Commitment and efficiency-inducing tax and subsidy scheme in the development of a clean technology

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1   Introduction

This paper analyses the optimal environmental policy design in situations where the regulator - hereafter a government - can not strongly commit to announcements about future tax and subsidy levels. The motivation is that long-run perspectives of environmental policies often face short run concerns. One consistent illustration in Europe is related to the french government which cancelled the carbon tax increase for year 2019 following recent demonstrations of the yellow vest movement. Other examples include the case of the australian government abolishing the carbon tax in 2014, or the spanish government abruptly cancelling the renewable energy subsidies in 2012.

I specifically consider environmental policies which aim at supporting the transition from the use of dirty technologies to clean technologies by subsidizing innovation. The interplay between innovation and environmental policies has been extensively addressed.[1] However, a large share of the literature abstracts from the issue of commitment. In most papers, the analysis consists in comparing the optimal policy and a business-as-usual scenario (see Bosetti et al., 2009 ; Edenhoffer et al., 2006 ; Popp, 2006). Yet several authors point out that the government lack of commitment may lead to inefficient environmental innovation (see Wirl, 2013 ; Montero, 2011). The question thus arises: if a government can not strongly commit to announcements about future tax and subsidy levels, is there an efficient policy design? And, if so, how does it differ from the case of strong commitment?

To tackle this issue, I analyse a model where two perfectly substitutable technologies are available to produce a final good. The "dirty" technology, used by a competitive industry, is associated with a flow of pollution. The "clean" technology, owned by a monopoly, is non-polluting but also initially more expensive than the dirty technology. The monopoly can invest to improve its clean technology. The improvement is modeled as the accumulation of a stock of knowledge, which reduces the cost associated with the clean technology as in Tsur and Zemel (2003). The government can tax the pollution flow and subsidy monopoly investments. In the

[1]See Popp (2019) for a recent review. See also Nakicenovic and Nordhaus (2011) for the specific case of global warming.

timeline of the game setting, the government is the first-mover (leader), it sets up its policy before the monopoly, which then chooses its production and investment levels as the second-mover (follower). This paper then characterizes the efficiency-inducing tax and subsidy schemes for two different commitment levels adopted by the government.

The analysis is divided into three steps. First, I describe the social optimum as a benchmark. Second, I characterize the Open-Loop Stackelberg Equilibrium (OLSE) of the game. This models a strong commitment level: the government announces and fully commits to the future path of the tax and subsidy rates. Third, I characterize the Global Stackelberg Equilibrium (GSE) of the game (Basar and Olsder, 1999). This models a weaker level of commitment than the OLSE: the government rather commits to a policy rule that characterizes the levels of tax and subsidy rates as a function of the stock of knowledge.

I first show that only a tax is necessary to induce efficiency under strong commitment (OLSE). Under weak commitment (GSE), I find that the policy rule that induces efficiency also features a subsidy to investment. The subsidy is used to deal with the anticipation by the monopoly that its choices of investment in knowledge will affect the future tax rates. The improvement of the clean technology being modeled via the stock of knowledge, a natural extension of the basic framework is to analyse what happens when two firms can contribute to this stock under technological spillovers. In this case, I show that a tax and a subsidy are necessary to induce efficiency under strong commitment (OLSE). Yet, compared to the case of the monopoly, a subsidy is needed to correct the free-rider problem that emerges from the existence of spillovers. Under weak commitment (GSE), the firms take into account that any new unit of knowledge decreases the future tax rate but also increases the future subsidy rate. This new effect implies a more complex form of the subsidy rate. A surprising result is that, in some situations, the subsidy rate may be negative in the short run, which implies that it may be optimal to initially tax investments in knowledge.

Regarding the related literature, the present framework is close to Wirl (2013), who studies the effect of government lack of commitment on the intertemporal monopolistic supply of a clean technology. In this paper, the inability of the government to commit leads to a suboptimally low improvement in the clean technology and, thus, to an inefficient allocation of investments. The result is equivalent for emission permits and emission taxes. There are two main differences with regard to my contributions. First of all, Wirl (2013) compares the OLSE to the Markovian Nash equilibrium of the game. As such, he compares what I refer to as a case of strong commitment level to a case where the government can not commit at all. By considering a GSE, I consider an intermediate case where the government commitment level is weak. I can characterize a policy rule that induces efficiency in the absence of a strong commitment, which is the first main contribution. Second, the present framework also differs from Wirl (2013) because he considers production capacity rather than knowledge. The stock variable relates to a physical capital in Wirl (2013). In this paper, it relates to a stock of knowledge that lowers the cost related to the clean technology. The immediate consequence is that, by contrast to Wirl (2013), the stock is not associated with well-defined property rights in the present model, and it can spill out between firms. The second main contribution of this paper is thus to analyse the effect of a lack

of commitment for policy design under technological spillovers and development of an alternative clean technology.

Also related to the present framework, Montero (2011) tackles the issue of environmental policy when the government can not commit. In a two-period model, one potential innovator invests, in the first period, to develop a clean technology that can be used in the second period. The analysis consists in comparing situations when the government does update (no commitment) or does not update (commitment) its policy in the second period. It shows that the performance of taxes and permits differs in such a setting. The present framework allows to further analyze the innovator's intertemporal investment dynamics in the development of the clean technology. In Montero (2011), innovator's decision only takes place in the first period.

Ulph and Ulph (2013) focus on a specific source of time-inconsistency of environmental policies, namely the fact that government are in power for a limited period of time and that next governments may give more or less weight to environmental damage costs. In this context, they find that a government may need to use additional R&D subsidies to induce immediate investments in the development of the clean technology. In the present article, I consider a different time-inconsistency, which emerges as a government may be tempted to update its policy and deviate from the originally decided policy path depending on the reaction of the firms. In this different context, I find, as Ulph and Ulph (2013), that a R&D subsidy is needed to make up for the government lack of commitment. However, this is more complex in the presence of multiple firms owning clean technologies under spillover effects. I show that, in some situations, the government may even need to tax firms investments in knowledge in the short run to slow down the pace of development of the clean technology.

In the remainder of this article, I introduce the basic framework and characterize the social optimum in section 2. Then, I characterize the OLSE and the GSE of the Stackelberg dynamic game in section 3. I extend the model to the case of a duopoly under technological spillovers in section 4. Last, I provide some concluding remarks in section 5.

