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Confronting climate change: Adaptation vs. migration strategies in Small Island Developing States

Lesly Cassin∗  Paolo Melindi Ghidi †  Fabien Prieur‡

Abstract

This paper examines the optimal adaptation policy of Small Island Developing States (SIDS) to cope with climate change. We build a dynamic optimization problem to incorporate the following ingredients: (i) local production uses labor and natural capital, which is degraded as a result of climate change; (ii) governments have two main policy options: control migration and/or conventional adaptation measures; (iii) migration decisions drive changes in the population size; (iv) expatriates send remittances back home. We show that the optimal policy depends on the interplay between the two policy instruments that can be either complements or substitutes depending on the individual characteristics and initial conditions. Using a numerical analysis based on the calibration of the model for different SIDS, we identify that only large islands use the two tools from the beginning, while for the smaller countries, there is a substitution between migration and conventional adaptation at the initial period.

Keywords: SIDS, climate change, adaptation, migration, natural capital.

JEL classification: Q54, Q56, F22.

∗EconomiX, University Paris Nanterre; 200, Avenue de la République 92000, Nanterre, France. E-mail: lcassin@parisnanterre.fr.
†EconomiX, University Paris Nanterre & AMSE, Aix-Marseille University, e-mail: paolo.melindighidi@parisnanterre.fr.
‡CEE-M, University of Montpellier, CNRS, INRAE, Montpellier SupAgro, Montpellier, France. Email: fabien.prieur@umontpellier.fr.
1 Introduction

Climate change affects regions all around the world. The effects of climate change on individual regions are heterogeneous and depend on the ability of different societal and environmental systems to respond to it. Two main (policy) responses are commonly put forward: mitigation and adaptation. Mitigation consists in limiting the extent of the change by reducing greenhouse gas emissions. Adaptation refers to all the measures – investment in protective infrastructure, management of endangered ecosystems, changes in production and consumption habits, etc. – intended to absorb the impact of climate change. In general, a combination of these two strategies seems warranted in order to address the climate issue in the most efficient way. But there is no one-fits-all solution: the best mix between adaptation and mitigation significantly differs across regions because of their heterogeneity.

The case of Small Island Developing States (SIDS) is emblematic of the inability to rest on these two pillars. This can be explained by their two unique characteristics. First, SIDS are not responsible for the ongoing increase in temperatures and have no means to stamp it out on their own. Second, they are among the most vulnerable to its repercussions.\footnote{Indeed SIDS will face an increase in the occurrence of extreme weather events (more frequent and severe storms and hurricanes, etc.), a rise in sea level accompanied by the degradation of natural capital, and health problems including infectious diseases (Nurse et al. (2014), Klöck and Nunn (2019)).} This implies that SIDS have no other option but to rely on adaptation measures. According to the Intergovernmental Panel on Climate Change (IPCC), climate change damages for these countries will be so large that adaptation is a necessary condition for a sustainable economic development (see for instance UN-OHRLLS (2017)).\footnote{http://unohrlls.org/custom-content/uploads/2017/09/SIDS-In-Numbers_Updated-Climate-Change-Edition-2017.pdf}

In this paper, we focus on the situation of SIDS and consider a third possible strategy to cope with climate change: migration, as a specific form of adaptation. If migration is a direct consequence of the worsening of living conditions due to climate change, then this process should be accompanied and eased by the design of appropriate public policies.
In other words, we want to adopt the perspective of a policy maker in SIDS who tries to figure out how to turn climate migration, which would otherwise be the last resort option, into a deliberate long term policy. The basic cost-benefit analysis is as follows. Climate-induced migration is, of course, disruptive on many grounds, as any form of migration. Nevertheless, in the context of SIDS, it also comes with benefits. On the one hand, migration is a means to release the pressure on scarce natural resources. On the other, migration will likely trigger financial transfers directed toward the SIDS, as migrants provide financial support to their family, in the form of remittances.

There is a tradition of studying the impact of climate change on migration (see among others Barrios et al. (2006), Marchiori and Schumacher (2011), Gray and Mueller (2012); Marchiori et al. (2012), Thiede et al. (2016)). We depart from this literature since our approach is normative, by construction. It is motivated by two observations. First, governments in SIDS are already considering migration as a credible adaptation strategy. For example, the “Migration with dignity” program by the Kiribati government aims at increasing investments in public education and schooling in order to make Kiribati migrants more attractive to receiving countries.\footnote{http://www.climate.gov.ki/category/action/relocation/} Moreover, international organizations such as the World Bank, the United Nations and more specifically the International Organization for Migration (IOM), present migration as an explicit tool to foster economic development. Here the emphasis is on the financial role of migration that endows origin countries with additional resources thanks to remittances (Agunias and Newland (2012), Clemens (2017)).\footnote{http://publications.iom.int/system/files/pdf/mecc_outlook.pdf} Second, migration is an important dimension of the demography of SIDS, that shapes their economic performance. For instance, Figure 1 displays the long term average—between 2000-2015—of the share of nationals living abroad by region. Figure 2 depicts the evolution of remittances in the percentage of GDP in SIDS and other developing countries. Both figures show that migration is massive and that it generates relevant economic returns for these developing islands.

Based on these observations, our main research question is: What is the optimal adaptation policy for SIDS? A corollary question being: Should they use migration as a
specific form of adaptation to climate change, and to what extent?

To address this issue, we develop a dynamic framework in which there are only two options to cope with climate change. On the one hand, the decision maker can choose to engage in adaptation to keep climate damages under control. As mentioned earlier, damages are multidimensional for SIDS, but we choose to focus on the degradation of natural assets. So adaptation measures are intended to slow down this degradation process. On the other hand, the policy maker can implement policies whose main target is to shape migration. The implicit assumption being that migration may, under some circumstances, be the appropriate response to the worsening of living conditions caused by global warming. It is especially a way to release the pressure on natural assets.

In this setting, the analysis of the optimal adaptation policy is conducted in two parts: the first develops a theoretical investigation; the second consists of a numerical calibration of the model. The other ingredients of our theory are the following. We consider two different sources of wealth determining the economic conditions on the island. Wealth has first a local component, that is, the production of a final good using labor and natural

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5We consider that natural capital is the most critical asset owned by SIDS from both an environmental and economic points of view.
capital. An active migration policy induces a contraction of the output thanks to the associated decrease in the labor force. However, as the local population decreases, the population of emigrants increases, which is associated with more remittances received from abroad. This yields the second external source of wealth. In line with the evidence provided earlier, remittances are large enough to involve a real economic trade-off in the management of the population. Overall, migration affects welfare both directly (cf. the total utility criterion), and indirectly by changing the amount of per capita consumption. As far as adaptation is concerned, there is a direct cost of adaptation in terms of foregone consumption. At the same time, the benefit stems from the capacity to maintain the stock of natural capital to a higher level and for a more extended time.

The analysis of the intertemporal decision problem is quite challenging because it generically produces a four-dimension dynamical system and can exhibit four different regimes, depending on whether the two instruments, migration and adaptation, are operative or not. To circumvent these difficulties, we first have a look at regimes in which the policy relies on one instrument only, and then combine our main findings to understand what is going on in the regime where both instruments are used. Considering first a regime with no adaptation, the SIDS suffers from the impacts of climate change and has no option but to adjust its population size. We find a critical condition on the fundamentals of the economy under which there exist migration incentives at the beginning of the planning period. In this situation, the optimal migration policy is characterized by a monotonically decreasing emigration rate, that vanishes eventually when the optimal population size is reached. When there is no migration incentive initially, but the SIDS bears an increasing environmental constraint because of climate change, we identify a condition that tells us if a switch to a positive migration regime will occur in finite time.

Next, we study a regime with no migration along the same line. In the absence of migration, the only possibility left is to undertake adaptation expenditures. We then find another critical condition, that also involves most of the economy’s fundamental, under which the SIDS starts to adapt from the origin. When the regime with positive adaptation is permanent, during the convergence to the saddle point, adaptation expenditures de-
crease monotonically over time but remain positive, which in turn ensures that the SIDS will enjoy a higher level of natural capital in the long run. Finally, merging both analyses, we conduct a formal discussion on the features of the optimal policy in general terms. When there is no adaptation initially but positive migration, the incentives to switch on the second instrument increase over time as the decrease in the population size reduces the marginal cost of adaptation. In the symmetric situation where there is adaptation but no migration initially, we highlight the condition that triggers a switch to positive migration in finite time. Moreover, if the regime with positive adaptation and migration hosts a steady state, then we show that the SIDS manages to stabilize natural assets to a constant and higher level than in the absence of adaptation. As a result, the population size is also larger in the long run.

To sum up, from the theoretical investigation, we obtain two critical conditions that shed some light on the SIDS preferred policy to deal with climate change. Using a specification of the model, we discuss which instrument the SIDS will deploy in priority to manage the damages optimally due to global warming, and how this choice depends on the critical parameters. To dig further into the analysis of the nature of the optimal policy, we finally resort to a calibration of the model to real-world data. This calibration is a means to emphasize the role of the initial conditions, that is, the initial size of the population and the initial endowment in natural capital. More importantly, this exercise ultimately helps us to understand which policy is optimal for which SIDS, given its characteristics. Last but not least, it allows us to explain when adaptation and migration, both seen as policy instruments, prove to be complements or substitutes.

The paper is organized as follows. In Section 2 we briefly review the literature on climate change damages and migration. Section 3 displays the model, which is then analyzed in Section 4. Section 5 is devoted to the calibration, while Section 6 concludes.
2 Related Literature

SIDS show a strong heterogeneity politically, economically, socially or culturally, however, according to the IPCC they face common constraints in terms of vulnerability and adaptation to climate change (Nurse et al., 2014). First of all, due to their high density of population, even spatially limited degradations may impact a large share of the population. Second, while the topology of these islands vary a lot according to their location or their geological formation, they all show a high concentration of their economic activities on the coastal areas. Finally, the weight of the natural assets in their economies is more likely to be high compared to other developing states. This is due to their specialization in tourism or fisheries (Nurse et al., 2014). Therefore, even if the various climate change risks do not affect the different SIDS in the same way, they all exhibit a high vulnerability to them. In order to cope with those risks, adaption is crucial. Klöck and Nunn (2019) propose a literature review on SIDS adaptation to climate change. Most of the articles described in this paper focus on a region, an island or a sector and show that adaptation efforts were inefficient (Dey et al., 2016b,a; Rosegrant et al., 2016; Valmonte-Santos et al., 2016; Weng et al., 2015; Mercer et al., 2012; Middelbeek et al., 2014; Vergara et al., 2015). Moreover, more general studies such as those of Scobie (2016) and Thomas and Benjamin, 2018 also find that the adaptation strategies in the SIDS are far from sufficient and that they lack of coherence. All in all, this seems to be accounted for by the technical and finance limits.

Besides conventional adaptation investments, migration seems to be a very plausible solution for small countries. Theoretical works such as Marchiori and Schumacher (2011), show that human displacements increase if no mitigation strategies are implemented by large emitters of GHG. According to this result, empirical papers try to predict the evolution of migration with climate change. They base their work on the past variations in human displacements according to environmental factors such as the rainfall variability, the precipitations volume or the temperature. Among others, Nawrotzki et al. (2015) predicts a climate-induced increase in the international out-migration from Mexico. Thiede et al. (2016) find that depending on the region, migration is correlated to climate vari-
ability for eight South-American countries. Moreover, papers as Marchiori et al. (2012) or Barrios et al. (2006) find a positive correlation between weather anomalies or climate change and migration in Sub-Saharan countries. Farbotko and Lazrus (2012), predicts a climate induced increase in the out-migration from Tuvalu, a Pacific island.

In all those papers, however, the effects of environmental degradations on economic outcomes and investments are neglected. Consequently, these works fail to take into account both optimization strategies to cope with climate change and the trade-off between adaptation and migration in order to adapt. On the contrary, Lilleor and den Broeck (2011) introduce the interaction between climate change and economic outcomes highlighting the demographic response to this interaction. They find a positive correlation between the loss of revenue due to climate change and migration, but no effects from the income variability due to the increasing weather variability.

A third group of papers explicitly introduces economic gains from migration for the sending economies. First, migration is a means to reduce the demographic pressure on the environment (Birk and Rasmussen (2014)). Second, migration could enhance investments in adaptation, especially in protective infrastructures, because remittances can help finance adaptation measures (Ng’ang’a et al. (2016)). In the same vein, Julca and Padidson (2010) concludes that migration and remittances are valuable levers for the SIDS economies. However, on the downside, the authors stress the dependence from remittances as a potential growing issue. More generally, in these papers migration and adaptation are seen as complements. Nevertheless, there is a bunch of papers that support the opposite view by observing that when migration is (already) very high, the need for adaptation actions is less urgent (see for instance Barnett and Adger (2003)). Embracing this argument boils down to considering migration and adaptation as substitutes. In any case, since the 1990s, the management of the diaspora strategy by the originating country is presented as a new policy tool that has been studied in the political geography literature. De Haas (2010) emphasizes that migration based on individual decisions could be ineffi-

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6 Hugo (2011) take a more nuanced position by claiming that sending areas might experiment many different economic, demographic and social adjustments that are difficult to anticipate.
cient. In this view, increases in the migration gains could be obtained thanks to public transnational policies based on the coordination (and the cooperation) with the diaspora (Faist (2008), De Haas (2010), Agunias and Newland (2012), Mullings (2012)). While all these papers are not directly in line with our approach, they acknowledge the existence of tools to shape migration. Of course, these tools are more relevant in the context of countries experiencing large scale migration like the SIDS.

