\phi - qx\right) dG$. In the same spirit an agent ϕ will be a (extra) supplier within the platform if, when $r < a$

$$
qx \geq \phi \geq \sigma\left(r\right) = qx - \frac{\delta}{a-r} \tag{4}
$$

and $\sigma(\bar{r}) = \underline{\phi}$ with $\bar{r} = a - \frac{\delta}{qx - \underline{\phi}}$. The aggregate supply at state x is such that

$$
S\left(r\right) = \begin{cases} \overline{s} & \text{if } r \geq \bar{r} \\ s\left(r\right) & \quad\;\, r < \bar{r} \end{cases} \tag{5}
$$

where $\overline{s} = \int_{\underline{\phi}}^{qx} \left(qx - \phi\right) dG$ and $s\left(r\right) = \int_{\sigma(r)}^{qx} \left(qx - \phi\right) dG$.

Market clearing and platform pricing In some state, $x > \underline{\phi}/q$, it may exist platform exchanges in the sense that the above demand and supply may meet. The market clearing price is then a couple of prices that equals demand and supply on the platform:

$$
\left(p, r\right) : D\left(p\right) = S\left(r\right)
$$

As the grid is a default option, the non served demands and supplies through the platform are served by the central grid. As a result, in any time, all energy flows are balanced. Let us now consider that at each state the platform chooses the prices (p, r) that maximize the total welfare of the participant in the platform that is the sum of prosumer' surpluses and the profit of the platform:

$$
W\left(x\right) = \int_{\underline{\phi}}^{\bar{\phi}} U\left(\phi, x, q\right) dG + \pi\left(x\right) = \left(\delta + v\right) \left\{G\left(\beta\left(p\right)\right) - G\left(\sigma\left(r\right)\right)\right\}
$$

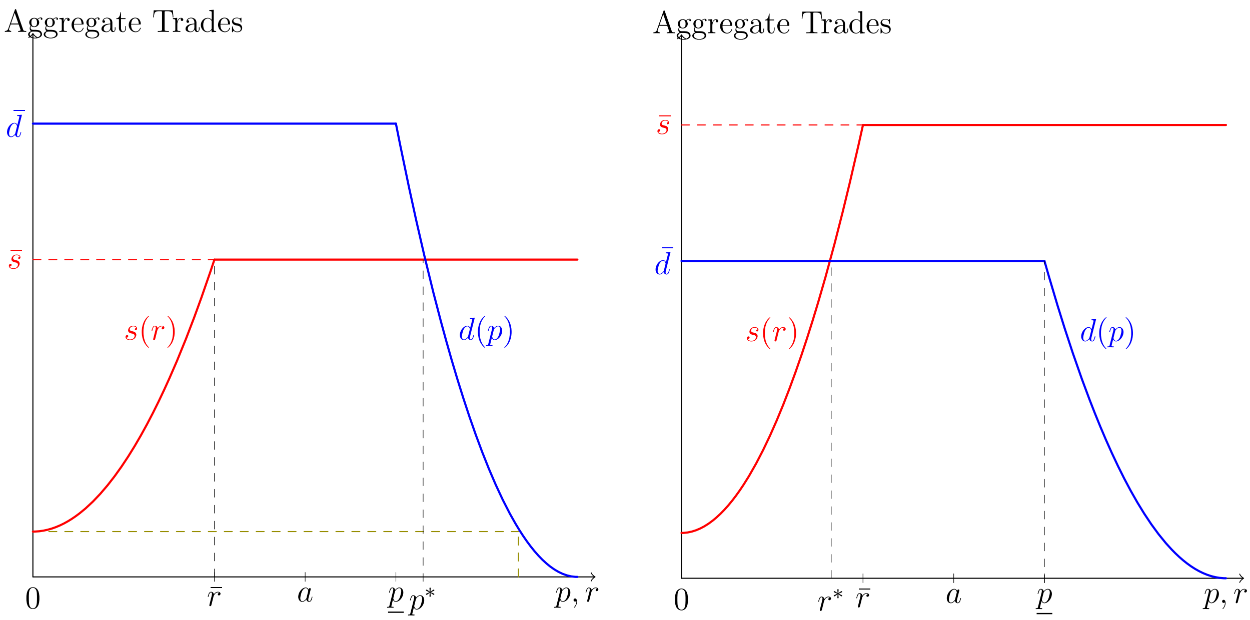
[8]More details are provided in the Appendix.

subject to $D(p) = S(r)$ and where $\pi(x) = pD(p) - rS(r)$ is the platform's profit. This leads to corner solutions[9] as depicted in the following Lemma, where $\hat{x} = \frac{\mathbb{E}[\phi]}{nq}$.

Lemma 2 *Optimal prices (p^*, r^*) are such that*

1. *$r^* = \bar{r}$ and $p^* > \underline{p}$ whenever $\bar{s} < \bar{d}$ that is for $x < \hat{x}$,*
2. *$p^* = \underline{p}$ and $r^* \leq \bar{r}$ whenever $\bar{s} \geq \bar{d}$ that is for $x \geq \hat{x}$.*

In unfavorable availability conditions, i.e. x low, the aggregate demand to the platform is structurally high and the supply low, so the selling price is stated at least to its maximum value[10] in order to attract all sellers to the platform. As a result, the demand price is the one that just clears the market. In favorable availability conditions, i.e. x high, the aggregate supply to the platform is structurally high and the demand low, so the demand price is stated to its minimum value to push possible local buyers to be active on the platform. As a result, the selling price just clears the market. The optimal market clearing is depicted



$d(p^*) = \bar{s}$ in a state $x \leq \hat{x}$ $\bar{d} = s(r^*)$ in a state $x \geq \hat{x}$

Figure 4: Market clearing

in Figure 4.

[9]Indeed, there are multiple solutions as they are depicted in the proof in the Appendix. We pick down the less favorable for prosumers.

[10]This is also equivalent is terms of demands or supplies to set alternatively the price equal to $a(x)$ or lower. But it is not in terms of net welfare as the platform generates a additional utility, i.e. δ

These optimal prices equilibrium put the agent in a trade set representing the states of nature in which the prosumer ϕ is a buyer to the grid ($\mathcal{B}^G$), to the platform ($\mathcal{B}^P$), a seller to the platform ($\mathcal{S}^P$) and finally a seller to the grid ($\mathcal{S}^G$). They write[11]

$$\begin{aligned}
\mathcal{B}^G &= \{x \in [0,1] : x \leq \xi_b(\phi)\} \\
\mathcal{B}^P &= \{x \in [0,1] : \xi_b(\phi) \leq x \leq \phi/q\} \\
\mathcal{S}^P &= \{x \in [0,1] : \xi_s(\phi) \geq x \geq \phi/q\} \\
\mathcal{S}^G &= \{x \in [0,1] : x \geq \xi_s(\phi)\}
\end{aligned}$$

and when[12] $x = \xi_b(\phi) : \beta(p^*) = \phi$ and $x = \xi_s(\phi) : \sigma(r^*) = \phi$. Note by definition that $\xi_b(\bar{\phi}) = \xi_s(\underline{\phi}) = \frac{\mathbb{E}[\phi]}{nq}$ as when $\beta(p^*) = \bar{\phi}$ and $\sigma(p^*) = \underline{\phi}$ then $\bar{d} = \bar{s}$, which occurs in $\hat{x} = \frac{\mathbb{E}[\phi]}{nq}$.