2 The model and the social optimum

2.1 Basic framework

First, in what follows, I always use linear quadratic specifications. This is standard in the literature on differential games (Dockner, 2000 ; Engwerda, 2005). As is known, this structure yields tractable analytical solutions. Second, I am being parsimonious in the use of parameters to ease the reading. Each parameter is introduced hereafter only if it does matter in the analysis. Last, the model is suitable to analyse any type of transition from the use of polluting techniques - or technologies - to non-polluting ones for the production of a good or a service. However, throughout the article, I focus on the energy sector to illustrate my framework.[2] For instance, the reader may think about the transition from technologies such as thermal power plants - which

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[2]Among others, an alternative application of the model could be, for instance, the agricultural sector. Nuclear technologies, which are supported and promoted by the Joint FAO/IAEA Division of Nuclear in Food and Agriculture (see Hendrichs et al., 2009 or Mabit et al., 2018), are examples of alternative and less polluting technologies.

involve the burning of fossils fuels such as coal or gas - to technologies such as hydro, wind or solar power in the production of electricity.

Consumers enjoy a gross surplus from consuming a homogeneous good q at a price p:[3]

$$U(q) = \left(q - \frac{q^2}{2} \right) \tag{1}$$

This expression implies a linear inverse demand with a choke price normalized to one:

$$p = 1 - q \tag{2}$$

Output q can be produced either using a clean, x, or a dirty technology, y. These two sources of supply are perfect substitutes, so that $q = x + y$. This modeling of the demand is in line with Wirl (2013). One may think of thermal power plants using coal as the dirty technology y and of solar energy as the clean technology x, both of them providing electricity q.

Dirty technology y is used on a competitive industry: the marginal cost of production is constant and normalized to zero. Its use induces a flow of pollution, which results in an external cost $D(y)$:

$$D(y) = \frac{\gamma}{2} y^2, \qquad \gamma > 0 \tag{3}$$

This constitutes a negative flow externality. For instance, the burning of coal required to operate a thermal power plant results in the emission of sulfur dioxide, a toxic gas which reacts with water molecules in the atmosphere, and causes acid rains.

Clean technology x is owned by a monopoly. This modeling of two different market structures for the technologies - competition for the dirty technology and monopoly for the clean one - is in line with Montero (2011) and Wirl (2013). The rationale is that the development of a clean technology proceeds in two steps. In the first step, many firms compete to find a new technology. In the second step, the owner of the new technology is protected by a patent. In what follows, the analysis is restricted to the second step, and the owner is a monopoly. The cost of using the clean technology depends on a stock of knowledge K:

$$C_x(x, K) = \left(\frac{1}{2} x + c_{x0} - c_{x1} K \right) x, \qquad c_{x0} > 0, \quad c_{x1} > 0, \quad c_{x0} - c_{x1} K > 0 \tag{4}$$

This expression implies that the marginal cost associated with the clean technology equals $x + c_{x0} - c_{x1} K$. The term x accounts for diseconomies of scale in the production process. The supply of a large amount of electricity requires the operation of a large number of plants, including the less efficient ones, which increases the cost. Consistently with Tsur and Zemel (2003), the term $c_{x0} - c_{x1} K$ models the progressive improvement of the clean technology. The stock of knowledge increases with the investment I undertaken by the monopoly:

$$\dot{K} = I, \quad K(0) = 0 \tag{5}$$

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[3]To keep notations as simple as possible, note that I generally omit the time variable "t".

4

This approach, referred to as R&D-induced technological change (Popp et al., 2010), is one of the main approaches used to endogenize technological change. An alternative is to consider Learning-by-doing (LBD), where the cumulative experience of using a technology induces cost reductions. My choice to focus on R&D-induced technological change is supported by the fact that, according to empirical studies, R&D activities seem to contribute more than LBD to technology improvements. Using a panel data for wind power installations in four western European countries, Söderholm and Sundqvist (2007) find, a LBD learning rate of around 5%, which is to be compared to 15% for R&D.[4] The absence of depreciation and the normalization of the initial stock of knowledge to zero are assumptions made for simplicity, and do not change the results.

For any investment in the stock of knowledge, the monopoly faces the following adjustment cost:

$$C_I(I, K) = \left(\frac{1}{2} I + c_I K \right) I \qquad (6)$$

First, this expression is quadratic in the level of investment I, which is a standard assumption in the theory of investment of the firm (see for instance Gould, 1968). Second, it depends on the stock of knowledge K. This models the fact that improvements of a new technology become increasingly more difficult as the level of R&D advances increases. Indeed, as the stock of accumulated knowledge increases, the marginal adjustment cost of investment rises, which means that it is increasingly costly to increase the stock of knowledge K by a given amount I. This specification is in consistent with the "fishing out effect" introduced by Jones (1995) in growth theory: the easiest ideas are always discovered first. Therefore, as technology becomes more and more complex, it takes more time and more effort to make further improvements.[5] Parameter c_I in (6) can be interpreted as a measure of the magnitude of the fishing out effect.

To summarize, my framework applies to a situation where a clean technology already exists and is owned by a firm, but it still needs R&D-induced technological change to be improved. The improvement decreases the cost, which makes the clean technology less expensive relative to the dirty technology. This implies a smooth transition from the dirty technology to the clean one.

Again, this applies well to the energy sector since governments use various environmentally related taxes and subsidy R&D activities in favor of renewable energies. OECD's policy instruments for the environment database (PINE) covers more than 1400 different environmentally related taxes (OECD, 2017). IEA's annual Energy Technology RD&D Budget Database estimates a total budget dedicated to RD&D activities in favor of renewable energy sources to 2,972 million dollars for IEA members (IEA, 2019).

2.2 Timeline of the game, equilibrium concepts and players' objectives

To tackle the issue of government commitment level, I consider a game with hierarchical play between a government and the monopoly owning the clean technology. The government (first-

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[4]The learning rate denotes the pourcentage change in cost reductions resulting from a doubling of either the stock of knowledge (for R&D) or the cumulative capacities (for LBD).

[5]Thomas Edisson invented the light bulb on his own. Today, improving a technology as solar panel requires research teams.

mover) can announce its policy before the monopoly (second-mover) makes its decisions. The government policy consists possibly in a tax τ on the dirty technology level and a subsidy σ to investment in the stock of knowledge. I compare two commitment levels from the government, each of them corresponding to a specific equilibrium concept.