In this paper, we depart from those three branches of the literature by developing a normative approach on migration as an alternative strategy to cope with climate change. The novelty of our work is to highlight both the potential complementarity and substitutability between migration and conventional adaptation strategies.

3 Model

We consider an infinite horizon SIDS economy and adopt a centralized perspective. Time is continuous and indexed by $t \in [0, \infty)$. Assuming away demographic growth, any change in the population size, $N(t)$, is the result of migration, with $m(t) \geq 0$ the emigration level:

$$\dot{N}(t) = -m(t).$$

(1)

The emigration rate simultaneously determines the evolution of the population of expatriates, $M(t)$:

$$\dot{M}(t) = m(t).$$

(2)

Three main ingredients are needed to characterize the SIDS: the definition of its welfare, the composition of its wealth, and the impacts of climate change.

Social welfare of the SIDS, $V(c(t), m(t)N(t))$, is made of two components. The first is total utility, $N(t)U(c(t))$, where $c(t)$ represents per capita consumption and $U(.)$ is increasing and concave. The second is the cost of migration, $D(m(t))$, with $D(.)$ increasing and strictly convex. If migration becomes a deliberate strategy to deal with climate change, then the decision maker should take into account the costs borne by those who
embark on the resettlement process. These costs are typically linked to the cultural differences, travel distance, and immigration policy in the destination country.\footnote{Defining $D(\cdot)$ more generally in terms of $m$ and $M(t)$ would allow us to also account for the social damage from migration in the origin country whose origin is the loss of social interactions, cultural transmission and family links that come with migration. This is left for future research.} Putting together these two elements, we get:

$$V(c(t), m(t), N(t)) = N(t)U(c(t)) - D(m(t)). \quad (3)$$

Two comments are in order here. First, according to this formulation, the planner cares about the local inhabitants while migrants no longer matter once their resettlement process is completed. The planner’s priority is to deal with the local situation that deteriorates thanks to the impacts of climate change. In this context, migration represents a form of long term adaptation. In other words, it is a specific instrument among the tools available to solve the problem. This characteristic prevails over the socio-economic dimension of migration (that encompasses the situation of migrants as individuals). In the literature review, we emphasized that international organizations present migration as a tool to foster economic development. This is not very different from the perspective we adopt here. Second, the question of how society should allocate its resources when the population is not constant is an old and important one. In welfare economics, utilitarianism allows to address this issue. In the utilitarian theory, there exist two conflicting approaches. According to the Benthamite view, society should care about the total utility of the members of the society. This is in contrast to the Millian perspective according to which this is the average utility, not the total utility, that matters. Neither classical nor average utilitarianism provides a satisfactory answer to the issue of how to choose the population size optimally.\footnote{Under fairly general conditions, they have implications that are ethically unacceptable, see Razin and Sadka, 2001, and references therein.} And both have their advocates and opponents.

Our purpose is not to enter this debate, though quite thrilling. The reason why we choose to use the classical version is the following. Let $C$ be aggregate consumption. Other things equal, the average utility $U(C_N)$ is decreasing in $N$ whereas if we take the
derivative of the total utility \(NU(\frac{C}{N})\) w.r.t \(N\), we get:

\[
\frac{\partial NU(\frac{C}{N})}{\partial N} = U\left(\frac{C}{N}\right)(1-\sigma_u)\text{ with }\sigma_u = \frac{C U'(\frac{C}{N})}{U(\frac{C}{N})}.
\]

Assuming that \(\sigma_u < 1\), we get that total utility is increasing in \(N\). Coming back to our motivation, given that we want to study migration driven by environmental (and financial) motives, it seems quite natural and logical to neutralize the other drivers of migration (and population change), and in particular the one related to the way society values the population size. Unlike average utilitarianism, classical utilitarianism allows us to do that since in the absence of remittances and climate change, the SIDS economy would not be willing to experience a decrease in the population thanks to migration.

Wealth in the SIDS, \(W(K(t),N(t))\), has two origins. It locally produces a unique final good, \(Y(t)\), by means of a constant returns to scale technology using natural capital, \(K(t)\), and labor, \(N(t): Y(t) = F(K(t),N(t))\), with \(F_i > 0, F_{ii} < 0\) for \(i = K, N\), and \(F_{KN} > 0\). What is worth noticing at this stage is that production capacities are limited by the amount of available natural assets, this is referred to as the environmental constraint. This constraint is expected to be increasing across time, as the negative repercussions of climate change will materialize. The SIDS also receives remittances, \(R(M(t))\), from abroad. They are supposed to be increasing and concave with respect to this population.

Combining (1)-(2), we get a direct connection between the two populations: \(M(t) = N_0 + M_0 - N(t)\) for all \(t\), with \(N_0 > 0\) and \(M_0 \geq 0\) the initial population size of respectively insular people and the diaspora. This allows us to express total wealth as follows:

\[
W(K(t),N(t)) = F(K(t),N(t)) + R(N_0 + M_0 - N(t)).
\]

Under fairly general conditions, the wealth function is either monotone increasing in \(N\), or inverted U-shaped, for \(K\) positive and given.\(^9\) We consider the latter case. Denote by \(N^*(K)\) the wealth maximizing population size, for \(K\) given. At the initial condition, we impose that

\(^9\)Take its first derive w.r.t. \(N\): \(W_N(K,N) = F_K(K,N) - R'(N_0 - N)\). Assuming \(\lim_{N \to 0} F_K(K,N) = \infty > R'(N_0 + M_0)\), \(W\) is either always increasing in \(N\) on \((0, N_0)\), or \(\exists N^*(K) \in (0, N_0) / W_N(K, N^*(K)) = 0\), with \(N^{'*}(K) > 0\).
Assumption 1 \( N_0 > N^*(K_0) \Leftrightarrow W_N(K_0, N_0) < 0 \). This is also equivalent to \( F_N(K_0, N_0) < R'(M_0) \).

This condition means that at the beginning of the planning program, the combination of the environmental constraint and the opportunity to resort the external funding of the economy through remittances is such that there exist incentives to undertake positive migration. This does not mean however that SIDS necessarily finds it optimal to go for positive migration from the origin because Assumption 1 only captures wealth motives, whereas migration is also accompanied by welfare effects. Since, everything else being equal, welfare is increasing in \( N \), the SIDS has no incentive to decrease its population size. In other words, Assumption 1 provides a necessary condition for a regime with positive migration.

This assumption seems to be the most appropriate, especially if one wants to describe the optimal migration policy (even) in the absence of climate damage. A glance at history shows us that the SIDS display a long tradition of migration, whose fundamental causes are partly environmental but not linked to climate change. We by no way claim that the migration flows that have been observed in these islands for decades can be attributed to any sort of optimal policy. Still, historical evidence suggests that positive migration may have been optimal for some SIDS. The key factor here is not the initial population size but the endowment of natural capital per capita. Thus we find it reasonable to give an account of the heterogeneity of SIDS, and resulting differences in migration patterns, which is what Assumption 1 allows.

Climate impacts show themselves in the degradation of the stock of natural asset, at a constant rate \( \delta > 0 \). Natural capital typically refers to the amount of arable lands, freshwater reserves, the endowment of the SIDS in marine and terrestrial ecosystems etc. To preserve these natural assets, the SIDS can by no way rely on mitigation since it has no capacity to affect the pattern of worldwide emissions on its own. Besides migration, the only option left to cope with climate change is then to invest in adaptation measures. For simplicity, we model adaptation as a decision variable, \( s(t) \geq 0 \), which suitably

\(^{10}\)This corresponds to the case in which \( K_0 \) is constant.
captures adaptation expenditures in ecosystems maintenance for instance.\(^{11}\) Adaptation is a means to slow down the ongoing process of deterioration of \(K(t)\). It however comes at an increasing and strictly convex cost, \(G(s(t))\). Defining as \(\varepsilon(s(t))\) the returns on adaptation, \(\varepsilon(.)\) being decreasing and convex with \(\varepsilon(0) = 1\), the law of motion of \(K(t)\) is given by:

\[
\dot{K}(t) = -\delta \varepsilon(s(t))K(t) + \delta K_{\infty}. \tag{5}
\]

Absent any climate change, \(K(t)\) would remain constant and equal to \(K_0\). Under ongoing climate change but without public adaptation, \(K(t)\) decreases exponentially at rate \(\delta\), going asymptotically to a strictly positive, though potentially very low, value \(K_{\infty}\).

Before summarizing the decision problem, it is worth formulating a general remark regarding our approach. In the literature review, we emphasized that the emigration policy has been considered as a tool for economic development for a long time. Moreover, we provided support for the perspective we adopt in this work, that consists in considering migration as an extreme form of adaptation. This of course supposes that decision makers can affect migration decisions and flows through targeted public policies.\(^{12}\) That being said, rather than modeling explicitly the education sector or interactions with the expatriates population and how they relate to migration, we make a shortcut by assuming that the decision maker directly chooses the number of emigrants, \(m(t)\).\(^{13}\)

In the end, the intertemporal decision problem can be written as follows:

\[
\max_{\{s(t), m(t)\}} \int_{t=0}^{\infty} V(c(t), m(t), N(t)) e^{-\rho t} dt, \tag{6}
\]

\(^{11}\)Therefore, by construction, our model is not designed to account for investments in adaptation infrastructure such as sea walls and dikes.

\(^{12}\)Policies that seem particularly relevant are the ones that deal with education and the management of the diaspora.

\(^{13}\)The assumption is made for simplicity and conveys the idea that governments in SIDS can ultimately control the decision to migrate. This is admittedly an oversimplified description of the real world. We however believe that this is the appropriate way to address the problem, especially because our aim is to study the optimal adaptation policy for SIDS. The alternative approach would have gone through the explicit modeling of individual migration decisions and their links with relevant public policies. This is an interesting extension of the current research that is left for future work.
with $\rho > 0$ the rate of pure time preference, subject to the resource constraint, $c(t) = \frac{W(K(t),N(t)) - G(s(t))}{N(t)}$, (1), (3), (4), and (5). Consumption is strictly positive whereas we have to account for the non-negativity constraints on $m(t)$ and $s(t)$. Finally, for the problem to be meaningful, we must focus on the situation where $\dot{K}(t) \leq 0$, i.e., there is no man-made natural capital. This would normally require to add another constraint to the optimization. For simplicity, we do not explicitly incorporate this constraint. But we will take care of it in the coming analysis.

4 Optimal policy

The optimization program above is a two-state two-control variable control problem. Taking into account the non-negativity constraints, the Lagrangian is:

$$\mathcal{L} = NU \left( \frac{W(K,N) - G(s)}{N} \right) - D(m) - \lambda_N m + \lambda_K (-\delta \varepsilon (s) K + \delta K_{\infty}) + \mu_m m + \mu_s s,$$

with $\lambda_N$ and $\lambda_K$ the co-state variables associated with $N$ and $K$, and $\mu_m, \mu_s \geq 0$ the Lagrange multipliers for $m$ and $s$.

Denote the elasticity of utility with respect to consumption and the elasticity of consumption with respect to $N$ respectively by $\sigma_u$ and $\sigma_c$:

$$\sigma_u = \frac{cU'(c)}{U(c)} \quad \text{and} \quad \sigma_c(N;K,s) = -\frac{Nc_N}{c},$$

where the elasticity $\sigma_u$ is assumed to be constant with $\sigma_u \in (0,1)$.

The set of (necessary) optimality conditions is given by:

$$\begin{cases}
D'(m) + \lambda_N \geq 0, \ m(D'(m) + \lambda_N) = 0 \\
G'(s)U'(c(N,K,s)) + \varepsilon'(s)\delta \lambda_K K \geq 0, \ s(G'(s)U'(c(N,K,s)) + \varepsilon(s)\delta \lambda_K K) = 0 \\
\dot{\lambda}_N = \rho \lambda_N - U(c(N,K,s))(1 - \sigma_c(N;K,s)\sigma_u) \\
\dot{\lambda}_K = (\rho + \delta \varepsilon(s))\lambda_K - F_K(K,N)U'(c(N,K,s)) \\
\dot{N} = -m \\
\dot{K} = \delta(K_{\infty} - \varepsilon(s)K)
\end{cases}$$

(7)

14The time index is omitted when there is no danger of confusion.
where $c(N, K, s)$ is the compact notation for the consumption function.

The first condition in (7) is related to the choice of $m$, and we immediately observe that $\lambda_N$ must be negative for experiencing a regime with $m > 0$. We come back to this point in a moment. The second optimality condition refers to the adaptation strategy. Adaptation involves the following trade-off. A marginal increase in adaptation expenditures is a means to slow down the deterioration of natural capital, a benefit that is measured at its social value. However, such an increase also implies that the economy has less resources available for consumption, this cost being measured in (marginal) utility terms.

Overall the optimality conditions in (7) define a four-dimension dynamical system that may encompass four regimes depending on whether $m, s \geq 0$. This kind of system is hardly manageable in general due to the dimensionality and non-linearity problems. Our aim in the following analysis is to get as much insight from the theoretical analysis as possible. For that purpose, we choose to work with projections in plans composed of a state variable and its corresponding control – or co-state – variable, taking the other variables as given. With the support of graphical illustrations, this should allow us to address the following questions: which regime can arise along the optimal solution? What are the dynamic features of these regimes? Is it possible for the economy to experience a switch from one regime to the other? We ultimately want to identify some critical conditions that may help us to understand which policy, in terms of the combination and timing of implementation of the two instruments, is optimal for which SIDS, based on its characteristics.