The Figure 5 represents the equilibrium net consumption for a prosumer ϕ and the way she will buy/sell the energy on or outside the platform. One can also represent the trade

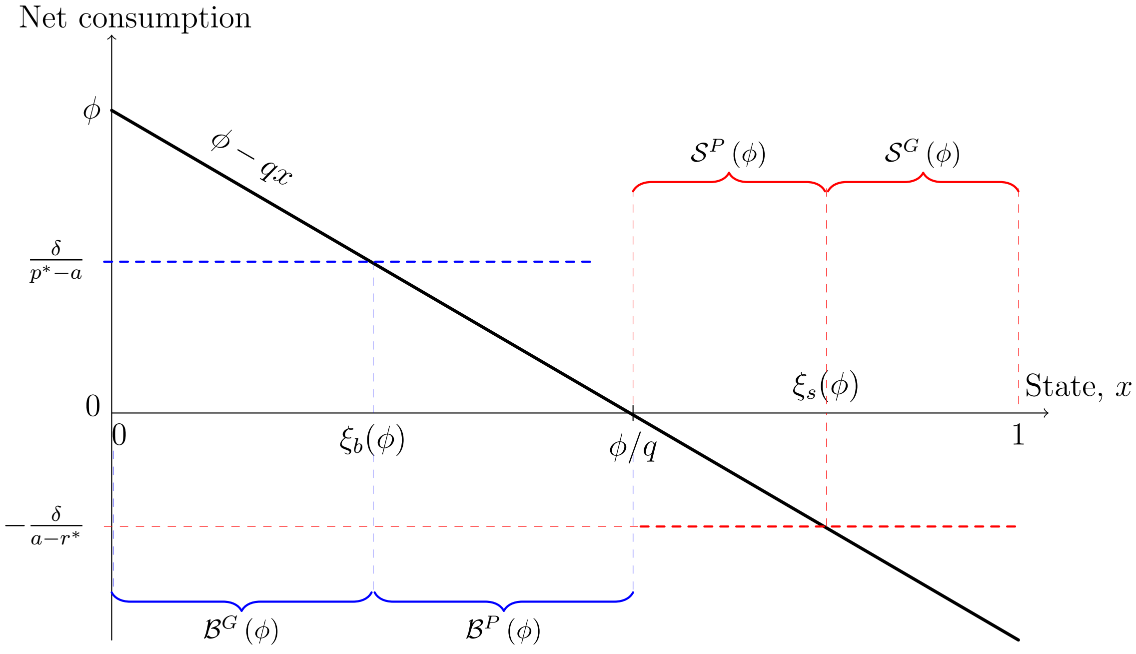


Figure 5: Individual trades within the platform

sets in the (ϕ, x) plane as depicted in Figure 6 where read/blue areas are such that agents buy/sell on the platform.

[11]These sets could further subdivided to take into account the pricing structure of the platform, as it is shown is Figure 6.

[12]Indeed we always have $\xi_b(\phi) \leq \phi/q$ as

$$\xi_b(\phi) = \frac{\phi}{q} - \frac{\delta}{q(p^* - a)} < \phi/q$$

Identically for $\xi_s(\phi) \geq \phi/q$. Moreover they are both increasing in ϕ.

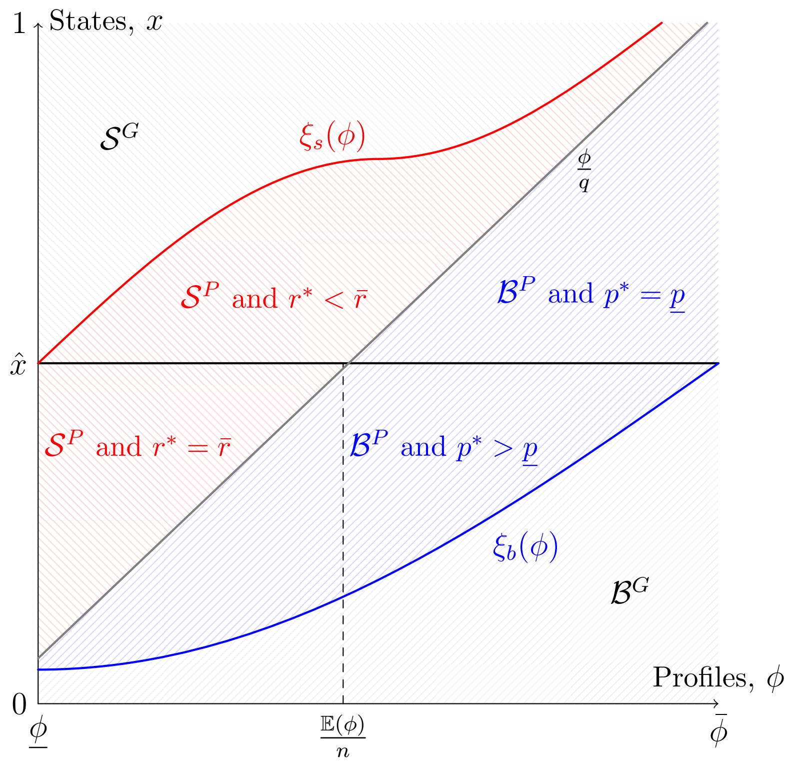


Figure 6: Trade regions

Incentives to install DPU Now, we analyze the incentives to install DPU created by the existence of the exchange platform. For an agent ϕ, the expected surplus for participating to the platform is then

$$\begin{aligned}\mathbb{E}\left[U\right] &= v - \mathbb{E}_{\mathcal{B}^G}\left[a\left(\phi - qx\right)\right] + \mathbb{E}_{\mathcal{B}^P}\left[\delta - p^*\left(\phi - qx\right)\right] \\ &\quad + \mathbb{E}_{\mathcal{S}^P}\left[\delta + r^*\left(qx - \phi\right)\right] + \mathbb{E}_{\mathcal{S}^G}\left[a\left(qx - \phi\right)\right]\end{aligned}$$

Actually she installs the capacity q when $\mathbb{E}\left[U\right] - k \geq \left[\underline{U}|q = 0\right]$ and looking for the indifferent prosumer ϕ^* such $\mathbb{E}\left[U\right] - k = \mathbb{E}\left[v - a\phi\right]$. Rearranging the terms, this leads to the equality:

$$\begin{aligned}\mathbb{E}\left[U\right] - k - \left(v - \mathbb{E}\left[a\phi^*\right]\right) &= 0 \\ &= q\mathbb{E}\left[ax\right] - k \\ &\quad + \mathbb{E}_{\mathcal{B}^P}\left[\delta - \left(p^* - a\right)\left(\phi^* - qx\right)\right] \\ &\quad + \mathbb{E}_{\mathcal{S}^P}\left[\delta + \left(r^* - a\right)\left(qx - \phi^*\right)\right]\end{aligned}$$

First, we now see that *in general* not all consumers are willing to participate to the platform and installing DPU. Indeed, we see that the load factor now is involved in the decision. Here the incentives to invest in DPU for an agent ϕ are $I_P\left(\phi\right) = \max\{\mathbb{E}\left[U\right] - k - \left(v - \mathbb{E}\left[a\phi\right]\right), 0\}$.

However, assume that $q\mathbb{E}[ax] = k - \varepsilon$, so that no agent would be a prosumer in the benchmark case (without platform). Then in that case we see that being prosumers connected to the platform all agents are not worse off, as

$$\mathbb{E}[U] - (v - \mathbb{E}[a\phi]) = \mathbb{E}_{\mathcal{B}^P}[\delta - (p^* - a)(\phi - qx)] + \mathbb{E}_{\mathcal{S}^P}[\delta + (r^* - a)(qx - \phi)] \geq 0 \quad (6)$$

Indeed, depending on price levels, mainly if the spread $p^* - r^*$ is large, the sets $\mathcal{B}^P$ and $\mathcal{S}^P$ may be empty and agents are in the same conditions as in the no platform case. But when the sets $\mathcal{B}^P$ and $\mathcal{S}^p$ are not empty, for an agent with a load factor ϕ, the inequalities (2) and (4) hold respectively in these sets. So for these states of nature, an agent with a load factor ϕ has a greater surplus trading with peers on the platform than with the grid so (6) holds.

Assume now that $q\mathbb{E}[ax] > k$, such that all agents install a DPU without a platform, as their incentives to invest are $I_0(\phi) = q\mathbb{E}[ax] - k > 0$. However connected to the platform, their incentives to invest $I_P(\phi)$ are never less than $I_0(\phi)$ as $I_P(\phi) = I_0(\phi) + \mathbb{E}[U] - (v - \mathbb{E}[a\phi])$ and (6) holds. The following proposition sums up the previous discussion.

Proposition 1 *If all agents install a DPU when there is no platform, they do and are not worse off when the dealing platform is active.*

Even if the energy prices are less favorable, the intrinsic and differentiated services provided by the platform (safer distribution, local trades, traceability or just sharing renewables sources) leads some consumers to use the platform to trade their domestic production.

Finally, now if $q\mathbb{E}[ax] < k$, no agent would be a prosumer without a platform. With the dealing platform, there is still a room for some agents to install the DPU. So it exist a set of agents $\Phi^* \subset [\underline{\phi}, \bar{\phi}]$, for which $I_P(\phi) > 0 > I_0(\phi)$. However one cannot state clearly what kind of agents will be concerned (low or high load factor). Indeed the variations of the incentives to install capacity is a non monotonic function of ϕ:

$$I'_P(\phi) = -\mathbb{E}_{\mathcal{B}^P}[p^* - a] + \mathbb{E}_{\mathcal{S}^p}[a - r^*]$$

It depends on the relative price spreads $p^* - a$ and $a - r^*$ at each state and also on the skewness of the distribution of states of nature. On one hand, agents with higher load profiles will be buyers more often (at the margin) and accordingly on the platform, then will have to pay the premium $p^* - a$ as a cost of sourcing, this reduces their incentives to invest i.e. $-\mathbb{E}_{\mathcal{B}^P}[p^* - a] < 0$. On the other, agents with higher load profiles will be sellers on the platform less often, then they will not have to bear shortfalls resulting from selling to the platform, this increases their incentives to invest i.e. $\mathbb{E}_{\mathcal{S}^p}[a - r^*] > 0$ at the margin.