The first equilibrium concept is the Open-Loop Stackelberg Equilibrium (Dockner et al., 2000, p. 113). It refers to a situation where the government announces at $t = 0$ a time-path strategy for the tax and the subsidy, i.e. $\{(\tau(t), \sigma(t)), t \in [0, \infty)\}$. It requires that the government can commit to the future path of tax and subsidy rates. It models a strong commitment level from the government.

The second equilibrium concept is the Global Stackelberg equilibrium (Basar and Olsder, 1999, p. 373).[6] It refers to a situation where the government announces a policy rule. Specifically, it is standard to consider a (stationary) Linear Markov Rule (Dockner, et al., 2000 ; Benchekroun and Long, 1998 ; Karp and Livernois, 1992), i.e. $\{(\tau(K) = \tau_0 + \tau_1 K, \sigma(K) = \sigma_0 + \sigma_1 K), K \in [0, \infty)\}$. This equilibrium concept is in general considered as more realistic, in the sense that the government is not asked to follow a specific path but it rather only observes the evolution of the stock of knowledge and updates the tax rate and the subsidy rate. It models a weak commitment level from the government.

In line with Wirl (2013), the tax τ induces the monopoly to set the following price:

$$p = \tau \tag{7}$$

Perfect substitution between the clean technology and the dirty technology implies that the monopoly must charge the same price than the competitive industry. Since the cost related to the dirty technology is normalized to zero, the competitive industry profit is $py - \tau y$ which yields the expression of the price in (7).

At each period, the monopoly instantaneous profits equal the revenue - with a price equal to the tax - plus the subsidy to investment, net of the production cost and the investment adjustment cost:

$$\pi = \tau x - \left(\frac{1}{2}x + c_{x0} - c_{x1}K\right)x - \left(\frac{1}{2}I + c_I K\right)I + \sigma I \tag{8}$$

Taking into account this announcement, the monopoly chooses production level x and investment level I. The monopoly chooses clean technology level x and investment I to maximize the net present value of its instantaneous profits (8) subject to the accumulation of the stock of knowledge (5):

$$\max_{x,I} \int_0^\infty \pi e^{-rt} dt, \quad \text{subject to} \quad \dot{K} = I \tag{9}$$

The benevolent government accounts for the profit of the monopoly (π), the net surplus of the consumers $(U(q) - pq)$, the tax revenue (τy) and the external cost of pollution $(D(y))$. Using

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[6]See also Dockner et al. (2000, p. 142) where this equilibrium concept is refered to as a "nondegenrate Markovian Stackelberg equilibrium".

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(7), I can write that $U(q) = (1 - \tau) - \frac{(1-\tau)^2}{2}$ and $D(y) = \frac{\gamma}{2}y^2 = \frac{\gamma}{2}(1 - x - \tau)^2$. Therefore, the social welfare equals, at each time t:

$$S = (1 - \tau) - \frac{(1 - \tau)^2}{2} - \left(\frac{1}{2}x + c_{x0} - c_{x1}K\right)x - \left(\frac{1}{2}I + c_I K\right)I - \frac{\gamma}{2}(1 - x - \tau)^2 \quad (10)$$

The government chooses tax rate τ and subsidy rate σ to maximize the aggregate welfare, i.e. the net present value of the stream of social welfare (10) subject to the accumulation of the stock of knowledge (5):[7]

$$\max_{\tau,\sigma} \int_0^\infty S e^{-rt} dt, \quad \text{subject to} \quad \dot{K} = I \quad (11)$$

2.3 The social optimum

Before analyzing the game between the government and the monopoly, I characterize the social optimum as a benchmark. To determine this allocation, I consider a benevolent social planner who chooses tax rate τ, clean technology level x and investment I to maximize the aggregate welfare subject to the evolution of the stock of knowledge:

$$\max_{\tau,x,I} \int_0^\infty S e^{-rt} dt, \quad \text{subject to} \quad \dot{K} = I \quad (12)$$

The current-value Hamiltonian corresponding to (12) is:

$$\mathcal{H} = (1 - \tau) - \frac{(1 - \tau)^2}{2} - \left(\frac{1}{2}x + c_{x0} - c_{x1}K\right)x - \left(\frac{1}{2}I + c_I K\right)I - \frac{\gamma}{2}(1 - x - \tau)^2 + \lambda I \quad (13)$$

where λ denotes the shadow value of the stock of knowledge. For an interior solution, necessary conditions are:

$$-\tau + \gamma(1 - x - \tau) = 0 \quad (14)$$

$$-x - c_{x0} + c_{x1}K + \gamma(1 - x - \tau) = 0 \quad (15)$$

$$I + c_I K = \lambda \quad (16)$$

The adjoint equation is:

$$\dot{\lambda} = r\lambda - c_{x1}x + c_I I \quad (17)$$

Last, the transversality condition is $\lim_{t \to +\infty} e^{-rt}\lambda(t)K(t) = 0$.

Conditions (14) and (15) yield:

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[7]At this stage, the reader may object that the subsidy rate σ is absent from (10). However, keep in mind that my analysis will further focus on a game with hierarchical play. The government being the first-mover, it takes into account the response of the monopoly to its policy. In such a context, subsidy rate σ does step in the government objective function.

$$\tau^\star(K) = \frac{\gamma(1 + c_{x0})}{1 + 2\gamma} - \frac{\gamma c_{x1}}{1 + 2\gamma}K \tag{18}$$

$$x^\star(K) = \frac{\gamma - (1 + \gamma)c_{x0}}{1 + 2\gamma} + \frac{(1 + \gamma)c_{x1}}{1 + 2\gamma}K \tag{19}$$

On the other hand, by combining (16) and (17), I get the following equation:

$$\dot{I} = rI + rc_I K - c_{x1}x \tag{20}$$

Then, substituting (19) into (20) and rearranging the terms, I get:

$$\dot{I} = rI + \left(rc_I - \frac{(1 + \gamma)c_{x1}^2}{1 + 2\gamma}\right)K - c_{x1}\frac{\gamma - (1 + \gamma)c_{x0}}{1 + 2\gamma}, \quad \dot{K} = I \tag{21}$$

Under conditions $\gamma - (1 + \gamma)c_{x0} > 0$ and $rc_I - \frac{\gamma}{1+2\gamma}\frac{c_{x1}^2}{c_{x0}} > 0$, equation (21) has a positive saddle-point stable steady state.[8] The solution is:

$$I^\star(K) = \theta^\star(K - K_\infty^\star) \tag{22}$$

where

$$K_\infty^\star = \frac{c_{x1}(\gamma - (1 + \gamma)c_{x0})}{rc_I(1 + 2\gamma) - (1 + \gamma)c_{x1}^2} \tag{23}$$

denotes the steady state of the stock of knowledge and:

$$\theta^\star = \frac{r}{2} - \sqrt{\left(\frac{r}{2}\right)^2 + \left(rc_I - \frac{1 + \gamma}{1 + 2\gamma}c_{x1}^2\right)} \tag{24}$$

relates to the speed of convergence to the steady state.