4.1 Insights from the theoretical analysis

We start with the analysis of a regime with no adaptation, $s = 0$, which means that the SIDS incurs the impacts of climate change and has no other option but to design its migration policy optimally in order to adapt to them. All proofs are gathered in the Appendix.
4.1.1 Regime with no adaptation

We first study the dynamics of population and migration when assuming that \( s = 0 \). Then we look at the joint evolution of the stock of natural capital and its shadow value.

The migration decision comes with a direct (marginal) cost, captured by \( D'(m) \), and potential benefits through the adjustment of the population size, that are captured by the shadow value of \( N \). Consider the interior solution first \( (m > 0, \mu_m = 0) \). Combining the first condition in (7) with the one characterizing the dynamics of the shadow value \( \lambda_N \) yields the following dynamic system in the \((N, m)\) plan:

\[
\begin{aligned}
\dot{m} &= m\sigma_d^{-1} \left( \rho + \frac{U(c(N;K(t),0))}{D''(m)} (1 - \sigma_u \sigma_c(N;K(t),0)) \right), \\
\dot{N} &= -m.
\end{aligned}
\]

with \( \sigma_d = \frac{mD''(m)}{D'(m)} > 0 \) the elasticity of the marginal damage, also assumed constant, and \( K(t) = (K_0 - K_\infty)e^{-\delta t} + K_\infty \).

Take \( K \) as given, and for the ease of discussion, equal to \( K_0 \). This means that there is no impact of climate change on the SIDS. In this situation, it is relatively easy to show that the condition

\[
\sigma_u \sigma_c(N_0, K_0, 0) > 1 \tag{8}
\]

is necessary and sufficient to get a permanent regime with \( m(t) > 0 \) for \( t < \infty \) (see the Appendix A.1.1). This condition involves both wealth and welfare effects. Wealth effects are captured by the elasticity \( \sigma_c(N_0;K_0) \) since \( \sigma_c(N_0;K_0) = 1 + \sigma_w(N_0;K_0) \) and \( \sigma_w \) is the elasticity of wealth w.r.t the population size. They have been discussed in detail following Assumption 1 that states that there exist migration incentives (as far as wealth is concerned). Welfare effects have to do with \( \sigma_u \). The size of \( \sigma_u \), which belongs to \((0, 1)\), tells us about how strongly society is affected by a change in the population size. Indeed, remember that \( \frac{\partial NU(C)}{\partial N} = U \left( \frac{C}{N} \right) (1 - \sigma_u) > 0 \). Now the inequality above indicates that the optimal policy features positive migration when the wealth benefit from migration (and the decrease in the population size) exceeds the welfare costs associated with it. Note that this is most likely to be true when \( \sigma_u \) is close to one, which means that society barely feels the impact of the decrease in population (as \( \frac{\partial NU(C)}{\partial N} \) is close to zero in this case).
Under condition (8), it cannot be optimal to switch to \( m = 0 \) in finite time. Starting from a positive level of migration, migration flows decrease monotonically. The migration process ends up eventually when the population size approaches its steady state value \( \hat{N}(K_0) \), that solves \( \sigma_u \sigma_c(N, K_0, 0) = 1 \).

Considering the exogenous degradation of \( K \) makes the dynamic analysis a bit more complex but does not change the main conclusion (see the Appendix A.1.2). Logically, and still assuming that condition (8) holds, as the burden imposed by the environmental constraint gets stronger and so is the dependence on remittances, the incentives to undertake positive migration are higher at any instant. The optimal migration policy displays the same qualitative features as the one depicted earlier. In the long run, natural capital converges to its degraded stationary value, \( K_\infty \). This in turn means that the population size has to decrease further in order to adapt to the much lower level of natural capital available to the SIDS. It will stabilize in the long run to the level \( \hat{N}(K_\infty) \) such that \( \sigma_c(\hat{N}(K_\infty); K_\infty, 0) = \sigma_u^{-1} \). See Figure 3 for an illustration.

When \( \sigma_c(N_0; K_0, 0) < \sigma_u^{-1} \), there is no migration incentive originally. Considering a constant \( K \), we would get a trivial stationary solution with no population change. Here however considering the degradation of the stock of natural capital (because of climate change) may lead to a different conclusion. Indeed, provided that the initial emigration ratio is low enough,

\[
\frac{M_0}{N_0} < \frac{\sigma_r(M_0)}{\sigma_u^{-1} - 1}
\]

with \( \sigma_r(M) = \frac{MR(M)}{R(M)} > 0 \) the elasticity of remittances w.r.t. the stock of expatriates,\(^\text{15}\) there exists a (unique) critical stock of natural capital such that it becomes optimal to initiate the migration process: let \( \hat{K}(N_0) \) be this stock, which solves \( \sigma_c(N_0; K, 0) = \sigma_u^{-1} \). Therefore a switch to migration will occur in finite time if and only if \( K_\infty < \hat{K}(N_0) \). The intuition of this result is quite simple. Under climate change, the environmental constraint incurred by the SIDS may become so high that at some point it is optimal to undertake positive migration in order to release the pressure imposed on natural assets. From that

\(^{15}\)This elasticity is equal to one – and the RHS of (9) does not depend on \( M_0 \) – when remittances are proportional to the size of the diaspora, \( R(M) = rM \), a specification that will be used later.
date on, migration follows an inverted-U shaped trajectory until the convergence to the same steady state. See Figure 4 for an illustration.

Let us now examine what is going on in the \((K, \lambda_K)\) plan. In this regime, the dynamics are simply given by:

\[
\begin{align*}
\dot{\lambda}_K &= (\rho + \delta)\lambda_K - F_K(K, N)U'(c(K, N, 0)) \\
\dot{K} &= \delta(K_\infty - K)
\end{align*}
\]

Replacing \(s = 0\) in the second condition in (7) and assuming that it holds with an equality, we can characterize the critical geometric locus that divides the \((K, \lambda_K)\) into two domains, the one with \(s = 0\) and the one with \(s > 0\): \(\lambda_K = \xi(K; N)\), with \(\xi_K(K; N) < 0\) and \(s = 0\) when \(\lambda_K < \xi(K; N)\).

Working first with \(N\) given, the locus \(\dot{\lambda}_K = 0\) defines another relationship between \(\lambda_K\) and \(K\), which is parameterized by \(N\): \(\lambda_K = \zeta(K; N)\), with \(\zeta_K(K; N) < 0\), and \(\dot{\lambda}_K > 0\) for \(\lambda_K > \zeta_K(K; N)\). Then we immediately obtain that the regime with no adaptation expenditure hosts a unique steady state with \(K_\infty = K_\infty\) and \(\lambda_{K\infty}(N) = \frac{F_K(K_\infty, N)U'(c(K_\infty, N))}{\rho + \delta}\).

During the convergence to the steady state, as the stock of natural capital deteriorates, its shadow value increases monotonically (see Figure 5 and the Appendix A.1.3).

Starting from the initial condition \((N_0, K_0)\), such a trajectory is feasible if and only if the domain where \(s = 0\) and \(\dot{\lambda}_K > 0\) is non-empty. Defining \(\tilde{K}(N_0)\) as the unique solution
to $\xi(K; N_0) = \zeta_K(K; N_0)$, this boils down to imposing:

$$\tilde{K}(N_0) > K_0.$$ (10)

This condition captures the initial trade-off embodied in the choice of going for adaptation, or not. It basically compares the marginal benefit from the first unit of adaptation with its marginal cost.

Considering the decrease in population size resulting from an active migration policy, things get more complicated. The two critical loci $\xi(K; N)$ and $\zeta(K; N)$ move down, which results in an increase in $\tilde{K}(N)$, and it proves difficult to assess the feasibility of a path featuring $s = 0$ for all $t$. One can however observe that as $N$ decreases the region of the $(K, \lambda_K)$ plan in which it is optimal not to adapt shrinks. This also comes as no surprise. Other things equal, with the decrease in $N$, the SIDS incentives to undertake $s$ get bigger as the opportunity cost of adaptation, in terms of foregone consumption, becomes lower.\footnote{Indeed, per capita consumption increases and marginal consumption increases when $N$ decreases.} This means that we cannot in general rule out the occurrence of a regime change from $s = 0$ to $s > 0$. As a rough illustration, Figure 6 depicts the optimal trajectory, in red, obtained for $N$ constant and equal to $N_0$. With $N$ decreasing and the frontiers moving down, it is possible that by following such a trajectory, the SIDS lies in the domain associated with $s > 0$ at a finite point in time and starts adapting to climate change thanks to this specific instrument.

We now turn to the analysis of the regime with positive adaptation.

### 4.1.2 Regime with positive adaptation

For the sake of simplicity, we continue the ongoing analysis by making use of specific functional forms:

- $U(c) = \sigma_u^{-1}c^{\sigma_u}$, $\sigma_u \in (0, 1)$;
- $D(m) = \frac{1}{1+\sigma_d}m^{1+\sigma_d}$, $\sigma_d \geq 1$;
- $Y = AK^\alpha N^{1-\alpha}$, $A > 0$, $\alpha \in (0, 1)$;
- $R(M) = rM$, $r > 0$;
- $G(s) = \gamma s$, $\gamma > 1$; and
- $\varepsilon(s) = e^{-\eta s}$, $\eta > 0$. For technical elements, refer to the Appendix A.2.
Using these specifications, we can define:

\[ \Phi(K; N) = \frac{\eta}{\gamma} \alpha AK^\alpha N^{1-\alpha} - 1, \]

and express the dynamical system as follows:

\[
\left\{ \begin{array}{l}
\dot{\lambda}_K = [\rho - \delta \varepsilon(s)\Phi(K; N)] \lambda_K \\
\dot{K} = \delta (K_\infty - \varepsilon(s)K)
\end{array} \right.
\]

Let us work with \( N \) constant, and equal to \( N_0 \) first. Noticing that the critical level \( \tilde{K}(N_0) \), defined just before, is also the solution to \( \Phi(K; N_0) = \frac{\rho}{\delta} \), the condition

\[ \tilde{K}(N_0) < K_0 \] (11)

is necessary and sufficient for the existence of a well-behaved regime with positive adaptation expenditures. If we further impose:

\[
\left\{ \begin{array}{l}
\tilde{K}(N_0) < K_\infty, \\
\Phi(K_0; N_0) < \frac{\rho}{\delta} \frac{K_0}{K_\infty},
\end{array} \right.
\]

then there exists a unique steady state parameterized by \( N_0, (K_\infty(N_0), s_\infty(N_0)) \), with \( K_\infty(N_0) \in [K_\infty, K_0] \), \( K'_\infty(N) > 0 \) and \( s'_\infty(N) > 0 \). During the transition to the steady state, the natural capital decreases monotonically and so do adaptation expenditures. We
can further impose a sufficient condition for the steady state to be a saddle point and represent the dynamical adjustment in the \((K, s)\) (see Figure 7).\(^{17}\)

The optimal level of expenditures, \(s = s(K, \lambda_K; N)\), is increasing in both \(K\) and \(\lambda_K\), while decreasing in \(N\). If its behavior w.r.t to \(\lambda_K\) is as expected, the same cannot be said of its behavior w.r.t \(K\). But this is very intuitive after all. Indeed, the returns on adaptation expenditures are larger the larger the stock of natural capital. In other words, it is when the stock of natural capital is high, and the negative impacts of climate change are felt the most (thanks to the exponential decrease of \(K\)), that it is worthwhile to invest a lot in protecting the natural capital. That why, in the regime with \(s > 0\), expenditure follows a monotone decreasing trajectory. With the degradation of its natural capital, the SIDS progressively reduces its investments until it manages to stabilize it to a degraded, tough better than \(K_\infty\), level.

Moreover, it is worth noting that a transition from the regime with \(s = 0\) to the regime with \(s > 0\) cannot take place in finite time. From an economic point of view and as long as the future matters, it does not make sense to maintain natural capital for a period of time and then stop the efforts as all the resources invested in adaptation will be wasted eventually as \(K\) converges to its lower bound \(K_\infty\).

Going back to the dynamics of population with positive adaptation, condition (8) is

\(^{17}\)We can get the explicit relationship between \(s\) and \(K\) corresponding to \(\dot{s} = 0\) in this regime. So we simply draw the loci \(\dot{\lambda}_K = 0\) and \(\dot{K} = 0\).
no longer necessary to have positive migration. Besides the usual wealth and welfare effects associated with migration, devoting a positive amount of resources to adaptation gives higher incentives to adjust the population size to get the highest possible income. In such a context, we expect that the dynamics in the \((N, m)\) remain similar to what we get in Section 4.1.1. In particular, as long as the SIDS starts with positive migration, the emigration rate should vanish only asymptotically.

The reverse inequality, \(\sigma_c(N_0; K_0, 0) < \sigma_u^{-1}\), is now necessary (but not sufficient) for a regime with \(m = 0\) together with \(s > 0\) to take place initially. In this case, it is relatively easy to provide a necessary and sufficient condition for a switch to \(m > 0\) in finite time:

\[
\sigma_c(N_0; K_\infty(N_0), s_\infty(N_0)) > \sigma_u^{-1}(> \sigma_c(N_0; K_0, 0)).
\] (13)

A last remark can be formulated. It is difficult to go further in the study of the dynamic behavior of the SIDS when located in the regime with positive adaptation and positive – but asymptotically going to zero – migration. A complete analysis would especially require to deal carefully with the issue of existence of a steady state for the general system (7). Rather, we simply want to make the following observation, assuming that a steady state exists. When the SIDS economy devotes resources to adaptation, it manages to maintain the stock of natural capital to a level above the lowest bound \(K_\infty\), which is compatible with a larger population size than in the absence of such expenditures. Not surprisingly, monitoring the speed at which natural capital deteriorates and managing to stabilize its level in the long run ultimately provides the SIDS economy with more latitude for ensuring a good enough standard of living for a larger number of inhabitants.