When $I'_P(\phi) < 0$, for all ϕ then $\Phi^* = [\underline{\phi}, \phi^*]$, prosumers connected to the platform are those who have low load profiles (i.e. small consumers), and they are motivated by a selling argument to participate and install DPU: the shortfall $a - r^*$ is not so important to them. Big consumers are not interested by participating the premium is $p^* - a$ is too costly for them.

When $I'_P(\phi) > 0$, then $\Phi^* = [\phi^*, \bar{\phi}]$, prosumers connected to the platform are those who have high load profiles (i.e. big consumers), they are motivated by a consuming argument to participate. Some kind of bell shapes can be easily for $I_P(\phi)$. [13]

Zero-pricing An argument sometimes put forward to justify the emergence of these platforms is that to some extent participants could exchange energy for free because first the short run marginal cost of production of DPU is near zero and also they could benefit from a certain reciprocity within the community. Of course, one could argue that zero pricing is detrimental for investments in local generation capacities.

First of all, a permanent zero pricing scheme is not generally possible, except in one (potential) state of nature for which $D(0) = \bar{d} = S(0) = s(0)$ which implies that it cannot be supported as an equilibrium for each states. Second, a unilateral zero pricing scheme (i.e. $p = 0$ or $r = 0$, $\forall x$) is not feasible as, for instance when $x \leq \hat{x}, D(0) = \bar{d} > \bar{s} > s(r)$, there are not enough sellers on the platform to serve the high demand.

However, a zero pricing "revolving" scheme can be achieved. Indeed, if $p = 0$ for all $x \geq \hat{x}$, it is equivalent in term of demand of a minimal pricing $p^* = \underline{p}$, and also in term of local welfare.[14] So the platform can propose an optimal selling price $r^* : S(r^*) = D(0) = \bar{d}$. However, the same *does not* apply if $r = 0$ for $x \leq \hat{x}$. Indeed a market equilibrium is achievable by posting a price $p_z : D(p_z) = S(0)$ as $\bar{d} > \bar{s} > S(0)$, but it is no more optimal and $p_z > p^*$. It is depicted in pale green in Figure 4. Then incentives to install DPU are now:

$$\begin{aligned} I_Z(\phi) &= \mathbb{E}_{\mathcal{B}_z^P}[\delta - (p_z - a)(\phi - qx)] + \mathbb{E}_{\mathcal{B}_0^P}[\delta + a(\phi - qx)] \\ &\quad + \mathbb{E}_{\mathcal{S}_0^P}[\delta - a(qx - \phi)] + \mathbb{E}_{\mathcal{S}_*^P}[\delta + (r^* - a)(qx - \phi)] \end{aligned}$$

where $\mathcal{B}^P$ and $\mathcal{S}^P$ are subdivided into $\mathcal{B}_z^P = [\xi_b^z(\phi), \hat{x}]$, $\mathcal{B}_0^P = [\hat{x}, \phi/q]$; $\mathcal{S}_0^P = [\phi/q, \hat{x}]$ and $\mathcal{S}_*^P = [\hat{x}, \xi_s(\phi)]$ where $\xi_b^z(\phi)$ is higher than $\xi_b(\phi)$ in the optimal case so $\mathcal{B}_z^P \subset \mathcal{B}_*^P$. First of all, we see that $I_Z(\phi)$ is positive for all ϕ as the trade sets are very empty all together. Second, compared to the optimal case, zero pricing reduces these incentives in selling periods (the shortfall is not smaller) but increases them during buying periods only when the DPU availability is high. For low availability, a zero selling price implies a huge purchase price increases that drives consumers to turn to the grid. As a result, it is clear that $I_Z(\phi) < I_P(\phi)$.

[13] We have just some evidence from simulations, using an uniform distributions for ϕ and x. They are available on demand.

[14] Of course the platform will not break even.

Proposition 2 *Zero pricing creates less than optimal incentives but more than without platform.*

To sum up, zero pricing is not detrimental for investments in local generation capacities, but creates low powered incentives. As a result, zero pricing cannot be a decisive argument to the creation and growth of energy platforms.

Policy tools Finally we look at the effects of policy tools that are usually implemented and how they can help promoting or deterring the development of energy P2P platforms.

For instance, one first can imagine that some subsidization schemes are implemented by governments in order to promote P2P platforms for environmental or innovative concerns. A simple lumpsum subsidy for each DPU installed will have the effect of reducing the installation cost k and of course will directly increase the incentives for prosuming. However, this effect is not amplified by the existence of a P2P platform.

Price subsidization schemes could be more effective. Indeed, a unit rebate ρ allowed for the purchasing price so that the paid price would be $p-\rho$ or a premium for the selling price so that the paid price would be $r+\rho$, would enhance demand and/or supply on the platform.[15] This premia and rebates have direct effects on the incentives for prosuming $I_P(\phi)$ as they influence positively the relative price spreads. However, they are bounded instruments as depending of the state of nature for DPU availability, a flat rebate or flat subsidy may be ineffective at some point. For instance, in unfavorable availability conditions, i.e. x low, Lemma 2 indicates that the selling price is set to the upper bound $\bar{r}$ for which all energy in excess is supplied within the platform. In this case adding a premium would not change the supply and then the selling price remains unchanged. The same applies for the purchasing price in favorable availability conditions, x high.

Finally, another way is to increase the grid price through directed taxation. This policy may have positive effects as it increases the total expected cost savings for a prosumer that had installed a DPU, i.e. $q\mathbb{E}[ax]$ and it decreases the purchase price spread. However, this also deflates the selling price spread which is a driver for prosuming, in favorable availability conditions.

5 Extensions

Some extensions of the basic framework are developed. First we consider that the DPU size is not fixed but can vary with the load profile in order to be adapted to the basic consumption profile of each agent. Second, we discuss about the effect of a for-profit

[15] These rebate or premium call for compensations for the platform

platform. Third we look at a more sophisticated way to realize trades for prosumers considering a matching platform.

Autarky

In our main approach, we assume that the grid is an outside option at each time or each state of DPU availability. What would be the picture if this option would not be possible? This the autarkic platform configuration in which the utility to trade with the (dealing) platform is $U(\phi, x, q)$ but it zero[16] if no trade is possible when prosumers demand energy but it is v when they do not sell they energy in excess. Then now demand $d(p)$ in (3) and supply $s(r)$ in (5) are modified in the sense that grid price is no more a base price so $\beta_a(p) = qx + \frac{\delta+v}{p}$ and now the floor purchase price is $\bar{p} = \frac{\delta+v}{\phi-qx}$. However whenever $r \geq 0$ then $s(r) = \bar{s}$ so when $x \geq \hat{x}$ at a selling zero market to not clear as $\bar{s} > \bar{d}$. So some (random) outage may arise on the consumers' side. Hence Lemma 2 is modified then $p^* = \underline{p}$ and $r^* = 0$: when DPU are highly available the demand is low and the selling price is at its minimal level here zero. Indeed if negative prices have been allowed, occasional suppliers would have been better off selling their energy in excess at a negative price. Then when $x \geq \hat{x}$, it is impossible for occasional suppliers to earn money from their energy in excess so they are obliged to give up at a zero price. Now, on the (occasional) consumers'side there are always outages or curtailment of electricity. Indeed when $x \leq \hat{x}$ the price is higher than $\bar{p}$ so large consumers prefer to reduce their needs and they are curtailed. When $x > \hat{x}$, the price is at its minimum $\bar{p}$ but it is not an equilibrium, so outages may occur for some of them.

Consequently, autarky has a detrimental effect on the incentives to install DPU to enter an autarkic platform. Now the incentives to invest in DPU for an agent ϕ is $I_A(\phi) = \max\{\mathbb{E}[U] - k - (v - \mathbb{E}[a\phi]), 0\}$ which writes

$$\begin{aligned} I_A(\phi) &= \mathbb{E}[qax] - k \\ &\quad + \mathbb{E}_{\mathcal{B}_A^P}[\delta - (p^* - a)(\phi - qx)] + \mathbb{E}_{\mathcal{S}}[\delta - a(qx - \phi)] \\ &\quad - \mathbb{E}_{\mathcal{B}_A^{\varnothing}}[v - a(\phi - qx)] \end{aligned}$$

where $\mathcal{B}_A^P$ and $\mathcal{S}_A^P$ refers to the set of availability states for which a agent ϕ is consumer/seller within the platform, $\mathcal{B}_A^{\varnothing}$ refers to the set of availability states for which agent cannot be served. Hence from the results above here $\mathcal{B}_A^P \subset \mathcal{B}^P$ and $\mathcal{S}_A^P = \mathcal{S}$. We see that when $q\mathbb{E}[ax] = k$, so that no agent would be a prosumer in the benchmark case, $I_A(\phi)$ is not necessarily positive due to the lossed from outages or curtailments $(-\mathbb{E}_{\mathcal{B}_A^{\varnothing}}[v - a(\phi - qx)])$ but also because the profitable states (i.e. when $x \in \mathcal{B}_A^P$) are less numerous.