The characterization of the social optimum goes as follows. In order to internalize the external cost of pollution, the benevolent social planner must use the tax rate (18). This tax yields a positive price for the output, which allows the development of a clean technology as given by (19). Thus, the social planner invests according to the feedback rule (22) to lower the marginal cost related to the clean technology. The development is implemented over time due to the adjustment costs of investment. The fishing out effect on R&D implies that investments are less and less profitable through time, so the stock of knowledge reaches a maximum level given by (23). Following the progressive development of the clean technology, the tax rate must decrease while the stock of knowledge accumulates.

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[8]The economic interpretation of these assumptions is the following. Condition $\gamma - (1 + \gamma)c_{x0} > 0$ means that the net benefit of the first unit of knowledge is positive (recall that $K(0) = 0$). Without this first assumption, the R&D is never profitable, and there is no way to reduce the cost differential between the two technologies without lowering the welfare. Condition $rc_I - \frac{\gamma}{1+2\gamma}\frac{c_{x1}^2}{c_{x0}} > 0$ ensures two properties. First, it means that, at each date, the net benefit of any infinitesimal quantity of knowledge is decreasing in K. It is consistent with the assumption of a fishing out effect: the stock of knowledge becomes less and less profitable as it accumulates. Second, it ensures that $c_{x0} - c_{x1}K_\infty^\star > 0$, which means that the cost can not become negative because of the accumulation of knowledge.

3 Analysis in a decentralized economy

3.1 Strong commitment: Open-loop Stackelberg equilibrium

In this section, I characterize the Open-Loop Stackelberg Equilibrium of the game, which corresponds to the case of strong commitment. In this equilibrium concept, the government announces a policy at $t = 0$, i.e. $\{(\tau(t), \sigma(t)), t \in [0, \infty)\}$, and commits to not to deviate from it. Given such a policy, the current-value Hamiltonian associated with the monopoly's maximization problem (9) is:

$$\mathcal{H} = \tau(t)x - \left(\frac{1}{2} + c_{x0} - c_{x1}K\right)x - \left(\frac{1}{2}I + c_I K\right)I + \sigma(t)I + \lambda I \tag{25}$$

where λ denotes the shadow value of the stock of knowledge. Necessary conditions are:

$$\tau(t) - x - c_{x0} + c_{x1}K = 0 \tag{26}$$

$$I + c_I K - \sigma(t) = \lambda \tag{27}$$

The adjoint equation is:

$$\dot{\lambda} = r\lambda - c_{x1}x + c_I I \tag{28}$$

Last, the transversality condition is $\lim\limits_{t \to +\infty} e^{-rt}\lambda(t)K(t) = 0$.

The goal is to find a policy $\{(\tau(t), \sigma(t)), t \in [0, \infty)\}$ such that equations (26) to (28) induce the efficient path characterized in the previous section. I notice that if the government sets the tax rate at each date as follows:

$$\tau(t) = \tau^\star(K(t)) = \frac{\gamma(1 + c_{x0})}{1 + 2\gamma} - \frac{\gamma c_{x1}}{1 + 2\gamma}K(t) \tag{29}$$

Then, the clean technology level is:

$$x(t) = \frac{\gamma - (1 + \gamma)c_{x0}}{1 + 2\gamma} + \frac{(1 + \gamma)c_{x1}}{1 + 2\gamma}K(t) \tag{30}$$

Expression (19) corresponds to socially optimal clean technology level. In other words, $x(t) = x^\star(K(t))$.

On the other hand, by combining (27) and (28), I get:

$$\dot{I} = rI + rc_I K - c_{x1}x + \dot{\sigma}(t) - r\sigma(t) \tag{31}$$

Therefore, it is straightforward that if $\sigma(t) = 0$, by substituting (30) into (31), I get the same equations than in (21) whose resolution yields a stock of knowledge following the efficient path. I summarize this result in the following Lemma:

Lemma 1. *Under a strong commitment level, the government can induce the efficient path in the decentralized economy by using the following tax and subsidy rates:*

$$\tau^{\star\star}(t) = \tau^{\star}(K(t)), \quad \sigma^{\star\star}(t) = 0 \tag{32}$$

Wirl (2013) finds a similar outcome (see Proposition 2, p. 7), keeping in mind that he only considers production capacities.

3.2 Weak commitment: Global Stackelberg equilibrium

Now, let us characterize the Global Stackelberg Equilibrium of the game, which corresponds to the case of weak commitment. In this equilibrium concept, the government announces a policy rule at $t = 0$. Such a policy is defined as a tax rate and a subsidy rate that both depend linearly on the stock of knowledge K, i.e. $\{(\tau(K) = \tau_0 + \tau_1 K, \sigma(K) = \sigma_0 + \sigma_1 K), K \in [0, \infty)\}$. Given such a policy, the current-value Hamitonian associated with monopoly's maximization problem (9) is:

$$\mathcal{H} = \tau(K)x - \left(\frac{1}{2} + c_{x0} - c_{x1}K\right)x - \left(\frac{1}{2}I + c_I K\right)I + \sigma(K)I + \lambda I \tag{33}$$

where λ denotes the shadow value of the stock of knowledge. Necessary conditions are:

$$\tau(K) - x - c_{x0} + c_{x1}K = 0 \tag{34}$$

$$I + c_I K - \sigma(K) = \lambda \tag{35}$$

The adjoint equation is:

$$\dot{\lambda} = r\lambda - c_{x1}x + c_I I - \tau'(K)x - \sigma'(K)I \tag{36}$$

Last, the transversality condition is $\lim_{t \to +\infty} e^{-rt}\lambda(t)K(t) = 0$.