4.2 Discussion

Let us now put together all the pieces of information we get so far. As far as the optimal policy is concerned, our analysis reveals that the SIDS has two qualitatively different options to cope with the negative repercussions of climate change. Interestingly, two conditions on the fundamentals of the economy help to explain which policy is optimal in which context. These conditions involve the ranking between \(K_0\) and \(\hat{K}(N_0)\) on the one
hand, given that \( \sigma_c(N_0; K_0, 0) \geq \sigma_u^{-1} \Leftrightarrow K_0 \leq \hat{K}(N_0) \), and between \( K_0 \) and \( \tilde{K}(N_0) \) on the other.

In order to ease the discussion, one possibility is to represent the situation in the plan of initial conditions \((N_0, K_0)\), which requires to learn a bit more about the properties of \( \hat{K}(N_0; r, M_0, \sigma_u, A) \) and \( \tilde{K}(N_0; \gamma, \rho, \eta, \delta, A) \).

First, we have already shown that \( K_0 < \hat{K}(N_0) \) is either sufficient (\( s > 0 \)) or necessary and sufficient (\( s = 0 \)) for an initial regime with positive migration. Moreover, it can easily be checked that \( \hat{K}_{N_0} > 0 \) and \( \hat{K}_{N_0 N_0} > 0 \). In other words, the critical locus triggering positive migration is higher the larger the initial population size. For very high \( N_0 \), it is optimal to start the migration process in order to release the pressure on natural assets for a larger the set of values of \( K_0 \). As to the comparative statics on \( \hat{K}(N_0; r, M_0, \sigma_u, A) \), we get:

\[
\hat{K}_r, \hat{K}_{\sigma_u} > 0 \text{ whereas } \hat{K}_{M_0}, \hat{K}_A < 0.
\]

Other things equal, the higher \( r \) and the lower \( M_0 \), the higher the returns on migration. On the contrary, a high productivity parameter \( A \) makes it worthwhile to maintain a large population to benefit from the local origin of wealth. Finally, the higher \( \sigma_u \), the lower the welfare cost of migration.

Second, one can note that \( \tilde{K}_{N_0} < 0 \) and \( \tilde{K}_{N_0 N_0} > 0 \). The critical initial level of the stock of natural capital at which it is optimal to invest in adaptation decreases as \( N_0 \) increases. Indeed, the larger the population, the stronger the incentives to undertake positive adaptation to slow down the degradation of the stock of natural capital. In addition, remember that when \( \tilde{K}(N_0) < K_0 \), the policy features \( s > 0 \), at least initially. The reason for this finds its roots in the properties of the optimal level of expenditures, discussed earlier. Regarding the question of how the location of the critical locus \( \tilde{K}(N_0; \gamma, \rho, \eta, \delta, A) \) changes with the fundamentals of the economy in the plan of initial conditions \((K_0, N_0)\), we further obtain:

\[
\tilde{K}_\gamma, \tilde{K}_\rho > 0 \text{ whereas } \tilde{K}_\eta, \tilde{K}_\delta, \tilde{K}_A < 0.
\]

\(^{18}\)Once working with the specifications introduced early, it is possible to make the dependence of these functions on the parameters explicit.
So, we can conclude that the larger $\gamma$ and/or the lower $A$, the higher the cost of the adaptation policy. In the same vein, the lower $\eta$, the lower the returns on adaptation expenditures. Finally, when $\rho$ is high, people attach less value to what happens in the long run, while a low $\delta$ means that climate change translates into a slow degradation of the natural capital. This all points to the fact that the set of initial conditions for which it is optimal to choose $s = 0$ expands.

Overall, we can conclude that there exist two main policy alternatives for the SIDS. Either the SIDS implements – at least initially and possibly permanently – a policy relying on only one of the two instruments, which makes them substitutes. Or the SIDS adopts a policy combining adaptation and migration from the origin, the two instruments being complements. Figure 8 provides a representation of the different possible combinations of the two instruments in the special case where $M_0 = 0$. For a large enough $N_0$ (larger than the intersection between $\hat{K}(N_0)$ and $\tilde{K}(N_0)$), we see that there are three possibilities depending on the initial endowment in natural capital. For a large enough $K_0$, it is optimal to go for adaptation first, while not using the migration tool. Migration may become operative at some later date, depending on whether condition (13) is met. Quite on the contrary, for $K_0$ low enough, there is no point at spending resources in adaptation and the only option left is migration. Finally in intermediate situations, the optimal policy consists of a mix between adaptation and migration right from the beginning.

Figure 8: Different optimal policies in the $(N_0, K_0)$ plan

---

19In this case, condition (9) is always fulfilled.
5 Calibration

In this section, we bring the model to the data in order to complement the theoretical analysis presented in the previous section. We calibrate the different parameters as well as the initial conditions for the main variables. In the coming exercise, a particular emphasis will be placed on international comparisons.

5.1 Parameters calibration

Among the parameters some values common to all the SIDS are taken from the economic literature or computed. In the theoretical analysis, we assume that the cost of infrastructure expenditure is linear: \( G(s) = \gamma s \), with \( \gamma > 1 \). The parameter \( \gamma \) can be interpreted as the marginal cost of public funds. To our knowledge, there is no paper providing an estimation of \( \gamma \) for SIDS countries. We therefore use a value in line with the estimations calculated by Auriol and Warlters (2012) using a sample of African countries, that is \( \gamma = 1.2 \).\(^{20}\) The share of labor in production—i.e. parameter \((1 - \alpha)\) in our model—is not the same in all countries. Therefore, we consider the average of values for SIDS given in the Pennsylvania World Table (PWT) (Feenstra et al. (2015)), \( (1 - \alpha) = 0.55 \).\(^{21}\)

The choice of the value of \( \rho \), the rate of pure time preference, has led to an intense debate in the literature (Tol (2006), Nordhaus (2007), Dasgupta (2008)). We use 0.04, but we have performed the numerical analysis for values ranging between 0.04 and 0.2 to be consistent with the literature (see Appendix A.4 for the robustness checks). For the sake of simplicity, we choose a quadratic function for the marginal cost of migration, which implies that \( \sigma_d = 1 \). As for the utility function, we take \( \sigma_u = 0.95 \), which means that we work with a function that displays features close to a linear one. We implement

\(^{20}\)Note that the results are not very sensitive to the value of \( \gamma \), when varying between 1 and 2. This claim has been tested in the supplementary comparative statics analysis provided in Appendix A.4.

\(^{21}\)Another solution would be to evaluate the value of \( \alpha \) accordingly to the natural capital stock values which are retained. However, in this case, the calibration of the rest of the model, especially for remittances function, would be less accurate. The robustness tests for the calibration with different definitions of natural capital stock are available upon request.
robustness tests for $\sigma_u = \{0.9; 0.95; 0.99\}$. The other parameters are determined using the following method:

- **The total factor productivity (TFP):** $A$

  The value of the Total Factor Productivity (TFP), $A$, is calibrated on data from the PWT and the World Development Indicators (WDI). We compute the Cobb-Douglas function of the model using the following data from the PWT: population, country level labor share and output stock in Purchasing Power Parity (PPP). Moreover, we define the natural capital stock as the sum of the value added from tourism as well as agriculture, forestry and fisheries (AFF) in the World Development Indicators (WDI). Note that tourism is incorporated in order to capture the economic gains from landscapes on these islands. We apply the standard growth accounting framework to our production function. According to it, economic growth can be decomposed into contributions from inputs, here labor and natural capital. Therefore, in order to obtain annual growth rate of the TFP for country $i$, we compute the following equations:

  \[
  g_{i, TFP} = g_{i, output} - (1 - \alpha)g_{i, labor} - \alpha g_{i, capital} \\
  TFP_i = 1 + g_{i, TFP}
  \]

  where the growth rate of the variable $x$ is defined as: $g_x = (x(t + 1) - x(t))/x(t)$. We use long-term average for SIDS, which is 1.01. Note that we conduct robustness tests on the value of the TFP according to the minimum and the maximum value computed for the SIDS, which are respectively 0.96 and 1.08 (cf. Appendix A.4).

- **Initial Values**

\[\text{Robustness results are given in the Appendix A.4.}^\text{22}\]

\[\text{We have also conducted a calibration where the natural capital is defined as the sum of the capital stock from the PWT and the value added from tourism and the AFF. For developing countries the latter methodology gives output that are less correlated to the GDP than the retained method, while for SIDS, the results remain the same (cf. Appendix A.4).}^\text{23}\]
We define the diaspora size as the average of the emigrant stock between 2000 and 2015, using the data-set of the United Nations, Department of Economic and Social Affairs, Population Division (POP/MIG). However, in order to have a better comparison between countries as well as to respect the scale of the model, we divide the values from the different dataset by $10^6$ to get the initial conditions for population ($N_{i0}$), the stock of natural capital ($K_{i0}$) and the stock of migrants ($M_{i0}$).

- The remittances coefficient: $r$

Next we want to compute the value of $r$, given the linear specification used: $R(M(t)) = rM(t)$. A preliminary step is to compute the level of remittances perceived in the economy. The total amount received from the diaspora is obtained using the variable “remittances perceived in percentage of GDP” from the WDI data-set, this variable is denoted by $\hat{R}_i(t)$ in our calibration. First, we calibrate the value of the remittances, $R_i(t)$, according to the following equation:

$$Y_i(t) = TFP_i(t) \times K_i(t)^\alpha \times N_i(t)^{1-\alpha}$$

$$R_i(t) = \hat{R}_i(t) \times Y_i(t)$$

Then, we regress $R_i(t)$ on the computed values of the diaspora size to get the value of $r$. The value of this coefficient for the SIDS is reported in Table 1.\textsuperscript{24} Note that the results displayed in the table tend to support our choice for the definition of natural capital. Indeed, the coefficient of correlation between the migrants stock and the remittances perceived is higher if capital is computed as the sum of the AFF and tourism added values than with the alternative definition.

Table 2 summarizes the parameters values used to calibrate the model.\textsuperscript{25} Table 3 gives the long term average values retained for the local population, the diaspora and the

\textsuperscript{24}This calibration works quite well for islands, with a $R^2 = 0.72$, while the correlation is not as good for other developing countries or developed countries, for the reasons explained above.

\textsuperscript{25}The values retained for $\sigma_u$, $\gamma$, $\delta$, $\eta$, and $\rho$ are in bold print, while the other computations are given in A.4.
Table 1: Correlation between the diaspora and the Remittances

<table>
<thead>
<tr>
<th>Variable</th>
<th>SIDS</th>
<th>Developing Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Diaspora</td>
<td>20.58</td>
<td>102.87</td>
</tr>
<tr>
<td>95% Conf. Interval</td>
<td>[17.29; 23.87]</td>
<td>[81.65; 124.09]</td>
</tr>
<tr>
<td>Count</td>
<td>63</td>
<td>62</td>
</tr>
<tr>
<td>R2</td>
<td>0.72</td>
<td>0.60</td>
</tr>
<tr>
<td>R2 Adjusted</td>
<td>0.71</td>
<td>0.60</td>
</tr>
</tbody>
</table>

(1) Capital is defined as the sum of added values of AFF and tourism

(2) Capital is defined as the sum of capital stock from the PWT and aggregate defined by (1).

stock of natural capital (with the values of tourism and AFF). Islands are classified in two groups, according to their geographical localization: Caribbean SIDS vs Other SIDS. In the analysis of the initial conditions, we compute $\hat{K}$ with $M(0) = 0$ to be able to have a comparison between countries.