[16] We assume in the basic model that 0 is the choke-off utility level.

Variable capacities

We now consider that agents can calibrate their DPU with respect to their load factor, that is now $q(\phi)$ is a variable depending on ϕ, at the equilibrium these prices will be impacted by the DPU choices made by the agents. We will seek at an continuous differentiable equilibrium path $q = q(\phi)$ where for each x, $1 - q'(\phi)x$ has a constant sign and we start by assuming $1 - q'(\phi)x > 0, \forall\phi$. As q is a choice of ϕ, we now assume that a capacity up-front cost $k(q)$ that is increasing and convex for a production capacity q kW.

Following similar developments as above, one can again derive the dealer prices that are now function of the entire path $\{q(\phi)\}_{\phi\in[\underline{\phi},\bar{\phi}]}$ where the aggregate demand is now $d(p) = \int_{\hat{\phi}_x}^{\beta(p)} (\phi - q(\phi)x)\, dG$ where the switching load profile is now $\hat{\phi}_x : \phi = q(\phi)x$, and the aggregate supply $s(r) = \int_{\sigma(r)}^{\hat{\phi}_x} (q(\phi)x - \phi)\, dG$. The result in Lemma 2 still holds with the main change that $q = q(\phi)$ for each ϕ. This implies that the switching state $\hat{x}$ is now defined as $\hat{x} = \frac{\mathbb{E}(\phi)}{n\mathbb{E}(q(\phi))}$. Therefore the optimal $q^*(\phi)$ that maximizes the expected surplus of a agent with a load factor ϕ is now driven by her marginal gains from increasing the capacity, taking as given those of others agents on the platform, that is

$$\frac{\partial\mathbb{E}[U]}{\partial q(\phi)} - k'(q(\phi))$$

When there is no platform, the marginal incentives imply that all agents will install the same capacity $q_0(\phi) = q_0$ such that

$$\mathbb{E}[ax] = k'(q_0) \text{ for all } \phi$$

Now when connected to a platform (applying Leibnitz derivation rule), in general agents with different load factor will install different levels of capacity as $q^*(\phi)$ is such that

$$\begin{aligned} & \frac{\partial\mathbb{E}[U]}{\partial q(\phi)} = k'(q(\phi)) \\ = \;& \mathbb{E}[ax] + \mathbb{E}_{\mathcal{B}^P}[(p^* - a)x] - \mathbb{E}_{\mathcal{B}^P}\left[\frac{\partial p^*}{\partial q(\phi)}(\phi - q(\phi)x)\right] \\ & + \mathbb{E}_{\mathcal{S}^P}[(r^* - a)x] + \mathbb{E}_{\mathcal{S}^P}\left[\frac{\partial r^*}{\partial q(\phi)}(q(\phi)x - \phi)\right] \end{aligned}$$

where $\mathcal{B}^G$ and $\mathcal{S}^P$ are subdivided into subsets depending on the platform pricing (as depicted in Lemma 3). Hence, on can see for each ϕ, $k'(q(\phi)) \neq \mathbb{E}[ax]$ so $q^*(\phi) \neq q_0$ in general.

We also see that the local impact on prices are $\frac{\partial p^*}{\partial q(\phi)}$ and $\frac{\partial r^*}{\partial q(\phi)}$ are key variables to promote or to dampen the installation of DPU by prosumers. One can prove that in general that (for $\phi \in \text{int}\mathcal{B}^P(x)$ or $\phi \in \text{int}\mathcal{S}^P(x)$).

Lemma 3 *For $x < \hat{x}$, then $r^* = \bar{r}$; $p^* > \underline{p}$ and*

$$\frac{\partial p^*}{\partial q\left(\phi\right)} \leq 0 \text{ for all } \phi \text{ while } \frac{\partial \bar{r}}{\partial q(\underline{\phi})} > 0 \text{ and } \frac{\partial \bar{r}}{\partial q\left(\phi\right)} = 0 \text{ for all } \phi > \underline{\phi}$$

For $x > \hat{x}$, then $p^* = \underline{p}$; $r^* < \underline{p}$ and

$$\frac{\partial r^*}{\partial q\left(\phi\right)} \leq 0 \text{ for all } \phi \text{ while } \frac{\partial \underline{p}}{\partial q\left(\bar{\phi}\right)} > 0 \text{ and } \frac{\partial \underline{p}}{\partial q\left(\phi\right)} = 0 \text{ for all } \phi < \bar{\phi}$$

Indeed, on one hand installing more DPU's reduces the demand on the buyer-side as self-consumption is more likely, but on the other hand it increases the supply on the seller-side. As a result, adjusted prices (i.e p^* or r^*) are reduced driven by the changing fundamentals of supply and demand. More surprising the corner prices $\bar{r}$ and $\underline{p}$ are positively impacted by investments for the extreme agents in terms of load. When the smallest consumer $\underline{\phi}$ invests she increases the maximal supply achievable $\bar{s}$ at a given state and then she also pushes up the maximum price. For the biggest consumer $\bar{\phi}$, investing reduces the maximal demand achievable $\bar{d}$ at a given state and then she pushes down the minimum price.

Now for an agent, depending on her load profile the marginal incentives to install DPU differ. For non extreme agents, the marginal incentives to install DPU are boosted by the positive marginal price effects they produce on the purchase price as[17]

$$I^+\left(\phi\right) = \mathbb{E}_{\mathcal{B}^P}\left[\left(p^* - a\right)x\right] - \mathbb{E}_{\mathcal{B}^P}\left[\frac{\partial p^*}{\partial q}\left(\phi - qx\right)\right] > 0$$

But they are dampened by the negative marginal price effects they produce on the selling price as

$$I^-\left(\phi\right) = \mathbb{E}_{\mathcal{S}^p}\left[\left(r^* - a\right)x\right] - \mathbb{E}_{\mathcal{S}^p}\left[\frac{\partial r^*}{\partial q}\left(qx - \phi\right)\right] < 0$$

For the smallest consumer $\underline{\phi}$, $I^+\left(\phi\right)$ is increased by $\mathbb{E}_{\mathcal{S}^p}\left[\frac{\partial \bar{r}}{\partial q}\left(qx - \underline{\phi}\right)\right] > 0$ and for the biggest consumer $\bar{\phi}$, $I^-\left(\phi\right)$ is decreased by $-\mathbb{E}_{\mathcal{B}^P}\left[\frac{\partial \underline{p}}{\partial q}\left(\bar{\phi} - qx\right)\right] < 0$.

As a result, compared to Proposition 1, it is now not so clear that all agents have strictly superior incentives to install more DPU then in the no platform case. Of course on average, the distribution of loads matters to identify is more or less total capacities will be installed.

[17] We drop the argument ϕ.

Pro-profit platform

In the main analysis, we consider a welfare maximizing dealing platform. Let us now suppose that the platform has a profit objective that writes

$$\pi(x) = p(x) D(p(x)) - r(x) S(r(x))$$

One can see that the platform as a dealer is a local node acting as an upstream monopsony and a downstream monopoly. The for-profit platform problem in x is then

$$\max_{p,r} \pi(x) \quad \text{s.t.} \quad D(p) = S(r)$$

which leads to an integrated monopsony-monopoly (interior) equilibrium[18]

$$\frac{p^d - r^d}{p^d} > \frac{1}{\eta_D} \text{ and } S\left(r^d\right) = D\left(p^d\right)$$

where η_D is the price elasticity of demand. As a result, one can state the following lemma.

Lemma 4 *Prices $\left(p^d, r^d\right)$ are such that*

$$p^d > p^m > p^* \geq \underline{p} > a > \bar{r} \geq r^* > r^d \geq 0$$

where p^m would be the monopoly-side price and $r = 0$,the monopsony-side price (free purchase).

As an non exclusive dealer, the platform has limited upstream and downstream market power. However compared to the grid price and the optimal prices, it both increases the energy price paid by consumers that are served through the platform and decreases the energy price received by prosumers that sell their energy in excess. These markups are possible as they incorporate partially the value δ of participating to the platform.