The goal is to find a policy rule $\{(\tau(K) = \tau_0 + \tau_1 K, \sigma(K) = \sigma_0 + \sigma_1 K), K \in [0, \infty)\}$ such that equations (34) to (36) characterize the social optimum. Again, I notice that if:

$$\tau(K) = \tau^{\star}(K) = \frac{\gamma(1 + c_{x0})}{1 + 2\gamma} - \frac{\gamma c_{x1}}{1 + 2\gamma}K \tag{37}$$

Then, equation (34) yields:

$$x(K) = \frac{\gamma - (1+\gamma)c_{x0}}{1 + 2\gamma} + \frac{(1+\gamma)c_{x1}}{1 + 2\gamma}K \tag{38}$$

This corresponds to the expression (19) of the clean technology level under the social optimum. In other words, $x(K) = x^{\star}(K)$. On the other hand, by combining (35) and (36), I get:

$$\dot{I} = rI + rc_I K - c_{x1}x - r\sigma(K) - \tau'(K)x \tag{39}$$

I notice that if $r\sigma(K) = -\tau'(K)x$, equation (39) simplifies to:

$$\dot{I} = rI + rc_I K - c_{x1}x \tag{40}$$

10

It is straightforward that substituting (38) into (40) yields the same equations (21) than under the social optimum, which ensures that the stock of knowledge follows the efficient path. I summarize this result in the following Proposition:

Proposition 1. *Under a weak commitment level, the government can induce the efficient path by setting both the following tax rule*

$$\tau^{\star\star\star}(K) = \tau^{\star}(K) = \frac{\gamma(1 + c_{x0})}{1 + 2\gamma} - \frac{\gamma c_{x1}}{1 + 2\gamma}K \tag{41}$$

and the following subsidy rule:

$$\sigma^{\star\star\star}(K) = -\frac{\tau^{'\star}(K)x^{\star}(K)}{r} = \frac{\gamma c_{x1}(\gamma - (1 + \gamma)c_{x0})}{r(1 + 2\gamma)^2} + \frac{\gamma c_{x1}^2(1 + \gamma)}{r(1 + 2\gamma)^2}K \tag{42}$$

This result states that if the government can not commit to announcements about future tax rates (strong commitment level) but can rather commit to announcements about a policy rule (weak commitment level), then it can induce the efficient path.

By comparison with Lemma 1, Proposition 1 highlights that, under weak commitment, a subsidy to investment in knowledge is required in addition to the tax. The reason is that the government lack of commitment implies that the firm takes into account that any new unit of knowledge reduces its revenue by decreasing the tax rate, as shown by the term $-\tau'(K)x$ in equation (39). Therefore, a subsidy is needed to make up for this future marginal loss in revenue.

Wirl (2013) considers the lowest possible level of commitment and finds the government can not induce the efficient path. By contrast, it is possible to induce the efficient path in this setting.

Returning to the case of the energy sector, the general message of Proposition 1 is as follows. Without strong commitment, the government may commit to follow a policy rule and, in order to make up for its lack of commitment, it should subsidy R&D activities in favor of the development of non-polluting sources of energies.

4 The case of a duopoly

An issue emerging in a game with a stock of knowledge is the issue of spillover effects. In this section, I extend the previous model by considering two identical firms, $i = 1, 2$, which own two clean technologies. Each firm i owns its private stock of knowledge K_i and I denote by I_i firm i's individual investment level. I assume that there are spillover effects in the evolution of both stocks. That is, the stocks of knowledge evolve as:

$$\dot{K_i} = I_i + \beta I_{-i}, \quad 0 \leq \beta \leq 1, \quad i = 1, 2 \tag{43}$$

This is a standard approach to knowledge spillovers in a dynamic context, see e.g Cellini and Lambertini (2009). Coefficient β denotes the technological spillover that each firm derives from the R&D activity of the opponent. This applies to a situation where there exist different clean technologies, each of them protected by patents, and when the improvement of each technology

benefits to the others. For instance, there exist several types of solar photovoltaic technologies (silicon solar cells, thin film solar cells...).

The clean technology level of firm i, denoted by x_i, is associated with a cost depending on the stock of knowledge as follows:

$$C_x^i(x_i, K_i) = \frac{1}{2}x_i^2 + (c_{x0} - c_{x1}K_i)x_i, \quad c_{x0} > 0, \quad c_{x1} > 0, \quad c_{x0} - c_{x1}K_i > 0, \quad i = 1, 2 \quad (44)$$

By investing in the stock of knowledge, firms face adjustment costs of investment:

$$C_I^i(I_i, K_i) = \left(\frac{1}{2}I_i + c_I K_i\right)I_i, \quad c_I > 0, \quad i = 1, 2 \quad (45)$$

The instantaneous profit of firm i is equal to the revenue plus the subsidy to investment, net of the production cost and the investment adjustment cost:

$$\pi_i = px_i - \left(\frac{1}{2}x_i + c_{x0} - c_{x1}K_i\right)x_i - \left(\frac{1}{2}I_i + c_I K_i\right)I_i + \sigma I_i, \quad i = 1, 2 \quad (46)$$

where $p = \tau$. Thus, its objective is:

$$\max_{x_i, I_i} \int_0^\infty \pi_i e^{-rt}dt, \quad \text{subject to} \quad \dot{K}_1 = I_1 + \beta I_2, \quad \dot{K}_2 = I_2 + \beta I_1, \quad i = 1, 2 \quad (47)$$

The benevolent government accounts for the firms' profits, the consumers' surplus, the tax revenue and the external cost of pollution:

$$S^d = (1-\tau) - \frac{(1-\tau)^2}{2} - \sum_{i=1,2}\left(\frac{1}{2}x_i + c_{x0} - c_{x1}K_i\right)x_i - \sum_{i=1,2}\left(\frac{1}{2}I_i + c_I K_i\right)I_i - \frac{\gamma}{2}(1-x_1-x_2-\tau)^2$$

$$(48)$$

The government chooses tax rate τ and subsidy rate σ to maximize the aggregate welfare:

$$\max_{\tau, \sigma} \int_0^\infty S^d e^{-rt}dt, \quad \text{subject to} \quad \dot{K}_1 = I_1 + \beta I_2, \quad \dot{K}_2 = I_2 + \beta I_1 \quad (49)$$

As in the case of a monopoly, I consider a game with hierarchical play between the government (first-mover) and the duopoly (second-mover). In the remainder of this section, I characterize the social optimum, the OLSE and the GSE.