Table 2: Parameters of the model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Factor Productivity</td>
<td>$A$</td>
</tr>
<tr>
<td></td>
<td>${0.96; \mathbf{1.01}; 1.08}$</td>
</tr>
<tr>
<td>Capital share in production</td>
<td>$\alpha$</td>
</tr>
<tr>
<td></td>
<td>0.45</td>
</tr>
<tr>
<td>Cost of Public expenditure</td>
<td>$\gamma$</td>
</tr>
<tr>
<td></td>
<td>${1; \mathbf{1.2}; 1.5}$</td>
</tr>
<tr>
<td>Elasticity of Utility with</td>
<td>$\sigma_u$</td>
</tr>
<tr>
<td></td>
<td>${0.9; \mathbf{0.95}; 0.99}$</td>
</tr>
<tr>
<td>Rate of pure time preference</td>
<td>$\rho$</td>
</tr>
<tr>
<td></td>
<td>${\mathbf{0.04}; 0.1; 0.2}$</td>
</tr>
<tr>
<td>Elasticity of the marginal damage</td>
<td>$\sigma_d$</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Remittances return rate</td>
<td>$r$</td>
</tr>
<tr>
<td></td>
<td>${15; \mathbf{20}; 25}$</td>
</tr>
<tr>
<td>Degradation rate</td>
<td>$\delta$</td>
</tr>
<tr>
<td></td>
<td>${0.02; \mathbf{0.1}; 0.25}$</td>
</tr>
<tr>
<td>Efficiency of adaptation</td>
<td>$\eta$</td>
</tr>
<tr>
<td></td>
<td>${0.2; \mathbf{0.4}; 0.6}$</td>
</tr>
</tbody>
</table>
Table 3: Initial Conditions

<table>
<thead>
<tr>
<th>Countries</th>
<th>Population</th>
<th>Mig. Stock</th>
<th>AFF</th>
<th>Tourism</th>
<th>Nat. cap.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Other SIDS</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>COM</td>
<td>653,547.13</td>
<td>97,925.75</td>
<td>471,785,696</td>
<td>54,433,940</td>
<td>526,219,648</td>
</tr>
<tr>
<td>CPV</td>
<td>487,067.75</td>
<td>145,873.25</td>
<td>224,986,560</td>
<td>553,566,592</td>
<td>778,553,152</td>
</tr>
<tr>
<td>FJI</td>
<td>844,610.19</td>
<td>168,499</td>
<td>684,939,776</td>
<td>1,424,057,600</td>
<td>2,108,997,376</td>
</tr>
<tr>
<td>GNB</td>
<td>1,480,094</td>
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<td>30,794,654</td>
<td>920,854,336</td>
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<td>2,139.75</td>
<td>240,465,408</td>
<td>2,589,127,936</td>
<td>2,829,593,344</td>
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<tr>
<td>MUS</td>
<td>1,231,074.25</td>
<td>141,176.75</td>
<td>770,540,736</td>
<td>2,943,838,464</td>
<td>3,714,379,264</td>
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<tr>
<td>STP</td>
<td>165,723.25</td>
<td>30,233</td>
<td>45,970,536</td>
<td>34,740,660</td>
<td>80,711,200</td>
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<td>SYC</td>
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<td>10,377.75</td>
<td>45,081,308</td>
<td>633,793,280</td>
<td>678,874,560</td>
</tr>
<tr>
<td><strong>Caribbean SIDS</strong></td>
<td></td>
<td></td>
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<tr>
<td>ABW</td>
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<td>15,343,271</td>
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<tr>
<td>ATG</td>
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<td>57,153.75</td>
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<td>BHS</td>
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<td>35,616.25</td>
<td>116,810,920</td>
<td>2,218,210,560</td>
<td>2,335,021,568</td>
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<td>BLZ</td>
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<td>53,181.25</td>
<td>298,639,648</td>
<td>438,550,592</td>
<td>737,190,272</td>
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<td>BRB</td>
<td>276,888.25</td>
<td>94,207</td>
<td>67,785,448</td>
<td>1,050,887,680</td>
<td>1,118,673,152</td>
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<tr>
<td>CUW</td>
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<td>65,692.75</td>
<td>13,903,565</td>
<td>938,107,072</td>
<td>952,010,624</td>
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<td>63,148.5</td>
<td>79,620,696</td>
<td>131,192,264</td>
<td>210,812,960</td>
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<td>574,474,560</td>
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<td>62,329,152</td>
<td>182,764,736</td>
<td>245,093,888</td>
</tr>
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</table>


*Source: PWT*

*Migrant Stock, source: UNSD*

*Source: WDI. Value added, PPP 2011$*

*Source: WDI. International receipt, PPP 2011$*

*Sum of AFF and Tourism*
5.2 Results

The first objective of our numerical analysis is to understand the position of each SIDS in the plan of the initial conditions and thus to answer to the following question: what is the optimal strategy of each island to cope with climate change? Figure 9 displays, in logarithmic scale plots, the position of each island according to their initial values for capital and population. We also represent on the same plan the two critical loci, $\tilde{K}(N)$ and $\hat{K}(N)$, derived in the theoretical analysis, that are given by the following equations:

$$\tilde{K}(N_0) = \left( \frac{\gamma(\rho + \delta)}{\delta \eta A(1 - \sigma_F)} N_0^{\alpha - 1} \right)^{\frac{1}{\alpha}},$$

(14)

$$\hat{K}(N_0) = \left( \frac{r N_0^{\alpha - 1} (N_0 - M_0(\sigma_u^{-1} - 1))}{A(\sigma_u^{-1} - \alpha)} \right)^{\frac{1}{\alpha}}.$$  

(15)

Note that these two curves are drawn by assuming that SIDS share what we call the common parameters, i.e., all the parameters except the initial conditions. We choose to focus on the heterogeneity with respect to the initial conditions because of the insights given by theoretical investigation. In particular, it has been emphasized that they are crucial to understand SIDS potential development path in a world with climate change.

Remind that if a country is located above the curve $\hat{K}$, optimal migration can be nil, while if it is below the curve, it will always be positive. Moreover, if the country position is below the curve $\tilde{K}$, investments in adaptation measures can be zero, while if it is above the curve there will be positive adaptation expenditures.

Several remarks arise from the observation of Figure 9. First of all, there exists a critical threshold for the population size, 1.5 million inhabitants, that determines whether it is optimal to systematically go for adaptation. When the population is larger than this threshold, the optimal policy is based on adaptation, that may be complemented with migration. On the contrary, if the population is very small, the island is likely located below the curve of $\tilde{K}$. In this scenario, using adaptation measures is optimal only for countries with very large stock of natural. For example, Barbados and the Bahamas are above the curve $\tilde{K}$ because of their large endowment in natural capital. Indeed, their
amount of natural capital, or the economic gains linked to this capital, is almost equal to countries’ capital which are ten times larger.

Second, what we can learn from these two figures is that half of the islands are in the region where neither adaptation measures nor migration are undertaken from the beginning and forever. However, most of these islands are very close to the frontiers defined by $\tilde{K}$ and $\hat{K}$. Put differently, switches to situations where migration and/or adaptation measures are implemented is very likely. Indeed, only countries with quite large population and stock of natural capital, combine both adaptation and migration from the origin, with no change expected in their optimal policy. This second group is composed of Dominican Republic, Haiti, Jamaica, Trinidad and Tobago, Suriname, Papua New Guinea, Cabo Verde and Comoros. For them, both tools will be used to cope with climate change. Finally, it appears that a supplementary analysis is necessary for small islands, those where investments in adaptation measures can be null or positive.

Figure 9: Distribution of the SIDS according to their initial conditions

Our analysis also contains robustness tests related to climate change parameters, such as the degradation rate, $\delta$, and the effect of the effectiveness of adaptation measures $\eta$. Figure 10 illustrates the effect of a change in $\delta$ on the curve $\tilde{K}$. It appears that when the degradation rate is high, almost all the countries implement adaptation measures in order to slow down the pace of natural capital degradation. If the degradation rate
is high, the gains from investing in adaptation are large compared to the cost of this policy. Figure 11 displays the effect of a change in $\eta$ on the curve $\tilde{K}$. As expected, the larger the effectiveness of the adaptation measures, the larger the incentives to undertake investments in adaptation.

Figure 10: Analysis of the effects of the parameter $\delta$

Figure 11: Analysis of the effects of the parameter $\eta$

Overall, the analysis of the impact of environmental parameters shows that the climate change scenario plays a critical role in understanding the dynamics of smallest islands. Indeed, because of their proximity with the two loci, a change in these parameters may
have a substantial impact on the characterization of their optimal strategy.

The last parameter that deserves a particular attention is $r$. Changes in $r$ induce large changes in the computation of $\hat{K}$. In the present analysis, we have tested values of the coefficient $r$ obtained in the regression according to approximately the 95% confidence interval. The interpretation of this parameter is not easy because we obtain very different values depending on the method to calibrate the production function. Here we have selected the values of $r$ which give the highest correlation with the remittances, knowing that they are the most conservative ones. However, $r$ might be larger with another definition of natural capital stock as shown in table 1. In that case, the model predicts that all the islands will implement migration to cope with climate change. Knowing the weight of migration for these islands, this is a plausible conclusion. In this context, the main question is whether or not it is optimal to use adaptation in complement with migration.

Figure 12: Analysis of the effects of the parameter $r$ (1)
6 Conclusion

In this paper we adopted a centralized perspective to assess the optimal policy of a SIDS facing the negative repercussions of climate change on its natural capital. We developed a dynamic framework in which economic conditions on the island are directly derived from two sources of wealth: production and remittances. To produce, the economy uses labor and natural capital, while remittances are sent by the diaspora. Therefore, under climate change and in the absence of any intervention, production follows an exogenous decreasing trend. Welfare in the SIDS is defined over the total utility derived from consumption. In this centralized model, the policy maker has two means to cope with climate-related damages: migration strategy in order to receive remittances or adaptation strategy, in order to slow the degradation process. Migration induces a social cost for the migrant and a contraction of the output because of a decreasing labor force. In this context, reducing the population size might be good for wealth at the origin, but it affects total welfare both directly (cf. the total utility criterion), and indirectly (by changing the amount of per capita consumption). The adaptation strategy, generates a direct cost in terms of foregone consumption, while the benefit stems from the capacity to maintain the stock of natural capital to a higher level, and for a longer period of time.

Our analysis of the optimal policy was conducted in two phases, the first one being devoted to a theoretical investigation, the second one consisting of a calibration of the model. From the theoretical investigation, we obtained two conditions that shed some light on the SIDS’s preferred policy to deal with climate change. When only migration is possible, we found a critical condition on the fundamentals of the economy under which there were migrations incentives at the beginning of the planning period. In this situation, migration decreases and vanishes only when the optimal population size is reached. In the absence of migration incentive initially, the increasing environmental constraint could lead to a switch to positive migration, according to another condition. On the contrary when only adaptation expenditures are possible, we found another critical condition under which the SIDS starts to adapt from the origin. When the regime with positive adaptation is permanent, adaptation expenditures decrease monotonically over
time but remains positive. Consequently, the natural capital remains at a higher level than in the case without adaptation measures.

Finally, merging both regimes analysis, we found that on one hand, if there is no adaptation initially but positive migration, the incentives to switch on the second instrument increase over time as the decrease in the population size reduces the marginal cost of adaptation. Moreover, if both tools are implemented, SIDS could stabilize natural assets to a constant and higher level than in the absence of adaptation. As a result, the population size is also larger in the long run.

In a second step, we calibrated the model. The main objective is to describe the initial conditions on the islands—i.e. the initial size of the population and the initial endowment in natural capital—and thus to determine their potential strategy. We found that SIDS could be distinguished in two groups. The first one, composed by countries with very small populations, that is the majority of the islands. The second group is composed of a few islands with larger population. For the latter, the calibration exercise suggests that at each period of time both policy tools should be implemented. In smaller islands results are however ambiguous. In this case, an analysis of the dynamics at a country level should be implemented to define what the optimal paths of these islands are. Indeed, while migration is always implemented during the transition, the adaptation measures could be positive or not.
References


Appendix

A  Facts

Total wealth is given by $W(K, N) = F(K, N) + R(N_0 + M_0 - N)$, with first derive w.r.t. $N$:

$W_N(K, N) = F_N(K, N) - R'(N_0 - N)$. For a given $K$, assuming $\lim_{N \to 0} F_N(K, N) = \infty$, $W$ is either always increasing in $N$ on $(0, N_0)$ or $\exists! N^*(K) \in (0, N_0) / W_N(K, N^*(K)) = 0$, with $N^*(K) > 0$. For an interesting problem, we consider the second case and further impose: $N_0 > N^*(K_0)$ (cf. Assumption 1).

The set of optimality conditions corresponding to the general problem is given by:

$$
\begin{aligned}
D'(m) + \lambda_N &\geq 0, \ m(D'(m) + \lambda_N) = 0 \\
G'(s)U'(c(N, K, s)) + \varepsilon'(s)\delta \lambda_K K &\geq 0, \ s(G'(s)U'(c(N, K, s)) + \varepsilon'(s)\delta \lambda_K K) = 0 \\
\dot{\lambda}_N & = \rho \lambda_N - (U(c(N, K, s)) + NU''(c(N, K, s))c_N(N, K, s)) \\
\dot{\lambda}_K & = (\rho + \delta \varepsilon(s))\lambda_K - F_K(K, N)U'(c(N, K, s)) \\
\dot{N} & = -m \\
\dot{K} & = \delta(K_\infty - \varepsilon(s)K)
\end{aligned}
$$

with $c(N, K, s) = \frac{W(K, N) - G(s)}{N}$, and $\lambda_K$ the shadow value of the stock of natural capital.

The system may go through four different regimes depending on whether $m \geq 0$ and $s \geq 0$. In any regime, we have to deal with four-dimension systems that are not easy to handle in general. To circumvent the difficulties posed by the analysis, we work hereafter with projections of these systems in two-dimension (sub)spaces, respectively in the plan $(K, \lambda_K)$ (for $s = 0$) or $(K, s)$ (for $s > 0$) by taking $(m, N)$ as given, and $(N, \lambda_N)$ (for $m = 0$) or $(N, m)$ (for $m > 0$) by taking $(s, K)$ as given. We then merge the results to characterize the overall solution.

Let us start the analysis of the dynamics in the regime with no adaptation: $s = 0$. First, we take a look at the population dynamics. Next, we consider the evolution of the pair $(K, \lambda_K)$. 