In this case, the incentives to invest in DPU for an agent ϕ are $I_P^d(\phi) = \max\{\mathbb{E}[U] - k - (v - \mathbb{E}[a\phi]), 0$ and compare to the for-profit platform (i.e. 6) this leads to (when $q\mathbb{E}[ax] = k - \varepsilon$) as

$$\begin{aligned} I_P^*(\phi) - I_P^d(\phi) &= \mathbb{E}_{\mathcal{B}_*^P}[\delta - (p^* - a)(\phi - qx)] + \mathbb{E}_{\mathcal{S}_*^P}[\delta + (r^* - a)(qx - \phi)] \\ &\quad - \mathbb{E}_{\mathcal{B}_d^P}\left[\delta - \left(p^d - a\right)(\phi - qx)\right] - \mathbb{E}_{\mathcal{S}_d^P}\left[\delta + \left(r^d - a\right)(qx - \phi)\right] \\ &\geq 0 \end{aligned}$$

where here lower indexes d and $*$ refer to the for-profit platform and welfare maximizing cases, respectively. Therefore $\mathcal{B}_d^P \subset \mathcal{B}_*^P$ and $\mathcal{S}_d^P \subset \mathcal{S}_*^P$ as $a < p^* < p^d$ and $r^d < r^* < a$. So

[18] This standard analysis of price setting by an intermediary can be found in Spulber (1999) for instance.

a for-profit platform generates less incentives to install DPU among prosumers that trade less "often"[19] within the platform as price are at their limit values (maximum selling price and minimum purchase price).

Matching platform

Following Goss *et al.* (2014), we look at a different way prosumers can find electricity through the platform, that is the dealer is now also matchmaker. Let's assume that the platform is a closed environment in which the , participants must declare themselves and install a DPU. In line with our main framework, we consider that the platform is nonprofit, in the sense that it is welfare maximizing. Doing so they can be technically connected to the local micro-grid and at that time the matching's technology will make it possible to carry out exchanges between the participants (peer-to-peer exchanges) or if there is no match made between the participants and the central grid. The problem is to know which agents will participate in this platform, depending on purchase and selling prices that the platform designer may choose, possibly one for all the states of nature.

The matching technology depends on the relative size of potential supplies and demands to be machted in state x with a counterpart within the platform. Hence if their a (endogeneous) mass of buyers participating on the platform that corresponds to a mass D of energy to be consumed and a mass of sellers that corresponds to a mass S of energy to be supplied, then we assume that the total number of matches is given by the well-known matching function[20]

$$M = M(S, D)$$

As is standard in the matching literature, the matching function $M(S, D)$ is assumed to be twice continuously differentiable, weakly increasing and concave such that $M(S, 0) = M(0, D) = 0$ and $M \leq \min\{S, D\}$. The platform is a random matchmaker such that all participants on the same side have the same probability of being matched

$$m_B = \frac{M(S, D)}{D} \text{ and } m_S = \frac{M(S, D)}{S}$$

Under these weak regularity conditions, it is easy to show that the match probability of buyers m_B is weakly decreasing in own-side participation D which captures a negative own-side externality, and weakly increasing in cross-side participation S which captures a positive cross-side externality. The same applies to m_S. A common example $M(S, D) = S\,(1 - \exp(-D/S))$. Here the presence of the grid provides an non-zero outside option.

[19]That is to say the are active on the platform in a narrower set of states of nature.

[20]This matching process is clearly exogenous in this context. A growing literature exists in order to ground one-to-one and one to many matching procedures (see Chade *et al.*, 2017). However the micro-foundations of our setting, that is many-to-many multidimensional matching with heterogeneous agents, are not yet established (see however Gomes and Pavan (2016) for a primer). It is left for future research.

It is useful to also define the matching elasticities for buyers and sellers respectively that write:

$$\psi^B = \frac{M'_D(S,D)D}{M(S,D)} \text{ and } \psi^S = \frac{M'_S(S,D)S}{M(S,D)}$$

There numbers lying in the interval $[0,1]$, they represents the percentage increase in the total number of matches for a percentage increase in own-side participation.

On the dealer side, the necessity to maintain an overall grid balance implies that the matched demands and supplies must be equalized by the platform,[21] the non-matched trade on the platform being ensured by the central grid. Hence, the platform proposes *ex ante* a menu of prices $(p(x), r(x))_{x\in[01,]}$ that balances energy exchanges within inner participants in each state x, that is[22]:

$$m_B D = m_S S \tag{7}$$

where here D is the potential energy demanded by participants to the platform in state x when price p is observed and S is the potential energy to be supplied when price r is observed in state x. As demand and supplies are in real time scale, potential demands and supplies can be viewed *ex post* as described by (3) and (5). Indeed, at each state of nature, the prosumer will prefer to trade within among peers or with the grid, depending upon price conditions (a, p, r), so she may demand or supply energy as in market conditions. For example, a smart contract can be signed with the matchmaker which states purchases and selling conditions for the prosumer.

In a matching process, the economic value rises through the fact of being matched to a peer only within the platform rather than being served through the grid. As a result now the intrinsic value is affected by the probability of being served within the platform.

So for an agent with a load factor ϕ the expected utility from trading through the platform in state x is

$$\mathbb{U}(\phi, x, q) = \begin{cases} v + m_B\delta - (m_B p + (1-m_B)a)(\phi - qx) \\ v + m_S\delta + (m_S r + (1-m_S)a)(qx - \phi) \end{cases} \text{if} \begin{array}{l} \phi \geq qx \\ \phi < qx \end{array}$$

Then *ex post* an agent will trade within the platform if her expected utility is greater then the surplus of trading with the grid only $\mathbb{U}(\phi, x, q) \geq v - a(\phi - qx)$ so as explained above, we find again the same demand and supply as described by (3) and (5). So we can state

$$D = D(p) \text{ and } S = S(r)$$

[21] On this point we rely on the analysis of Benjaafar *et al.* (2018) concerning P2P car sharing.

[22] At the "rational-expectations" equilibrium, as suggested by Caillaud and Jullien (2003), this is always true has it leas to $M^* = M^*$.

As a result m_B and m_S depend on both (p, r) as

$$m_B(p,r) = \frac{M(S(r), D(p))}{D(p)} \text{ and } m_S(p,r) = \frac{M(S(r), D(p))}{S(r)}$$

Indeed, the probability of being matched for buyers is increasing function p and r and the probability of being matched for sellers is decreasing function r and p.

Let us denote the *net* expected utility of trading within the platform

$$\mathbb{V}(\phi, x, q) = \begin{cases} m_B(\delta - (p-a)(\phi - qx)) & \text{if } \phi \geq qx \\ m_S(\delta + (r-a)(qx - \phi)) & \phantom{\text{if }} \phi < qx \end{cases}$$

Therefore the platform pricing is now impacted by the matching process as the expected local welfare when agents are matched with peers prosumers[23]

$$\mathbb{W}(x) = \int_{\underline{\phi}}^{\bar{\phi}} \mathbb{V}(\phi, x, q)\, dG + \pi(x)$$

where

$$\pi(x) = (p-a)\, m_B D(p) - (r-a)\, m_S S(r) = (p-r)\, M(S(r), D(p))$$

so we can rewrite the platform welfare as:

$$\mathbb{W}(x) = m_B(p,r)\,\delta\,\{G(\beta(p)) - G(qx)\} + m_S(p,r)\,\delta\,\{G(qx) - G(\sigma(r))\}$$

Compared to the pure dealing platform, the matching process implies two-sided effects of pricing schemes that create countervailing forces that may operate. Resolving the matching platform problem which is to maximize $\mathbb{W}(x)$ for each state x, one can state the following lemma for pigovian pricing

Lemma 5 *Matching prices (p^μ, r^μ) are driven by the underlying matching technology and entail :*

$$\begin{aligned} p^\mu &= \max\{a + \left(1 - \psi^B\right) A^B - \psi^B A^S, \bar{p}\} \\ r^\mu &= \min\{a + \psi^S A^B - \left(1 - \psi^S\right) A^S, \bar{r}\} \end{aligned}$$

where ψ^B, ψ^S is the matching elasticities for a buyer and a seller respectively and A^B, A^S stands for the weighted net match valuation of buyers and sellers respectively.

[23] We go on with the convention that no markups are possible when the platform trades with the grid.

For the matching platform increasing purchase price or selling price helps attracting buyers but it repels sellers. Decreasing prices do the reverse. Hence depending on the relative strength of the matching elasticities the matchmaker will prefer to push up a price than another. So it can be the case that for some state of nature (mainly for intermediate values of x) that both prices admit mark-ups in the sense that $p^{\mu} > \underline{p} > \underline{r} > r^{\mu}$.