4.1 The social optimum

A benevolent social planner chooses tax rate τ, clean technology levels (x_1, x_2), investment levels (I_1, I_2) in order to maximize the aggregate welfare subject to the evolution of the stocks of knowledge:

$$\max_{\tau, x_1, x_2, I_1, I_2} \int_0^\infty S^d e^{-rt}dt, \quad \text{subject to} \quad \dot{K}_1 = I_1 + \beta I_2, \quad \dot{K}_2 = I_2 + \beta I_1 \quad (50)$$

The current-value Hamiltonian corresponding to (50) is:

$$\mathcal{H} = (1-\tau) - \frac{(1-\tau)^2}{2} - \sum_{i=1,2}\left(\frac{1}{2}x_i + c_{x0} - c_{x1}K_i\right)x_i - \sum_{i=1,2j}\left(\frac{1}{2}I_i + c_I K_i\right)I_i \tag{51}$$

$$-\frac{\gamma}{2}(1 - x_1 - x_2 - \tau)^2 + \lambda_1(I_1 + \beta I_2) + \lambda_2(I_2 + \beta I_1)$$

where λ_1 and λ_2 respectively denote the shadow values of the stocks of knowledge K_1 and K_2. For an interior solution, necessary conditions are:

$$-\tau + \gamma(1 - x_1 - x_2 - \tau) = 0 \tag{52}$$

$$-x_i - c_{x0} + c_{x1}K_i + \gamma(1 - x_1 - x_2 - \tau) = 0, \quad i = 1, 2 \tag{53}$$

$$I_i + c_I K_i = \lambda_i + \beta\lambda_{-i}, \quad i = 1, 2 \tag{54}$$

Adjoint equations are:

$$\dot{\lambda}_i = r\lambda_i - c_{x1}x_i + c_I I_i, \quad i = 1, 2 \tag{55}$$

Last, the transversality conditions are $\lim_{t \to +\infty} e^{-rt}\lambda_i(t)K_i(t) = 0$ $(i = 1, 2)$.

Equations (52) and (53) yield:

$$\tau^\star(K_1, K_2) = \frac{\gamma(1 + 2c_{x0} - c_{x1}(K_1 + K_2))}{1 + 3\gamma} \tag{56}$$

$$x_i^\star(K_1, K_2) = \frac{\gamma - (1 + \gamma)c_{x0} + c_{x1}((1 + 2\gamma)K_i - \gamma K_{-i})}{1 + 3\gamma}, \quad i = 1, 2 \tag{57}$$

On the other hand, by combining (54) to (55), I get:

$$\dot{I}_i = rI_i + rc_I K_i - c_{x1}x_i - \beta c_{x1}x_{-i}, \quad i = 1, 2 \tag{58}$$

Since firms are identical, I only consider symmetric equilibria where all firms behave identically. It implies that $K_1 = K_2 = K/2$, $I_1 = I_2 = I/2$ and $x_1 = x_2 = x/2$ where K, I and x denote the total stock and control variables. The expression of the tax rate (56) and the total clean technology level (57) become:

$$\tau^\star(K) = \frac{\gamma(1 + 2c_{x0})}{1 + 3\gamma} - \frac{\gamma c_{x1}}{1 + 3\gamma}K \tag{59}$$

$$x^\star(K) = \frac{2\gamma - 2(1 + \gamma)c_{x0}}{1 + 3\gamma} + \frac{(1 + \gamma)c_{x1}}{1 + 3\gamma}K \tag{60}$$

Futhermore, equation (58) can be written as:

$$\dot{I} = rI + rc_I K - c_{x1}(1 + \beta)x \tag{61}$$

Finally, substituting (60) into (61), I get:

$$\dot{I} = rI + \left(rc_I - (1+\beta)\frac{(1+\gamma)c_{x1}^2}{1+3\gamma}\right)K - c_{x1}(1+\beta)\frac{2\gamma - 2(1+\gamma)c_{x0}}{1+3\gamma} \tag{62}$$

Under conditions $\gamma - (1+\gamma)c_{x0} > 0$ and $rc_I - (1+\beta)\frac{\gamma}{1+3\gamma}\frac{c_{x1}^2}{c_{x0}} > 0$, equation (62) has a positive saddle-point stable steady state. The solution is:

$$I^\star(K) = \frac{1}{1+\beta}\theta^\star(K - K_\infty^\star) \tag{63}$$

where

$$K_\infty^\star = \frac{2(1+\beta)c_{x1}(\gamma - (1+\gamma)c_{x0})}{rc_I(1+3\gamma) - (1+\beta)(1+\gamma)c_{x1}^2} \tag{64}$$

denotes the steady state stock of knowledge and:

$$\theta^\star = \frac{r}{2} - \sqrt{\left(\frac{r}{2}\right)^2 + (1+\beta)\left(rc_I - \frac{(1+\beta)(1+\gamma)}{1+3\gamma}c_{x1}^2\right)} \tag{65}$$

relates to the speed of convergence to the steady state.

4.2 Strong commitment: Open-loop Stackelberg equilibrium

In this section, I characterize the Open-Loop Stackelberg Equilibrium of the game, which corresponds to a strong commitment level from the government. The government announces a policy at $t = 0$, i.e. $\{(\tau(t), \sigma(t)), t \in [0, \infty)\}$. Given such a policy, and given the strategy of the other firm (q_{-i}, I_{-i}) the current-value Hamiltonian associated with the firm maximization problem (47) is:

$$\mathcal{H}_i = \tau(t)x_i - \left(\frac{1}{2}x_i + c_{x0} - c_{x1}K_i\right)x_i - \left(\frac{1}{2}I_i + c_I K_i\right)I_i + \sigma(t)I_i + \lambda_i(I_i + \beta I_{-i}) + \mu_i(I_{-i} + \beta I_i), \quad i = 1, 2 \tag{66}$$

where λ_i denotes the shadow value of the stock of knowledge K_i and μ_i the shadow value of the stock of knowledge K_{-i}. The necessary conditions are:

$$\tau(t) - x_i - c_{x0} + c_{x1}K_i = 0, \quad i = 1, 2 \tag{67}$$

$$I_i + c_I K_i - \sigma(t) = \lambda_i + \beta\mu_i, \quad i = 1, 2 \tag{68}$$

The adjoint equations are:

$$\dot{\lambda}_i = r\lambda_i - c_{x1}x_i + c_I I_i, \quad i = 1, 2 \tag{69}$$

$$\dot{\mu}_i = r\mu_i, \quad i = 1, 2 \tag{70}$$

Last, the transversality conditions are $\lim\limits_{t \to +\infty} e^{-rt}\lambda_i K_i(t) = 0$ and $\lim\limits_{t \to +\infty} e^{-rt}\mu_i K_{-i}(t) = 0$.