40
A.1 Regime with no adaptation

A.1.1 Population dynamics under a constant $K$

For $K$ constant and equal to $K_0$, the FOCs are given by:

$$
\begin{aligned}
D'(m) + \lambda_N &\geq 0, \\
\dot{\lambda}_N &= \rho \lambda_N - (U(c(N; K_0, 0)) + NU'(c(N; K_0, 0))c_N(N; K_0, 0)), \\
\dot{N} &= -m.
\end{aligned}
$$

where $\lambda_N$ is the shadow price of the population size and $c(N; K_0, 0) = \frac{W(N; K_0)}{N}$ the consumption rate, with $c_N = \frac{NW_N - W}{N^2} < 0 \leftrightarrow N > N^*(K_0)$. Note that $m > 0 \iff \lambda_N < 0$.\(^{26}\)

Denote by $\sigma_u$, the elasticity of the utility function w.r.t $c$: $\sigma_u = \frac{U'(c)}{U(c)} \in (0, 1)$, constant. Denote also by $\sigma_c(N; K_0)$ (respectively $\sigma_w(N; K_0)$) the elasticity of consumption (resp. wealth) w.r.t $N$: $\sigma_c(N; K_0, 0) = -\frac{NC_N}{c}$ (resp. $\sigma_w(N; K_0) = -\frac{NW_W}{W}$). $\sigma_w(N; K_0) > 0$ on the interval $(N^*(K_0), N_0)$. Given that $\sigma_c(N; K_0, 0) = 1 + \sigma_w(N; K_0)$, $\sigma_c(N; K_0, 0) > 1$ on this interval. Finally define $\sigma_d$ as the elasticity of the marginal damage w.r.t $m$:

$$
\sigma_d = \frac{mD''(m)}{D'(m)} > 0; \text{ also assumed to be constant.}
$$

Consider first a regime with $m > 0$. From the associated dynamics

$$
\begin{aligned}
\dot{m} &= m\sigma_d^{-1} \left( \rho + \frac{U(c(N; K_0)))}{D'(m)} (1 - \sigma_u \sigma_c(N; K_0)) \right), \\
\dot{N} &= -m,
\end{aligned}
$$

we can define the locus $\dot{m} = 0$, for positive $m$, (if) only if $1 \leq \sigma_u \sigma_c(N; K_0, 0)$. It then yields a relationship between $m$ and $N$:

$$
m = H(N; K_0) \text{ with } H(N; K_0) = (D')^{-1} \left( -\frac{U(c(N; K_0, 0))}{\rho} (1 - \sigma_u \sigma_c(N; K_0, 0)) \right).
$$

The derivative of $\sigma_w$ (and $\sigma_c$) w.r.t $N$ are:

$$
\frac{\partial \sigma_w}{\partial N} = -\frac{1}{W} (W_N (1 + \sigma_w(N; K_0)) + NW_{NN}) > 0,
$$

\(^{26}\)For the sake of exposition, we omit the arguments of the functions (especially the derivatives) when they are unnecessary.
because \( W_{NN} = F_{NN} + R''(N_0 + M_0 - N) < 0 \).

Now imposing

\[
\sigma_c(N_0; K_0, 0) > \sigma_u^{-1} \iff \sigma_u(1 + \sigma_w(N_0; K_0)) > 1
\]  

denotes necessary and sufficient for the existence of this regime with \( m > 0 \). Noticing that \( \sigma_c(N^*(K_0); K_0, 0) = 1 \), we can also define a unique \( \hat{N}(K_0) \in (N^*(K_0), N_0) \) that solves \( \sigma_c(\hat{N}(K_0); K_0, 0) = \sigma_u^{-1} \). The next step is to study the features of \( H(N; K_0) \). We get

\[
\frac{\partial H}{\partial N} = \frac{U(c)\sigma_u}{\rho ND''(m)} \left( \sigma_c(1 - \sigma_u \sigma_c) + N \frac{\partial \sigma_c}{\partial N} \right),
\]

\[
\iff \frac{\partial H}{\partial N} = \frac{U(c)\sigma_u \sigma_c}{\rho ND''(m)} \left( \sigma_c(1 - \sigma_u) - \frac{NW_{NN}}{W} \right) > 0.
\]

In addition, we get that \( H(\hat{N}(K_0); K_0) = 0 \).

Starting from a positive level of migration below the locus \( H(N; K_0) \), migration flows decrease monotonically until either \( H(N; K_0) \) is hit in finite time, which would then lead to the second corner regime \( (m = 0) \), or it approaches asymptotically the critical level \( \hat{N}(K_0) \) at which \( \dot{m} = m = 0 \). It is easy to show that the first option cannot coincide with the optimal policy. Suppose that there exists \( T < \infty \) such that \( m(T) = 0 \iff \lambda_N(T) = 0 \) and \( m(t) = 0, N(t) = \tilde{N} > \hat{N}(K_0) \) for all \( t \geq T \). Solving for the differential equation in \( \lambda_N \) in the corner regime, we get:

\[
\lambda_N(t) = \frac{U(c(\tilde{N}; K_0, 0))(1 - \sigma_u \sigma_c(\tilde{N}; K_0, 0))}{\rho} \left( 1 - e^{\rho(t-T)} \right).
\]

But then, the transversality condition, \( \lim_{t \to \infty} e^{-\rho t} \lambda_N(t) \tilde{N} = 0 \), cannot hold. So we get a contradiction.

Under (17), there exists a permanent regime with \( m > 0 \). Under the opposite of (17),

\[
\sigma_c(N_0; K_0) < \sigma_u^{-1},
\]

we get the trivial solution with \( m = 0, N = N_0 \) and \( \lambda_N = \frac{U(c(N_0; K_0, 0))(1 - \sigma_u \sigma_c(N_0; K_0, 0))}{\rho} > 0 \) for all \( t \).

\[27\] Actually \( \sigma_c(N^*(K); K, s) = 1 \) whatever \( s \).
A.1.2 Population dynamics under a decreasing natural capital

To analyze this case, we further need: \( c_K = \frac{W_K}{N} > 0 \), \( \frac{\partial \sigma_u}{\partial K} = \frac{\partial \sigma_c}{\partial K} = -\frac{N}{W^2}(W_KW_N - W_KW_N) < 0 \) since \( F_{KN} > 0 \).

Let us again consider the regime with \( m > 0 \): following the same approach – under Assumption 1 – we can define \( N^*(K) \), for any \( K \in (K_\infty, K_0) \) that solves \( W_N(N^*(K), K) = 0 \). We have \( 0 < N^*(K) < N^*(K_0) \) for all \( K \) in this interval. Assume that condition (17) holds, the dynamics are given by:

\[
\begin{align*}
\dot{m} &= m\sigma_u^{-1}\left(\rho + \frac{U(c(N;K(t),0))}{D'(m)}(1 - \sigma_u \sigma_c(N;K(t),0))\right), \\
\dot{N} &= -m.
\end{align*}
\]

The locus \( \dot{m} = 0 \), for \( m > 0 \), still yields a relationship between \( m \) and \( N \), parameterized by \( K(t) \):

\[
H(N;K(t)) = (D')^{-1}\left(-\frac{U(c(N;K(t),0)}{\rho}(1 - \sigma_u \sigma_c(N;K(t),0))\right),
\]

that exists iff \( \sigma_u \sigma_c(N;K(t),0) > 1 \). It is possible to show that the derivative of \( H \) w.r.t to \( K \) is negative: \( \frac{\partial H}{\partial K} = -\frac{\sigma_u U}{\rho D'(m)}(\frac{\varepsilon_K}{c}(1 - \sigma_u \sigma_c(N;K(t),0)) - \frac{\partial \sigma_c}{\partial K}) < 0 \) since \( \frac{\varepsilon_K}{c}(1 - \sigma_u \sigma_c(N;K(t),0)) - \frac{\partial \sigma_c}{\partial K} = \frac{1}{W}((1 - \sigma_u)\sigma_cW_K + NW_{NK}) > 0 \). The region featuring \( \dot{m} < 0 \) expands as \( K \) decreases. If this regime is permanent, \( K \) asymptotically converges its lower bound \( K_\infty \). In turn, the population size approaches the value \( \dot{N}(K_\infty) \) that solves \( \sigma_c(N;K_\infty) = \sigma_u^{-1} \).

Again we have to determine whether it is possible to enter the second corner regime \( (m = 0) \) in finite time. Let us proceed as we did before: Suppose that there exists \( T < \infty \) such that \( m(T) = 0 \iff \lambda_N(T) = 0 \) and \( m(t) = 0 \), \( N(t) = \dot{N} = \dot{N}(K_\infty) \) for all \( t \geq T \). The solution for \( \lambda_N \) in this corner regime is: \( \lambda_N(t) = -e^{\rho t}\int_T^t U(c(\dot{N};K(\tau))(1 - \sigma_u \sigma_c(\dot{N},K(\tau)))e^{-\rho \tau}d\tau > 0 \). But then \( \lim_{t \to \infty} e^{-\rho t}\lambda_N(t)\dot{N} = -\dot{N} \int_T^\infty U(c(\dot{N};K(\tau))(1 - \sigma_u \sigma_c(\dot{N},K(\tau)))e^{-\rho \tau}d\tau \neq 0 \). So the transversality condition is not satisfied and we get a contradiction.

Now, let us assume that (18) holds. This implies that originally, there is no incentive
to undertake migration: \( m = 0 \) and \( N = N_0 \). The evolution of \( \lambda_N \) is given by:

\[
\dot{\lambda}_N = \rho \lambda_N - U(c(N_0; K(t), 0)(1 - \sigma_u \sigma_c(N_0; K(t), 0))
\]

The locus \( \dot{\lambda}_N = 0 \) gives a relationship:

\[
\mathcal{H}(N_0; K) = \frac{U(c)(1 - \sigma_u \sigma_c)}{\rho}
\]

with \( \mathcal{H}_{N_0} = \frac{1}{\rho} \left( c_u U'(c)(1 - \sigma_u \sigma_c) - \sigma_u U(c) \frac{\partial \sigma_c}{\partial N} \right) = \frac{U(c)\sigma_u}{\rho W} (W_N(1 - \sigma_c(\sigma_u - 1)) + N_0 W_{NN}) < 0 \) and \( \dot{\lambda}_N > 0 \) for \( \lambda_N > \mathcal{H}(N_0; K) \). Under (18), this locus satisfies \( \mathcal{H}(N_0; K_0) > 0 \). Now, with \( K \) decreasing, noticing that \( \frac{\partial \sigma_u}{\partial K} < 0 \), we have \( \sigma_c \leq \sigma_u^{-1} \Leftrightarrow F(K_0, N_0) (\sigma_u^{-1} - 1 + \sigma_f) \geq R'(M_0) - (\sigma_u^{-1} - 1) \frac{R(M_0)}{N_0} \), with \( \sigma_f = \frac{NF_N}{F} \) the share of labor in production. Let us work with technologies exhibiting a constant \( \sigma_f \in (0, 1) \). Then, either \( M_0 \) is so large that the RHS is negative and the inequality is always satisfied: the regime with \( m = 0 \) is permanent and has the same features as the ones studied in Appendix A.1.1. Or, \( M_0 \) is low enough – and the initial emigration ratio satisfies: \( \frac{M_0}{N_0} < \frac{\sigma_u(M_0)}{\sigma_u^{-1} - 1} \) with \( \sigma_u(M) = \frac{MR'(M)}{R(M)} \) – for the RHS to be positive. In this case, there exists a unique \( \hat{K}(N_0) \) that solves the equation above and \( \sigma_c \geq \sigma_u^{-1} \Leftrightarrow K \leq \hat{K}(N_0) \). In turn and provided that \( \dot{K}(N_0) > K_\infty \), this implies that the economy will experience a switch in finite time to the regime with \( m > 0 \). For \( K \) given, in a regime with \( m = 0 \), the shadow value of \( N \) should be constant and equal to \( \mathcal{H}(N_0; K) \). Now, given the continuous decrease in \( K \), this shadow value will decrease too still while being given by \( \lambda_N(t) = \mathcal{H}(N_0; K(t)) \).\(^{28}\) Let \( T \) be the date at which \( \lambda_N \) is equal to 0: \( \lambda_N(T) = 0 \), or equivalently \( K(T) = \hat{K}(N_0) \). From \( T \) onwards, we expect that \( \lambda_N(t) < 0 \) first, experiences a phase where it decreases and then becomes increasing and behaves according to the interior solution analyzed above. This would in turn results in a non-monotone trajectory for \( m \), \( m \) being increasing first before it starts to decrease to asymptotically approach 0.

\(^{28}\)This confirms that the system cannot start in the regime where \( m > 0 \) (which features \( \lambda_N < 0 \) and \( \dot{\lambda}_N > 0 \)) in this case.
A.1.3 Dynamics of natural capital and its shadow value

Take first \( N \) as given. In the regime with \( s = 0 \), the dynamics are:

\[
\begin{align*}
\dot{\lambda}_K &= (\rho + \delta)\lambda_K - F_K(K, N)U'(c(K, N, 0)) \\
\dot{K} &= \delta(K - K) 
\end{align*}
\]  

(19)

The steady state of this regime is located at the (unique) intersection between the vertical line \( K = K \) and the locus \( \dot{\lambda}_K = 0 \), which now defines a relation between \( K \) and \( \lambda_K \): \( \lambda_K = \frac{F_K(K, N)U'(c)}{\rho + \delta} \equiv \zeta(K; N) \), with \( \zeta_K(K; N) = \frac{F_KU'(c) + F_KcKU''(c)}{\rho + \delta} < 0 \), \( \zeta_K(K; N) > 0 \), and \( \dot{\lambda}_K \geq 0 \Leftrightarrow \lambda_K \geq \zeta(K; N) \). In addition, it is quite easy to check that this steady state is saddle point stable. This means that convergence can occur only along the part of the stable branch originating in the domain where \( \dot{\lambda}_K > 0 \) and \( \dot{K} < 0 \). This makes sense: as natural capital depreciates, its social value increases during the transition to the StS.

The next question is: can a transition from \( s = 0 \) to \( s > 0 \) occur in finite time? To provide an answer, we have to define the critical locus that divides the \( (K, \lambda_K) \) into two domains, one with \( s = 0 \), the other with \( s = 0 \). To do so, simply replace \( s = 0 \) in the second FOC of system (16), and suppose that it holds with an equality. This gives another relation between \( K \) and \( \lambda_K \): \( \lambda_K = -\frac{G'(0)U'(c)}{\delta \delta'(0)K} \equiv \xi(K; N) \), with \( \xi_K(K; N) = \frac{G'(0)cN U''(c)}{\delta \delta'(0)K} > 0 \). Moreover, we get that \( s = 0 \) when \( \lambda_K < \xi(K; N) \).