This well-known balancing mechanism is only possible if the matching technology exhibits decreasing and limited return to scale, that is when $\psi^B + \psi^S < 1$. If not, the pricing scheme will be bounded by the price limits $\bar{p}$ or $\bar{r}$, as demand and supply is also bounded in the platform.

Lemma 6 *Compared to optimal prices, there exists elasticities thresholds* $\psi_*^B \leq \bar{\psi}^B$ *and* $\psi_*^S \leq \bar{\psi}^S$ *such they are*

$$\begin{cases} p^{\mu} \geq p^* \\ p^* \geq p^{\mu} \geq \bar{p} \end{cases} \textit{for} \begin{array}{l} \psi^B \leq \psi_*^B \leq \bar{\psi}^B \\ \psi_*^B \leq \psi^B \leq \bar{\psi}^B \end{array}$$

$$\begin{cases} r^{\mu} \leq r^* \\ r^* \leq r^{\mu} \leq \bar{p} \end{cases} \textit{for} \begin{array}{l} \psi^S \leq \psi_*^S \leq \bar{\psi}^S \\ \psi_*^S \leq \psi^S \leq \bar{\psi}^S \end{array}$$

The last Lemma is quite intuitive. When the matching technology is rigid (i.e. $\psi^B \leq \psi_*^B$ and/or $\psi^S \leq \psi_*^S$) negative own-side externalities have a greater impact than positive cross-side externalities, as a result this calls for increasing the purchase price towars the weighted net match valuation of buyers or decreasing to the one of sellers respectively. When the matching technology is sufficiently elastic (i.e. $\psi^B \geq \psi_*^B$ and/or $\psi^S \geq \psi_*^S$) positive cross-side externalities are more effective so this calls for decreasing the purchase price towars the ceiling price or increasing the selling price to price cap.

Finally, we turn to the incentives to install DPU created by the existence of the matching platform. For an agent ϕ, these incentives (if positive) are defined again by $I_M(\phi) = \mathbb{E}[\mathbb{U}] - k - (v - \mathbb{E}[a\phi])$ but now write

$$\begin{aligned} I_M(\phi) &= q\mathbb{E}[ax] - k \\ &\quad + \mathbb{E}_{\mathcal{B}}\left[m_B^{\mu}\left\{\delta - (p^{\mu} - a)(\phi - qx)\right\}\right] \\ &\quad + \mathbb{E}_{\mathcal{S}}\left[m_S^{\mu}\left\{\delta + (r^{\mu} - a)(qx - \phi)\right\}\right] \end{aligned}$$

where $m_j^{\mu} = m_j(p^{\mu}, r^{\mu})$ for $j = B, S$. Again, compared to the no platform benchmark, the prosumers are not worse off. However, it is not clear if prosumers are more or less better off than with a dealing (welfare maximizing) platform, that is if $I_M(\phi) \geq (\leq) I_P(\phi)$. Indeed, first *ex ante* in all state of nature a possible match is possible, this has a positive effect on the incentives to install the unit. Second, if the matching technology is sufficiently elastic (i.e. $\psi^B \geq \psi_*^B$ and $\psi^S \geq \psi_*^S$) then prices tend to their respective bounds which also may boost prosumer's investments. Of course, the reverse holds if the matching technology is rigid. Finally, the matching itself as a uncertain process creates a depressive effect on the incentives to invest. As a result we cannot directly assess which effect will dominate. A least an elastic matching technology is a factor that can enhance the prosumer's investments.

6 Conclusion

In this paper, we have provided a first economic analysis of how new models of exchanges in the electricity sector may be viable and may yield sufficient incentives to invest in domestic production units based on renewable energy sources. We analyzed a P2P energy trading community connected to the national grid acts as market-dealer. We provided a simple model by considering heterogeneity among prosumers who offer variables quantities. In this context, we discussed about the price levels on the platform and we offer a comparison with the price on the central (national) grid, but also about incentives to invest in a domestic production unit (hereafter DPU) in the P2P energy trading platform. We have shown that the existence of dealing platform can boost the installation process of DPU's. This comes from the fact that these platforms are able to generate economic intrinsic values for participant that are the fundamentals of trade. A consequence is that energy is purchased at a higher price and sold at a lower price than the grid reference. However, these spreads become the drivers of the incentives to install DPU.

Some issues have been left aside. To conclude we just discuss them in the following.

Grid pricing. Grid pricing issues when prosumers are active have been analyzed recently by Gautier *et al.* (2018). Here, we look at the retroactive effect of the existence of the platform and its equilibrium on the way the (regulated or market-based) grid pricing may be impacted. Following Abada *et al.* (2020b), we seek at a snowball effect in the co-existence of platforms and the central grid. The main idea is that depending on whether the grid pricing is average-cost based or marginal cost based, the contraction of exchanges due to the existence of the trading platform may respectively increase or decrease the supply price to the grid, $a(x)$. In return, this modifies the incentives to install DPU from potential prosumers.

Size of the platform. Indeed, the coexistence of the platform connected agents and grid dependent agents, begs the question as to whether the latter may join the former. The question of the endogenous joining to the platform (i.e. determining n^*) implies to enter further the "blackbox" we suppose so far to represent the external central grid.

Variable demand. One can argue that the load factor would be better described by a load profile $\phi(x)$. This would implies that the distribution of load profiles G is generated by the distribution of state F.

Batteries and Storage. Microgrids are mainly the places were these kinds of platforms are nowadays is likely to be built. As a result, an issue is how the platform or the agents connected may provide electricity backups (batteries and storage capacities) instead of withdrawing electricity from the grid.

Some *miscellaneous* and energy oriented issues may also be addressed, for instance those which touch upon the effects of ITC and Blockchain. What are the improvements expected with ITC and blockchain technologies with smart contracting? Also the issue of reactive energy: can the platform be able to provide ancillary services?

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Appendix

Demand and Supply

Demands. If $p \leq a$, then all the agents connected to the platform will demand energy within it and then the demand is rigid i.e. $\bar{d} = \int_{qx}^{\bar{\phi}} (\phi - qx)\, dG$. If $p > a$, the demand is price-sensitive and we denote $\beta(p)$, the value of ϕ such that

$$\beta(p) = qx + \frac{\delta}{p-a} \geq 0$$

with $\beta'(p) = -\frac{\delta}{(p-a)^2} < 0$ and $\beta''(p) = \frac{2\delta}{(p-a)^3} > 0$. Moreover, it may exist a maximum value of $\underline{p} > a$ such that $\beta(\underline{p}) = \bar{\phi}$ with $\frac{dp}{da} = 1$, more precisely

$$\underline{p} = a + \frac{\delta}{\bar{\phi} - qx} \tag{8}$$

Using the notation $\dot{y} = \frac{dy}{dx}$, note that $\dot{\underline{p}} - \dot{a} = \frac{\delta q}{(\bar{\phi} - qx)} > 0$. So if $p \geq \underline{p}$, the demand to the platform is

$$d(p) = \int_{qx}^{\beta(p)} (\phi - qx)\, dG = \int_{qx}^{\beta(p)} [G(\beta(p)) - G(\phi)]\, d\phi$$

with $d'(p) = \beta'(p)(\beta(p) - qx)\, g(\beta(p)) < 0$. As expected, the demand is normal (downward sloping). Note that this demand is always positive as we assumed that $\bar{\phi}/q \geq 1$.Note that with this vertical differentiation framework, the demand is never choked off. So the demand at state x is such that

$$D(p) = \begin{cases} \bar{d} & \text{if } p \leq \underline{p} \\ d(p) & \phantom{\text{if }} p > \underline{p} \end{cases}$$

Note that for all p, $\dot{\bar{d}} = -q(n - G(qx)) < 0$ with $\bar{d} = \mathbb{E}(\phi)$ if $x \leq \underline{\phi}/q$.