The goal is to find a policy $\{(\tau(t), \sigma), t \in [0, \infty)\}$ such that equations (67) to (70) correspond to the social optimum. First of all, condition (70) has no use in this case since the transversality

14

condition implies that $\mu_i = 0$ for all t. Furthermore, I notice that if:

$$\tau(K_1, K_2) = \tau^\star(K_1, K_2) = \frac{\gamma(1 + 2c_{x0} - c_{x1}(K_1 + K_2))}{1 + 3\gamma} \tag{71}$$

Then the clean technology levels are:

$$x_i(K_1, K_2) = \frac{\gamma - (1 + \gamma)c_{x0} + c_{x1}((1 + 2\gamma)K_i - \gamma K_{-i})}{1 + 3\gamma}, \quad i = 1, 2 \tag{72}$$

This corresponds to the expression (57) of the clean technology levels under the social optimum: $x_i(K_1, K_2) = x_i^\star(K_1, K_2)$. Now, by combining (68) and (69), I get:

$$\dot{I}_i = rI_i + rc_I K_i - c_{x1} x_i - \beta c_I I_{-i} + \dot{\sigma}(t) - r\sigma(t), \quad i = 1, 2 \tag{73}$$

The firms being identical, equation (73) becomes:

$$\dot{I} = rI + rc_I K - \beta c_{x1} x - \beta c_I I + 2\dot{\sigma}(t) - 2r\sigma(t), \quad \dot{K} = (1 + \beta)I \tag{74}$$

I notice that if $\sigma(t)$ is solution to:

$$2\dot{\sigma}(t) - 2r\sigma(t) = \beta(c_I I(K(t)) - c_{x1} x(K(t))) \tag{75}$$

Then, equation (74) simplifies to the same equation than under the social optimum (61), which solution yields a stock of knowledge following the efficient path. Finally, I obtain the following Lemma:

Lemma 2. *Under a strong commitment level, the government can induce the efficient path in the decentralized economy by using both a tax rate and a subsidy rate satisfying:*

$$\tau^{\star\star}(t) = \tau^\star(K(t)), \quad \sigma^{\star\star}(t) = -\frac{\beta}{2(1 + \beta)} \left[\frac{\theta^\star}{1 + \beta} K_\infty^\star + \left(c_I + \frac{\theta^\star}{1 + \beta} \right) K(t) \right] \tag{76}$$

The proof of Lemma 2 is provided in Appendix A. By comparison with the case of a monopoly, the government policy includes a subsidy even under a strong commitment level. This latter is needed to internalize the free-rider problem that emerges from the existence of spillovers. One may check that in the absence of spillover effects, i.e. if $\beta = 0$, no subsidy is needed.

4.3 Weak commitment: Global Stackelberg Equilibrium

Now, let us characterize the Global Stackelberg Equilibrium of the game, which corresponds to the case of weak commitment. The government announces a policy rule at $t = 0$: such a policy is defined as a tax rate and a subsidy rate which both depend linearly on the two stocks of knowledge K_1 and K_2, that is $\tau(K_1, K_2) = \tau_0 + \tau_1^1 K_1 + \tau_1^2 K_2$ and $\sigma(K_1, K_2) = \sigma_0 + \sigma_1^1 K_1 + \sigma_1^2 K_2$. Given such a policy, and given the strategy of the other firm (x_{-i}, I_{-i}) the current-value Hamiltonian associated with the firm maximization problem (9) is:

$$\mathcal{H}_i = \tau(K_1, K_2)x_i - \left(\tfrac{1}{2}x_i + c_{x0} - c_{x1}K_i\right)x_i - \left(\tfrac{1}{2}I_i + c_I K_i\right)I_i + \sigma(K_1, K_2)I_i$$

$$+\lambda_i(I_i + \beta I_{-i}) + \mu_i(I_{-i} + \beta I_i), \quad i = 1, 2 \tag{77}$$

where λ_i denotes the shadow value of firm i's stock of knowledge and μ_i the shadow value of firm $-i$'s stock of knowledge. The necessary conditions are:

$$\tau(K_1, K_2) - x_i - c_{x0} + c_{x1}K_i = 0, \quad i = 1, 2 \tag{78}$$

$$I_i + c_I K_i - \sigma(K_1, K_2) = \lambda_i + \beta\mu_i, \quad i = 1, 2 \tag{79}$$

The adjoint equations are:

$$\dot{\lambda}_i = r\lambda_i - c_{x1}x_i + c_I I_i - \tau_1^i x_i - \sigma_1^i I_i, \quad i = 1, 2 \tag{80}$$

$$\dot{\mu}_i = r\mu_i - \tau_1^{-i}x_i - \sigma_1^{-i}I_i, \quad i = 1, 2 \tag{81}$$

The transversality conditions are $\lim_{t \to +\infty} e^{-rt}\lambda_i(t)K_i(t) = 0$ and $\lim_{t \to +\infty} e^{-rt}\mu_i(t)K_{-i}(t) = 0$.

The goal is to find a policy rule $\{(\tau(K_1, K_2) = \tau_0 + \tau_1^1 K_1 + \tau_1^2 K_2, \sigma(K_1, K_2) = \sigma_0 + \sigma_1^1 K_1 + \sigma_1^2 K_2), (K_1, K_2 \in [0, \infty))\}$ such that equations (34) to (36) correspond to the social optimum. Again, I notice that if:

$$\tau(K_1, K_2) = \tau^\star(K_1, K_2) = \frac{\gamma(1 + 2c_{x0} - c_{x1}(K_1 + K_2))}{1 + 3\gamma} \tag{82}$$

Then the clean technology levels are:

$$x_i(K_i, K_{-i}) = \frac{\gamma - (1 + \gamma)c_{x0} + c_{x1}((1 + 2\gamma)K_i - \gamma K_{-i})}{1 + 3\gamma}, \quad i = 1, 2 \tag{83}$$

This levels correspond to the expression (57) of the clean technology levels in the social optimum: $x_i(K_i, K_{-i}) = x_i^\star(K_i, K_{-i})$.