Then we obtain \( \zeta(K; N) \leq \xi(K; N) \Leftrightarrow K F_K \leq -\frac{G'(0)(\rho + \delta)}{\delta \delta'(0)} \). Given that the LHS of this inequality is equal to \((1 - \sigma_f) F(K, N)\), so increasing in \( K \), this is equivalent to \( K \leq \tilde{K}(N) \) with \( \tilde{K}(N) = F^{-1}\left(\frac{-G'(0)(\rho + \delta)}{(1 - \sigma_f) \delta \delta'(0)}; N\right) \). In addition, as it also decreasing in \( N \), we can conclude that: \( \tilde{K}'(N) < 0 \).

Suppose that \( K^\infty < \tilde{K}(N_0) < K_0 \), then the economy with no adaptation must be originally located in the region with \( \dot{\lambda}_K < 0 \) and \( \dot{K} < 0 \), which is not compatible with the existence of an optimal trajectory leading to the corner steady state. In other words, \( \tilde{K}(N_0) > K_0 \) is a necessary and sufficient condition for the existence of a corner solution with \( K^\infty = K^\infty \). This also implies that a transition from the corner regime with \( s = 0 \) to
the interior regime with \( s > 0 \) cannot take place ever.

With \( N \) decreasing, that is under positive migration, all the critical loci go down since \( \zeta_N(K; N), \xi_N(K; N) > 0 \) while \( \bar{K}(N) \) moves up because of the different speeds of adjustment of \( \zeta(K; N) \) and \( \xi(K; N) \). This first means that the region where it is optimal not to adapt shrinks over time. Clearly, in this situation, starting within the regime with \( s = 0 \), it may be possible that a switch to \( s > 0 \) occurs in finite time. But it proves very difficult to provide formal conditions that guarantee such a transition.

### A.2 Regime with positive adaptation

#### A.2.1 Dynamics of natural capital and adaptation

We first work with for \( N \) given. Consider the dynamics in the regime with \( s > 0 \), represented in the \((K, s)\) plan:

\[
\begin{align*}
\dot{\lambda}_K &= \left[ \rho + \delta \varepsilon(s) \left( 1 + \frac{K F_K(K; N)}{G(s)} \varepsilon'(s) \right) \right] \lambda_K \\
\dot{K} &= \delta (K_\infty - \varepsilon(s) K) \\
\end{align*}
\]

where \( \dot{\lambda}_K = \frac{\varepsilon'(s)}{\varepsilon(s)} \frac{1}{G'(s)} K F_K - 1. \)

For a tractable analysis, from now on we consider a specification of the model with the following functional forms: \( U(c) = \sigma_u^{-1} c^{\sigma_u}, \sigma_u \in (0, 1); D(m) = \frac{1}{1+\sigma_d} m^{1+\sigma_d}, \sigma_d \geq 1; Y = AK^\alpha N^{1-\alpha}, A > 0, \alpha \in (0, 1); R(M) = rM, r > 0; G(s) = \gamma s, \gamma > 1; \) and \( \varepsilon(s) = e^{-\eta s}, \eta > 0. \)

This expression above then reduces to:

\[
\Phi(K; N) = \frac{\eta}{\gamma} AK^\alpha N^{1-\alpha} - 1,
\]

with \( \Phi_K = \frac{2}{\gamma} \alpha^2 AK^{\alpha-1} N^{1-\alpha} > 0 \) for \( K \in [K_\infty, K_0]. \)

We want to deal with the features of the \( \dot{K} = 0 \) and \( \dot{\lambda}_K = 0 \) loci, (from which we can infer those of \( \dot{s} = 0 \) locus) given that the latter is parameterized by \( N \). This leads to the
we further have

\[ \epsilon \] does not belong to the interval \( K \] on the interval \[ N \] well-defined, \( K \] is decreasing in \( \epsilon \) and \( \phi \). Thus \( K \) is necessarily non-increasing in our problem, \( \epsilon \) is decreasing in \( \phi \). As to the other locus, \( \epsilon' = \frac{1}{\epsilon'(\epsilon)} - \frac{\Phi(K,N)}{\Phi(K,N)^2} > 0 \). Again, referring to the functional forms we use, we obtain \( \epsilon(K,N) = \frac{1}{\eta} \ln \left( \frac{\delta \Phi(K,N)}{\rho} \right) = \epsilon_K(K,N) > 0 \) and \( \epsilon_{KK}(K,N) = \frac{\Phi_{KK} \Phi - \Phi_{K}^2}{\Phi_{K}^2} < 0 \). Both loci are increasing in concave in \( K \). Given that from the second FOC in (16), \( \lambda_K \geq 0 \), we further have \( \lambda_K \geq 0 \) if \( s \) is the optimal \( \epsilon(K,N) \) because \( \epsilon_N(K,N) = \frac{\Phi_N}{\Phi} > 0 \).

Assume for now that \( \varphi(K) \) and \( \epsilon(K,N) \) have a unique intersection. By construction, this steady state belongs to the \( s = 0 \) locus. By differentiating the second FOC in (16), that defines the optimal \( s \) as \( s = s(\lambda_K,K,N) \), we get

\[
\left( \frac{\epsilon''(s)}{\epsilon'(s)} - \frac{cU''(c) c_s}{U'(c)} \right) ds = - \left( 1 - \frac{cU''(c) K c_K}{U'(c) c} \right) \frac{dK}{K} - \frac{d\lambda_K}{\lambda_K},
\]

Define \( K(N) \) such that \( \Phi(K(N);N) = 0 \); \( K(N) = \left( \frac{\gamma N^\alpha - 1}{\delta \eta (1-\sigma_p) A N^\alpha - 1} \right)^{\frac{1}{3}} \), with \( K'(N) < 0 \). We have \( K(N) > K(0) \) for all \( N < N_0 \). For the first relation to exist, we must focus on the interval \( [K(N),K_0] \), which is non-empty only if the following condition holds: \( \epsilon(N) < K_0 \). Moreover, by definition \( \epsilon(s) \in [0,1] \) for \( s \geq 0 \), and \( \epsilon(0) = 1 \). This imposes a stronger restriction on the domain of definition of \( K \): for a solution with \( s > 0 \) to be well-defined, \( K \) should belong to \( [K(N),K_0] \), with \( K(N) = \left( \frac{\gamma N^\alpha - 1}{\delta \eta (1-\sigma_p) A N^\alpha - 1} \right)^{\frac{1}{3}} > K(N) \) for the functional forms used. This interval is non-empty only if \( \epsilon(N) < K_0 \), as \( K(.) \) is decreasing in \( N \). So we impose

\[
K(N_0) < K_0.
\]

Remind that \( K \) is necessarily non-increasing in our problem, so we must restrict the analysis to pairs \( (K,s) \) such that \( \dot{K} \leq 0 \) if \( s \leq \varphi(K) \). The \( \dot{K} = 0 \) satisfies:

\[
\varphi'(K) = \frac{1}{\epsilon'(\epsilon)} - \frac{\Phi_K}{\Phi(K,N)^2} > 0.
\]

Thus, \( \varphi(K) = \frac{1}{\eta} \ln \left( \frac{K}{K_{\infty}} \right) \geq 0 \) and \( \varphi''(K) = -\frac{1}{\eta K} < 0 \). As to the other locus, \( \epsilon'(K,N) = \frac{1}{\epsilon'(\epsilon)} - \frac{\Phi_K(\Phi)N}{\Phi(K,N)^2} > 0 \). Again, referring to the functional forms we use, we obtain \( \epsilon(K,N) = \frac{1}{\eta} \ln \left( \frac{\delta \Phi(K,N)}{\rho} \right) = \epsilon_K(K,N) > 0 \) and \( \epsilon_{KK}(K,N) = \frac{\Phi_{KK} \Phi - \Phi_{K}^2}{\Phi_{K}^2} < 0 \). Both loci are increasing in concave in \( K \). Given that from the second FOC in (16), \( \lambda_K \geq 0 \), we further have \( \lambda_K \geq 0 \) if \( s \) is the optimal \( \epsilon(K,N) \) because \( \epsilon_N(K,N) = \frac{\Phi_N}{\Phi} > 0 \).

\[
\text{Assume for now that } \varphi(K) \text{ and } \epsilon(K,N) \text{ have a unique intersection. By construction, this steady state belongs to the } s = 0 \text{ locus. By differentiating the second FOC in (16), that defines the optimal } s \text{ as } s = s(\lambda_K,K,N), \text{ we get}
\]

\[
\left( \frac{\epsilon''(s)}{\epsilon'(s)} - \frac{cU''(c) c_s}{U'(c)} \right) ds = - \left( 1 - \frac{cU''(c) K c_K}{U'(c) c} \right) \frac{dK}{K} - \frac{d\lambda_K}{\lambda_K},
\]

\[
\text{Indeed, } K(N) \text{ solves } \Phi(K(N);N) = \frac{\epsilon}{s} \iff \epsilon(K(N);N) = 0 \text{ and it must hold that } \Phi(K(N);N) \geq \frac{\epsilon}{s}.
\]

\[
\text{A restriction that we didn’t incorporate in the optimization program for simplicity, but that has to be verified ex-post.}
\]
noticing that \( \frac{\varepsilon''(s)}{\varepsilon'(s)} = -\eta - \frac{dU''(c)}{U'(c)} = 1 - \sigma_u \) and using the specifications, this expression simplifies to:

\[
ds = \left( \eta + \frac{\gamma(1 - \sigma_u)}{Nc} \right)^{-1} \left[ \left( 1 + \frac{KF_K(1 - \sigma_u)}{Nc} \right) \frac{dK}{K} + \frac{d\lambda_K}{\lambda_K} \right],
\]

which means that \( s_K > 0 \), \( s\lambda_K > 0 \) and \( s_N < 0 \). From that, we directly obtain the expression of \( \dot{s} \), whose sign is undetermined when \( \dot{\lambda}_K > 0 \), given that \( \dot{K} < 0 \) (except at the steady state). We come back to this later on. In the meantime, the steady state analysis is conducted.

We still work for \( N \) given and look at the conditions for the existence of a steady state \((K_\infty(N), s_\infty(N))\). From (21), a steady state solves: \( \varphi(K) = \epsilon(K; N) \). This boils down to finding the solution to: \( \frac{\rho}{\delta} K_\infty = \Phi(K; N) \). Under (22), given that \( \Phi_K > 0 \) and \( \Phi(\tilde{K}(N), N) = \frac{\rho}{\delta} < \frac{\rho \tilde{K}(N)}{\delta K_\infty} \Leftrightarrow \tilde{K}(N) > K_\infty \), if we impose:

\[
\begin{align*}
\tilde{K}(N_0) < K_\infty, \\
\Phi(K_0; N_0) < \frac{\rho}{\delta} K_0,
\end{align*}
\]

then we know that there exists a unique \( K_\infty(N) \in (\tilde{K}(N), K_0) \) solving the equation above. Actually, \( K_\infty(N) \) must be larger than \( K_\infty \): \( K_\infty(N) \in (K_\infty, K_0) \). In addition, \( K_\infty(N)'(N) > 0 \) as \( \Phi_N(K; N) > 0 \). We finally obtain \( s_\infty(N) \) by replacing \( K \) with \( K_\infty(N) \) in for instance \( \varphi(K) \), with \( s_\infty'(N) = \varphi'(K)K_\infty'(N) > 0 \).

Let us now assess the local stability conditions, for \( N \) given. Equation (23) yields \( \dot{s} = 0 \) in terms of \( \dot{\lambda}_K \) and \( \dot{K} \), replacing them with the corresponding law of motion, we get:

\[
\dot{s} = \delta \left( \eta + \frac{\gamma(1 - \sigma_u)}{Nc} \right)^{-1} \left( \Lambda(K; N) + \frac{(1 - \sigma_u)(1 - \sigma_f)F(K, N)}{Nc} \left( \frac{K_\infty}{K} - \varepsilon(s) \right) \right)
\]

with \( \Lambda(K, s) = \frac{\rho}{\delta} + \frac{K_\infty}{K} + \frac{(1 - \sigma_f)\varepsilon'(s)F(K, N)}{\gamma} \). Combining this equation with the expression for \( \dot{K} \), and linearizing around the steady state, we obtain the Jacobian matrix, \( J \):

\[
J = \begin{pmatrix}
-\delta(1 - \sigma_f)\varepsilon'(s)F(K, N) & \delta \left( \eta + \frac{\gamma(1 - \sigma_u)}{Nc} \right)^{-1} \left( \frac{(1 - \sigma_f)^2\varepsilon'(s)F(K, N)}{\gamma} - (1 + \frac{(1 - \sigma_u)(1 - \sigma_f)F(K, N)}{Nc} \frac{K_\infty}{K} \right) \\
-\delta\varepsilon'(s)K & -\delta\varepsilon'(s)K
\end{pmatrix}
\]

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Direct calculations yield:
\[
\det J = \delta^2 \varepsilon'(s_\infty(N))\varepsilon(s_\infty(N)) (\sigma_f(1 + \Phi(K_\infty(N); N)) - 1) < 0 \Leftrightarrow \Phi(K_\infty(N); N)) > \frac{1 - \sigma_f}{\sigma_f},
\]
and under this condition, the steady state \((K_\infty(N), s_\infty(N))\) is a saddle point.