Supplies. An agent ϕ will be a (extra) supplier within the platform if

$$\delta + r(qx - \phi) \geq a(qx - \phi)$$

we denote $\sigma(r)$, the value of ϕ such that

$$\sigma(r) = qx - \frac{\delta}{a-r} \geq 0 \text{ if } r < a$$

with $\sigma'(r) = -\frac{\delta}{(a-r)^2} < 0$. If $r > a$ then $\sigma(r) = \underline{\phi}$ and all potential suppliers to the platform. Moreover, it may exist a maximum value of $\bar{r} < a$ such that $\sigma(\bar{r}) = \underline{\phi}$ with $\frac{d\bar{r}}{da} = 1$. Namely

$$\bar{r} = a - \frac{\delta}{qx - \underline{\phi}} \tag{9}$$

note that $\dot{\bar{r}} - \dot{a} = \frac{\delta}{qx - \underline{\phi}} > 0$. Then if $r \geq \bar{r}$, the supply is rigid and equal to

$$\bar{s} = \int_{\underline{\phi}}^{qx} (qx - \phi)\, dG$$

When $0 \leq r \leq \bar{r}$, the supply to the platform is

$$s(r) = \int_{\sigma(r)}^{qx} (qx - \phi)\, dG = \int_{\sigma(r)}^{qx} [G(\phi) - G(\sigma(r))]\, d\phi$$

Note that $s'(r) = -\sigma'(r)(qx - \sigma(r))\, g(\sigma(r)) > 0$, the supply is upward sloping. Of course, this supply is nil if $x \leq \underline{\phi}/q$. Note that as $\sigma(0) < qx$ then $S(0) > 0$: there are always sellers willing to sell electricity for free. Finally the supply at state x is such that

$$S(r) = \begin{cases} \bar{s} & \text{if } r \geq \bar{r} \\ s(r) & \phantom{\text{if }} r < \bar{r} \end{cases}$$

Note that for all r, $\dot{\bar{s}} = qG(qx) > 0$ and $\bar{s} = 0$ if $x \leq \underline{\phi}/q$.

Proof of Lemma 2

If $p < \underline{p}$ then $D(p) = \bar{d}$ and $r^* = S^{-1}(\bar{d})$ so $W(x) = (\delta + v)\{n - G(\sigma(r^*))\}$. If $r > \bar{r}$ then $S(r) = \bar{s}$ and $p^* = D^{-1}(\bar{s})$ so $W(x) = (\delta + v)\, G(\beta(p^*))$. When $p \geq \underline{p}$ and $r \leq \bar{r}$, and $D(p) = S(r)$ one can extract

$$G(\sigma(r)) = \frac{a - r}{\delta} \int_{\sigma(r)}^{\beta(p)} G(\phi)\, d\phi - G(\beta(p))\, \frac{a - r}{p - a}$$

so the welfare can be rewritten as

$$W(x) = (\delta + v) \left\{ G(\beta(p)) \left(\frac{p - r}{p - a} \right) - \frac{a - r}{\delta} \int_{\sigma(r)}^{\beta(p)} G(\phi)\, d\phi \right\}$$

Consequently derivatives imply;

$$\begin{aligned} \frac{\partial W}{\partial p} &= (\delta + v)\, \beta'(p)\, g(\beta(p)) \left(\frac{p - r}{p - a} \right) \\ \frac{\partial W}{\partial r} &= 0, \forall r \end{aligned}$$

As $p > a > r$ for our admissible demands and supplies then $\frac{\partial W}{\partial p} < 0$ and two cases are possible

1. $p^* = \underline{p}$ so $r^* = S^{-1}(\bar{d})$ where $r^* \leq \bar{r}$ whenever $\bar{s} = S(\bar{r}) \geq \bar{d} = D(\underline{p})$. In that case the local welfare is also equal to
$$W(x) = (\delta + v)\{n - G(\sigma(r^*))\}$$

2. $r^* = \bar{r}$ so $p^* = D^{-1}(\bar{s})$ where $p^* > \underline{p}$ whenever $\bar{s} < \bar{d}$. In that case the local welfare is also equal to
$$W(x) = (\delta + v)\, G(\beta(p^*))$$

Then it may exist a level of $x = \hat{x} : \bar{s} = \bar{d}$, such that

$$\bar{s} - \bar{d} = \int_{\underline{\phi}}^{\bar{\phi}} (\phi - qx)\, dG = \mathbb{E}(\phi) - nqx = 0$$

here expectations are over ϕ, so

$$\hat{x} = \frac{\mathbb{E}(\phi)}{nq}$$

As a result there are a multiple of solutions. If $x \leq \hat{x}$ then $p^* = D^{-1}(\bar{s}) > 0$ and $r^* \in [\bar{r}, +\infty]$ and if $x \geq \hat{x}$ then $p^* \in [0, \underline{p}]$ and $r^* = S^{-1}\left(\bar{d}\right) > 0$. Of course if we add a breakeaven constraint for the plateform account $\pi(x) = (p - r)\min\{\bar{d}, \bar{s}\} \geq 0$ then this restrict the set of optima to $p^* \geq r^*$. This restrict selling prices to $r^* \in [\bar{r}, p^*]$ when $x \leq \hat{x}$ and purchase prices to $p^* \in [r^*, \underline{p}]$ when $x \geq \hat{x}$.

Proof of Lemma 3

From (8) and (4) in the Proof of Lemma 2 one can derive that

$$\begin{aligned} \frac{\partial \underline{p}}{\partial q(\bar{\phi})} &= \frac{\delta x}{\left(\bar{\phi} - q(\bar{\phi})\, x\right)^2} > 0 \text{ and } \frac{\partial \underline{p}}{\partial q(\phi)} = 0 \text{ for all } \phi \neq \bar{\phi} \\ \frac{\partial \bar{r}}{\partial q(\underline{\phi})} &= \frac{\delta x}{\left(q(\underline{\phi})\, x - \underline{\phi}\right)^2} \geq 0 \text{ and } \frac{\partial \bar{r}}{\partial q(\phi)} = 0 \text{ for all } \phi \neq \underline{\phi} \end{aligned}$$

Moreover if $0 \leq x \leq \hat{x}$ then $d(p^*) = \bar{s}$ then for a given ϕ such that

$$d'(p)\frac{\partial p^*}{\partial q(\phi)} = \frac{\partial \bar{s}}{\partial q(\phi)} - \frac{\partial d(p)}{\partial q(\phi)}$$

- $\phi \in [\underline{\phi}, \sigma(r^*)[$ or $\phi \in [\beta(p^*), \bar{\phi}[$ then $\frac{\partial p^*}{\partial q(\phi)} = 0$
- $\phi \in [\sigma(r^*), \hat{\phi}_x]$ then

$$d'(p)\frac{\partial p^*}{\partial q(\phi)} = \frac{\partial \bar{s}}{\partial q(\phi)} - 0 = xg(\phi) \geq 0 \Rightarrow \frac{\partial p^*}{\partial q(\phi)} \leq 0$$

- $\phi \in [\hat{\phi}_x, \beta(p^*)]$ then

$$d'(p)\frac{\partial p^*}{\partial q(\phi)} = 0 - \frac{\partial d(p)}{\partial q(\phi)} = xg(\phi) \geq 0 \Rightarrow \frac{\partial p^*}{\partial q(\phi)} \leq 0$$

If $1 \geq x \geq \hat{x}$ then $s(r^*) = \bar{d}$ then for a given ϕ such that

$$s'(r)\frac{\partial r^*}{\partial q(\phi)} = \frac{\partial \bar{d}}{\partial q(\phi)} - \frac{\partial s(r)}{\partial q(\phi)}$$

- $\phi \in [\underline{\phi}, \sigma(r^*)[$ or $\phi \in [\beta(p^*), \bar{\phi}[$ then $\frac{\partial r^*}{\partial q(\phi)} = 0$
- $\phi \in [\sigma(r^*), \hat{\phi}_x]$ then

$$s'(r)\frac{\partial r^*}{\partial q(\phi)} = 0 - \frac{\partial s(r)}{\partial q(\phi)} = -xg(\phi) \leq 0 \Rightarrow \frac{\partial r^*}{\partial q(\phi)} \leq 0$$

- $\phi \in [\hat{\phi}_x, \beta(p^*)]$ then

$$s'(r)\frac{\partial r^*}{\partial q(\phi)} = \frac{\partial \bar{d}}{\partial q(\phi)} - 0 = -xg(\phi) \leq 0 \Rightarrow \frac{\partial r^*}{\partial q(\phi)} \leq 0$$

Proof of Lemma 4

From the market clearing condition $D(p) = S(r)$ one can define a locus $\hat{r}(p)$ such that

$$S(\hat{r}(p)) = D(p)$$

which entails $r^*(p)$ decreasing in $p \in [\underline{p}, \infty[$ whenever $S(0) < D(\underline{p})$ i.e.

$$D(\underline{p}) - S(0) = \int_{qx-\frac{\delta}{a}}^{\bar{\phi}} (\phi - qx)\, dG > 0$$

So the dealer problem writes $\max_{p \geq \underline{p}} (p - \hat{r}(p)) D(p)$ and the first order condition gives:

$$\begin{aligned} \frac{p^d - \hat{r}(p^d)}{p^*} &= \frac{1 - \hat{r}'(p^d)}{\eta_D} > \frac{1}{\eta_D} \\ S(\hat{r}(p^*)) &= D(p^*) \end{aligned} \tag{10}$$

where

$$\eta_D = -\frac{D'(p)\, p}{D(p)} > 0$$

so $p^d \geq p^m$. As a result

$$p^d > p^* \geq \underline{p} \text{ and } \bar{r} \geq r^* > r^d$$

Proof of Lemma 5

Assume that $p > \underline{p}$ and $r < \bar{r}$, then (interior) first order conditions write:

$$\frac{\partial \mathbb{W}(x)}{\partial p} = 0 \text{ and } \frac{\partial \mathbb{W}(x)}{\partial r} = 0$$

Derivatives write

$$\begin{aligned} \frac{\partial \mathbb{W}(x)}{\partial p} &= m_B(p,r)\, \delta g(\beta(p))\, \beta'(p) \\ &\quad + \frac{\partial m_S(p,r)}{\partial p} \delta [G(qx) - G(\sigma(r))] + \frac{\partial m_B(p,r)}{\partial p} \delta [G(\beta(p)) - G(qx)] \\ \frac{\partial \mathbb{W}(x)}{\partial r} &= -m_S(p,r)\, \delta g(\sigma(r))\, \sigma'(r) + \frac{\partial m_B(p,r)}{\partial r} \delta [G(\beta(p)) - G(qx)] \\ &\quad + \frac{\partial m_S(p,r)}{\partial r} \delta [G(qx) - G(\sigma(r))] \end{aligned}$$

with

$$\begin{aligned} \frac{\partial m_B(p,r)}{\partial p} &= m_B(p,r) \left(\psi^B - 1\right) \frac{D'(p)}{D(p)} \;;\; \frac{\partial m_B(p,r)}{\partial r} = m_B(p,r)\, \psi^S \frac{S'(r)}{S(r)} \\ \frac{\partial m_S(p,r)}{\partial r} &= m_S(p,r) \left(\psi^S - 1\right) \frac{S'(r)}{S(r)} \text{ and } \frac{\partial m_S(p,r)}{\partial p} = m_S(p,r)\, \psi^B \frac{D'(p)}{D(p)} \end{aligned}$$

where

$$\psi^B = \frac{M'_D(S,D)D}{M(S,D)} \;;\; \psi^S = \frac{M'_S(S,D)S}{M(S,D)}$$

are the matching elasticities for buyers and sellers respectivelyn such that

$$0 \le \psi^j \le 1 \text{ for } j = B, S$$

and $\psi^B + \psi^S = 1$ if the matching technology is characterized by constant returns to scale (i.e. $M(S, D)$ is homogeneous of degree one)

So one can rewrite FOC using $D'(p) = \beta'(p)(\beta(p) - qx)\, g(\beta(p))$ and $S'(r) = -\sigma'(r)(qx - \sigma(r))\, g(\sigma(r))$:

$$\begin{aligned}\frac{\partial \mathbb{W}(x)}{\partial p} &= 0 = \frac{D'(p)}{D(p)} \left\{ m_B(p,r)\,\delta \frac{D(p)}{\beta(p) - qx} \right. \\ &\quad \left. + m_S(p,r)\,\psi^B \delta\left[G(qx) - G(\sigma(r))\right] + m_B(p,r)\left(\psi^B - 1\right)\delta\left[G(\beta(p)) - G(qx)\right] \right\}\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathbb{W}(x)}{\partial r} &= 0 = \frac{S'(r)}{S(r)} \left\{ m_S(p,r)\,\delta \frac{S(r)}{qx - \sigma(r)} \right. \\ &\quad \left. + m_B(p,r)\,\psi^S \delta\left[G(\beta(p)) - G(qx)\right] + m_S(p,r)\left(\psi^S - 1\right)\delta\left[G(qx) - G(\sigma(r))\right] \right\}\end{aligned}$$

Using definitions of $\beta(p)$ and $\sigma(r)$, this leads to define (p^μ, r^μ) as :

$$p^\mu = a + \left(1 - \psi^B\right) A^B - \psi^B A^S \tag{11}$$

$$r^\mu = a + \psi^S A^B - \left(1 - \psi^S\right) A^S \tag{12}$$

where

$$A_B = \frac{G(\beta(p^\mu)) - G(qx)}{D(p^\mu)}\delta > 0 \text{ and } A_S = \frac{G(qx) - G(\sigma(r^\mu))}{S(r^\mu)}\delta > 0$$

These expressions stand for the weighted net match valuation of buyers and sellers respectively. Note that this solution is not valid for matching technology characterized by constant or incrasing returns to scale (Cobb Douglas technology for instance), indeed $\psi^B + \psi^S \ge 1$:

$$r^\mu \ge a + \left(\psi^S A^B - \psi^B A^S\right) \ge p^\mu$$

so one cannot verify $p^\mu > \underline{p} > \underline{r} > r^\mu$.

Conditions (11) and (9) are reminiscent of (17) in Goss *et al.* (2014) in a different context. Existence for the interior solution is ensured by the "rational-expectations" equilibrium we adopted as suggested by Caillaud and Jullien (2003).

By the mean theorem we see that $d(p) = (\varphi - qx)(G(\beta(p)) - G(qx))$ where $\varphi < \beta(p) \le \bar{\phi}$, so

$$a + A_B > \bar{p}$$

Identically, $s(r) = (qx - \xi)(G(qx) - G(\sigma(r)))$ where $\xi > \rho(r) \ge \underline{\phi}$, so

$$a - A^S < \bar{r}$$

So it exists value of the elasticies (i.e. forms of the underlying matching technology) such that the interior solution is valid for some x

$$\begin{aligned}
\psi^B &\leq \bar{\psi}^B = \frac{(qx-\xi)(\bar{\phi}-\varphi)}{(\varphi-\xi)(\bar{\phi}-qx)} \geq 0 \\
\psi^S &\leq \bar{\psi}^S = \frac{(\varphi-qx)(\xi-\underline{\phi})}{(\varphi-\xi)(qx-\underline{\phi})} \geq 0
\end{aligned}$$

As $\bar{\psi}^B$ is monotonically increasing with respect to x and maps $[\frac{\xi}{q}, \min\{1, \frac{\bar{\phi}}{q}\}]$ into $[0, +\infty]$ so it exists aunique $x_b \in]\frac{\xi}{q}, \min\{1, \frac{\bar{\phi}}{q}\}[$: $\bar{\psi}^B = 1$. Identically, $\bar{\psi}^S$ is monotonically decreasing with respect to x and maps $[\frac{\underline{\phi}}{q}, \min\{1, \frac{\varphi}{q}\}]$ into $[0, +\infty]$ so it exists a unique $x_s \in]\frac{\underline{\phi}}{q}, \min\{1, \frac{\varphi}{q}\}[$: $\bar{\psi}^S = 1$. As a result it existe a unique $x_e \in]x_s, x_b[$ such that $\bar{\psi}^B = \bar{\psi}^S$ when $x = x_e$.

As a result, the interior solution is valid for some underlying matching technologies (with decreasing return to scale) and some state of nature. Otherwise a corner solution applies which implies either $p = \underline{p}$ or $r = \bar{r}$.

Proof of Lemma 6

One can see that it exists levels of $\left(\psi_*^B, \psi_*^S\right)$ such that

$$\begin{aligned}
p^\mu &= a + \left(1-\psi_*^B\right) A^B - \psi_*^B A^S = p^* \\
r^\mu &= a + \psi_*^S A^B - \left(1-\psi_*^S\right) A^S = r^*
\end{aligned}$$

Indeed when at p^* such that $D\left(p^*\right) = \bar{s}$ we have $p^* = a + \frac{\delta}{\beta(p^*)-qx}$ and one can rewrite

$$p^\mu\left(\psi^B\right) = a + \left(1-\psi^B\right)\frac{\delta}{\varphi-qx} - \psi^B\frac{\delta}{qx-\xi}$$

so $p^\mu(0) > p^*$. As $p^\mu\left(\psi^B\right)$ is linear decreasing in ψ^B it exists $\psi_*^B : p^\mu\left(\psi_*^B\right) = p^*$. Same reasoning applies for r^μ.

Hence one can depicts prices as

$$\begin{aligned}
&p^\mu \geq p^* && \text{for} \quad \psi^B \leq \psi_*^B \leq \bar{\psi}^B \\
&p^* \geq p^\mu \geq \bar{p} && \text{for} \quad \psi_*^B \leq \psi^B \leq \bar{\psi}^B \\
&r^\mu \leq r^* && \text{for} \quad \psi^S \leq \psi_*^S \leq \bar{\psi}^S \\
&r^* \leq r^\mu \leq \bar{p} && \text{for} \quad \psi_*^S \leq \psi^S \leq \bar{\psi}^S
\end{aligned}$$