The convergence to the steady state takes place along the stable branch, which means that both \(s\) and \(K\) follow a monotone path. Note that the second term between parenthesis in (25) is always negative. Now assume that \(\dot{s} > 0\) for all \(t\). We have \(\Lambda(K_\infty(N), s_\infty(N)) = 0\), and \(s < s_\infty(N)\) and \(K > K_\infty(N)\) for any pair \((K, s)\) in the neighborhood of the steady state. Given that \(\Lambda_K < 0\) and \(\lambda_s > 0\), \(\Lambda(K, s) < \Lambda(K_\infty(N), s_\infty(N)) = 0\). This in turn implies that \(\dot{s} < 0\) at the instant when the pair \((K, s)\) is achieved, which yields a contradiction. So we can claim that \(s\) is monotone decreasing over time.

### A.2.2 Migration and population dynamics

From here, we do not really provide a formal proof but rather a discussion on the possible outcomes.

In the last step of the resolution, we move back to the analysis in the \((N,\lambda_N)\), or \((N,m)\), plan. Following the same lines as in the Appendices A.1.1 and A.1.2, we already know that the properties of \(\sigma_c\) are decisive to characterize the solution. Before we proceed, it is worth noticing that the general definition of \(\sigma_c\) has changed:

\[
\sigma_c(N; K, s) = -\frac{1}{N_c} (G(s) - W(1 + \sigma_w(N, K))) \Leftrightarrow \sigma_c(N; K, s) = 1 + \frac{W\sigma_w(N, K)}{W - G},
\]

with

\[
\frac{\partial \sigma_c}{\partial N} = -\frac{c_N}{c} \left(\sigma_c - 1 + \frac{W_N N}{c_N}\right) > 0,
\]

\[
\frac{\partial \sigma_c}{\partial K} = -\frac{1}{(Nc)^2} \left(GW_K\sigma_w + \frac{N(W - G)}{W}(W_N W - W_K W_N)\right) = -\frac{N}{(Nc)^2} ((W - G)W_N W - W_K W_N) \leq 0,
\]

\[
\frac{\partial \sigma_c}{\partial s} = \frac{G'(s) W \sigma_w}{(Nc)^2} = -\frac{\gamma NW_N}{(Nc)^2} > 0.
\]

We observe that \(\sigma_c(N; K, s) > \sigma_c(N; K, 0)\) for all \(s > 0\). Assume that condition (17) holds. This implies that \(\sigma_c(N_0; K_0, s_0) > \sigma_u^{-1}\): there will be incentive to migrate
initially. For \( s > 0 \) and \( K \) given, we further know that \( \exists! N^*(K) \in (0, N_0) \) such that \( \sigma_c(N^*(K); K) = 1 < \sigma_u^{-1} \). Taking the pair \((K, s)\) as given, it must exists a unique solution to \( \sigma_c(N; K, s) = \sigma_u^{-1} \). Denote this solution as \( \hat{N}(K, s) \). But this does not solve the existence issue: we want to be sure that there exists \( N \in (0, N_0) \) that solves \( \sigma_c(N; K_{\infty}(N), s_{\infty}(N)) = \sigma_u^{-1} \). This is not obvious as if we know that \( \sigma_c \) may be varying for at least some period of time in the interval \((1, \sigma_u^{-1})\) and is continuous w.r.t. \( t \), it is difficult to understand how it will change over time because it depends on \( N, K \) and \( s \).

Next we can use the relation \( s = \varphi(K) \equiv s(K) \) to define the steady state population size as a function of \( K \) only: \( \hat{N}(K) = \hat{N}(K, s(K)) \). Then, we compute

\[
\hat{N}'(K) = -\left( \frac{\partial \sigma_c}{\partial N} + \frac{\partial \sigma_c}{\partial s} s'(K) \right)
\]

Direct manipulations give:

\[
\frac{\partial \sigma_c}{\partial K} + \frac{\partial \sigma_c}{\partial s} s'(K) < -\frac{\gamma \Phi(K; N)}{\eta K (W - G)^2} \leq 0 \text{ for } K \geq \hat{K}(N).
\]

Thus, the numerator above is non-negative, which means that a necessary and sufficient condition for \( \hat{N}'(K) > 0 \) is \( \frac{\partial \sigma_c}{\partial N} > 0 \), which is indeed the case. There is a clear parallel to draw between the current analysis and the one conducted in the Appendix A.1.2. Indeed, the solution we get in this benchmark case can be rewritten (with slight abuse of notation) as \( \hat{N}(K_{\infty}, 0) \). Moreover, the solution is clearly continuous in \((K, s)\). As \( \hat{N} \) is increasing in \( s \) but decreasing in \( K \) we then conclude that \( \hat{N}(K, s(K)) > \hat{N}(K_{\infty}, 0) \) for \( K > K_{\infty} \) and \( s(K) > 0 \).

Now consider that condition (18) holds. Compared to the benchmark, this is only necessary for a regime \( m = 0 \) to take place initially. Assume that this is indeed the case, which requires \( s(0) \) be low enough: \( \sigma_c(N; K, s(0)) < \sigma_u^{-1} \). Then the question is: would a transition in finite time be possible in this situation? In other words, we want to check to which extent the analysis conducted in the Appendix A.1.2 can still apply to the general case. With \( N \) constant and equal to \( N_0 \) initially (as \( m = 0 \)), we have to study the impacts of the dynamic adjustments in \((K, s)\) on migration incentives. Basically, this boils down
to tracking the evolution of $\sigma_c$ across time, given that: $\dot{\sigma}_c = \frac{\partial \sigma_c}{\partial \dot{K}} \dot{K} + \frac{\partial \sigma_c}{\partial \dot{s}} \dot{s}$. Rearranging (that is, using the expression of $\dot{s}$, $\dot{K}$ and of the partial derivatives), we get

$$\dot{\sigma}_c = -\frac{NK}{(Nc)^2} \left( \eta + \frac{\gamma(1 - \sigma_u)}{Nc} \right)^{-1} \times \left[ \left( (W - G) \left( \eta + \frac{\gamma(1 - \sigma_u)}{Nc} \right) - \frac{\gamma WN \Phi(K,N)}{K} \right) \frac{\dot{K}}{K} + \frac{\gamma WN \lambda_K}{K} \right]$$

This expression is positive if $\lambda_K > 0$ in this regime. The steady state, analyzed in the $(K, \lambda_K)$, is a saddle point, which implies that $\lambda_K$ must be monotone. In addition, it is clear that $\ddot{K} > 0$ in the neighborhood of the steady state, with

$$\ddot{K} = -\delta\varepsilon(s)K \left( -\eta \dot{s} + \frac{\dot{K}}{K} \right)$$

$$= \delta\varepsilon(s)K \left( \eta + \frac{\gamma(1 - \sigma_u)}{Nc} \right)^{-1} \left( \frac{\lambda_K}{\lambda_K} + \frac{\gamma(1 - \sigma_u)\Phi(K,N)}{Nc} \frac{\dot{K}}{K} \right),$$

a necessary condition for this to be true is $\lim_{t \to \infty} \dot{\lambda}_K > 0$. Thus $\dot{\lambda}_K > 0$ for all $t < \infty$. In sum, $\sigma_c$ is increasing over time in this regime. The last question is, will it reach the threshold $\sigma_u^{-1}$, that triggers migration, in finite time? The answer depends on the features of the steady state $(K_\infty(N_0), s_\infty(N_0))$. Actually, we can conclude that a transition to the regime with $m > 0$ will occur in finite time if and only if

$$\sigma_c(N_0; K_\infty(N_0), s_\infty(N_0)) > \sigma_u^{-1}. \quad (27)$$

A.3 Further elements

A.3.1 Natural capital dynamics, with and without adaptation

Denote respectively by $K^*$ and $K^0$ the stock of capital corresponding to a situation with permanent adaptation and no adaptation at all. We want to show that $K^*(t) > K^0(t)$ for all $t$. Let us work by contradiction.
We know that $K^s\infty = K\infty(N) > K^s\infty = K\infty$ and $K^s(0) = K^0(0) = K_0$. Moreover $\dot{K}^s(0) = \delta(K\infty - \varepsilon(s(0))K_0) > \dot{K}^0(0) = \delta(K\infty - K_0)$ because $\varepsilon(s(0)) < 1$ for $s(0) > 0$. So in the neighborhood of $t = 0$, $K^s > K^0$. If there exist instants at which $K^s$ and $K^0$ take the same value, the number of such instants must be even. Assume that there exists two instants $(t_1, t_2)$, with $t_1 < t_2$ such that $K^s(t_1) = K^0(t_1)$ and $K^s(t_2) = K^0(t_2)$. We necessarily have $\dot{K}^s(t_1) < \dot{K}^0(t_1)$. This is equivalent to $\varepsilon(s(t_1)) > 1$, which is impossible. This yields the contradiction and we can claim that $K^s(t) > K^0(t)$ for all $t$.

A.3.2 Critical threshold $\dot{K}(N_0)$ for the specified model

For our specifications, we get:

$$\dot{K}(N_0) = \left[ \frac{r(N_0 - (\sigma_d^{-1} - 1)M_0) N_0^{\alpha - 1}}{A(\sigma_u^{1-\alpha})} \right]^{\frac{1}{\alpha}},$$

it is linear in $N_0$ for $M_0 = 0$, and well defined for $M_0 > 0$ provided that $M_0$ is low enough.

A.4 Calibration

In this section we provide details on the calibration of the theoretical analysis developed in section 5.

A.4.1 Model Specification

The functional forms used at the different stages of the analysis are the following:

- Power functions with $D(m) = \frac{1}{1+\sigma_d} m^{1+\sigma_d}$, $\sigma_d \geq 1$, and $U(c) = \frac{1}{\sigma_u} e^{\sigma_u}$, $\sigma_u \in (0, 1)$.
- Cobb-Douglas technology: $Y = AK^\alpha N^{1-\alpha}$, and linear remittances: $R(M) = rM$. Then $\sigma_F = 1 - \alpha$, $\sigma_R = \frac{N}{N_0-N}$, and $\sigma'_R = \frac{N_0}{N_0-N} = \frac{(1+\sigma_R)}{N_0-N} > 0$.
- Linear cost of infrastructure expenditure $G(s) = \gamma s$, with $\gamma > 1$ the cost of public funds, and $\varepsilon(s) = e^{-\eta s}$. 

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A.4.2 Parameters description

A.4.3 Comparative statics

In this section we show that the model is robust to changes in the parameters, and that the conclusion for the analysis of the initial conditions are the same for reasonable changes in the parameters values.

Figure 13: Analysis of the effects of the parameter $A$ on $\hat{K}$

Figure 14: Analysis of the effects of the parameter $\sigma_u$
Table 4: Country level of the TFP

<table>
<thead>
<tr>
<th>Countries</th>
<th>Cap. (AFF and Tourism) (1)</th>
<th>Cap. (AFF and Tourism) (2)</th>
<th>Cap. (All Stocks) (1)</th>
<th>Cap. (All Stocks) (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comoros</td>
<td>1</td>
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<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>Cabo Verde</td>
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<td>1.02</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>Fiji</td>
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<td>1.02</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>Guinea-Bissau</td>
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<td>1.00</td>
<td>1.00</td>
<td>1.01</td>
</tr>
<tr>
<td>Maldives</td>
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<td>1.04</td>
<td>1.00</td>
<td>1.02</td>
</tr>
<tr>
<td>Mauritius</td>
<td>1.00</td>
<td>1.01</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>Sao Tome and Principe</td>
<td>1.02</td>
<td>1.02</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Seychelles</td>
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<td>1.02</td>
<td>1.03</td>
</tr>
<tr>
<td>Aruba</td>
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<td>0.99</td>
<td>0.97</td>
<td>0.97</td>
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<tr>
<td>Antigua and Barbuda</td>
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<td>1.02</td>
<td>1.04</td>
<td>1.01</td>
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<tr>
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<tr>
<td>Belize</td>
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<td></td>
</tr>
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<td>0.8</td>
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<tr>
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<td>0.96</td>
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</tr>
<tr>
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<td>0.98</td>
</tr>
<tr>
<td>Saint-Lucia</td>
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<td>1.01</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>Suriname</td>
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<td>1.03</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Trinidad and Tobago</td>
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<td>0.99</td>
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<tr>
<td>Saint-Vinc. and the Gren.</td>
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<td>1.00</td>
<td>0.95</td>
<td>0.96</td>
</tr>
<tr>
<td><strong>Average</strong></td>
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<td><strong>1.01</strong></td>
<td><strong>0.98</strong></td>
<td><strong>0.99</strong></td>
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<tr>
<td><strong>Maximum</strong></td>
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<td><strong>1.08</strong></td>
<td><strong>1.04</strong></td>
<td><strong>1.03</strong></td>
</tr>
<tr>
<td><strong>Minimum</strong></td>
<td><strong>0.83</strong></td>
<td><strong>0.96</strong></td>
<td><strong>0.8</strong></td>
<td><strong>0.92</strong></td>
</tr>
</tbody>
</table>

**Legend:** (1): Labor is given by Employment (PWT), (2) Labor is given by the Population (PWT)
Table 5: Correlation coefficient between computed output and observed output (PWT)

<table>
<thead>
<tr>
<th>Countries group</th>
<th>Cap. (All stocks) (1)</th>
<th>Cap. (AFF and Tourism) (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIDS</td>
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<td>0.93</td>
</tr>
<tr>
<td>Developing Countries</td>
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<td>Developed Countries</td>
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<td>0.86</td>
</tr>
</tbody>
</table>

Legend: (1): Labor is defined as the Employment from the PWT, (2): Labor is defined as the Population from the PWT.

Figure 15: Analysis of the effects of the parameter $\gamma$
Figure 16: Analysis of the effects of the parameter $\rho$

Figure 17: Analysis of the effects of the parameter $A$ on $\tilde{K}$
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