On the other hand, by combining (79), (80) and (81), I get:

$$\dot{I}_i = rI_i + rc_I K_i - (c_{x1} + \tau_1^i + \beta\tau_1^{-i})x_i + (\sigma_{-i} + \beta\sigma_i - \beta c_I)I_{-i} - r\sigma(K_1, K_2), \quad i = 1, 2 \tag{84}$$

The firms being identical, equation (84) becomes:

$$\dot{I} = rI + rc_I K - (c_{x1} + (1 + \beta)\tau_1)x + (-\beta c_I + (1 + \beta)\sigma_1)I - 2r(\sigma_0 + \sigma_1 K), \quad \dot{K} = (1 + \beta)I \tag{85}$$

where $\tau_1 = \tau_1^i = \tau_1^{-i}$ and $\sigma_1 = \sigma_1^i = \sigma_1^{-i}$.

I notice that if subsidy rate $\sigma(K) = \sigma_0 + \sigma_1 K$ satisfies:

$$(1 + \beta)\sigma_1 I(K) - 2r(\sigma_0 + \sigma_1 K) = \beta\left(c_I I(K) - c_{x1}x(K)\right) + (1 + \beta)\tau_1 x(K) \tag{86}$$

16

Then, equation (85) simplifies to the same equation under the social optimum (61), which solution yields a stock of knowledge following the efficient path. Finally, I obtain the following Proposition:

Proposition 2. *Under a weak commitment level, the government can induce the efficient path by setting both the following tax rule*

$$\tau^{\star\star\star}(K) = \tau^{\star}(K) = \frac{\gamma(1 + 2c_{x0} - c_{x1}K)}{1 + 3\gamma} \tag{87}$$

and the following subsidy rule:

$$\begin{cases} \sigma^{\star\star\star}(K) = \sigma^{\star\star}(K) - \frac{(1+\beta)}{2}\frac{\tau_1}{r}x^{\star}(K) + \phi(K) \\ \\ with \quad \phi(K) = -\frac{\theta^{\star}/2}{r - \theta^{\star}/2}\left(\frac{\beta}{2(1+\beta)}\left(c_I + \frac{\theta^{\star}}{1+\beta}\right) + \frac{(1+\beta)}{2}\frac{\tau_1}{r}\frac{(1+\gamma)c_{x1}}{1+3\gamma}\right)(K - K_{\infty}^{\star}) \end{cases} \tag{88}$$

The proof of Proposition 2 is provided in Appendix B.

When the government commits to a policy rule, the subsidy rate (88) is made of three components. The first component, i.e. $\sigma^{\star\star}(K)$, corrects the free-rider problem as under strong commitment, see (76). The second component, i.e. $-\frac{(1+\beta)}{2}\frac{\tau_1}{r}x^{\star}(K)$, corrects the fact that firms take into account that any new unit of knowledge reduces their future revenues because of the evolution of the tax rate as in the case of a monopoly, see (42).

The third effect, related to $\phi(K)$, is novel. It is used to correct an indirect effect on the subsidy. As for the tax rate, it results from the fact that both firms anticipate that any new unit of knowledge affects the future subsidy rate. This induces the firms to invest faster in the short run. The component $\phi(K)$ of the subsidy rate corrects this indirect effect in the short run. One may check that this component equals zero in the long run, i.e. $\phi(K_{\infty}^{\star}) = 0$.

A consequence of this novel effect is that, in some situations, one may have $\sigma^{\star\star\star}(K) < \sigma^{\star\star}(K)$ in the short run. Surprisingly, it can even be the case that $\sigma^{\star\star\star}(K)$ be initially negative to slow down the inefficiently high pace of investments. This is what I summarize in the following corollary:

Corollary 1. *Under a weak commitment level, the government may initially tax investments in knowledge made by the firms in order to slow down an otherwise too fast accumulation of knowledge.*

Figure 1 illustrates this last result. In the case depicted here, the pace of knowledge investment is too high. Then, the government subsidies less under weak commitment (GSE) than under strong commitment (OLSE) in the short run. Finally, in the very short run, it actually taxes investment in the GSE case.
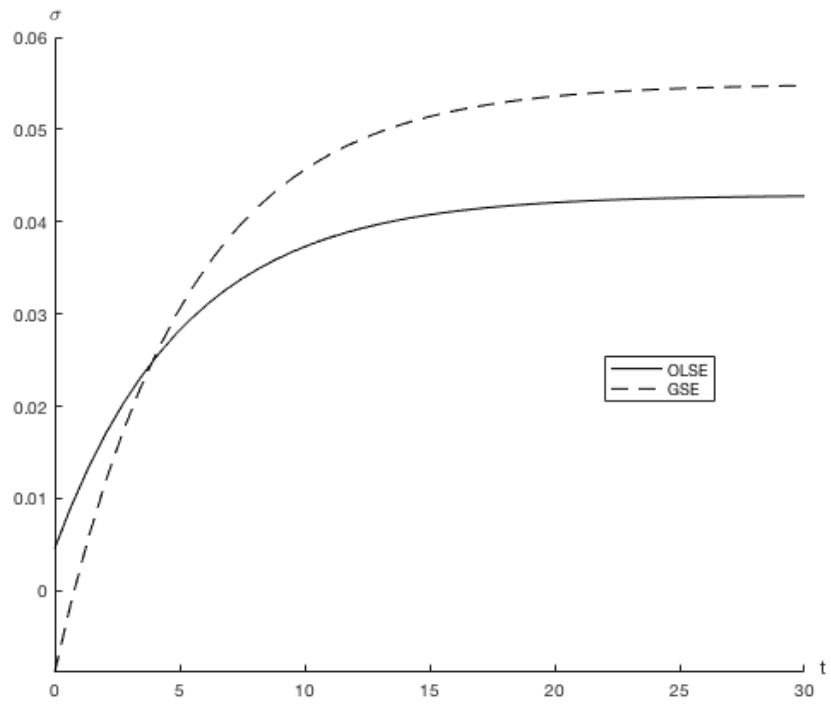
Figure 1: Comparison of the subsidy rates under strong commitment level (OLSE) and under weak commitment level (GSE) in the case of a duopoly with $c_{x0} = 0.1$, $c_{x1} = 0.05$, $\gamma = 0.2$, $c_I = 1$, $r = 0.05$ and $\beta = 0.8$.

5 Concluding remarks

I show that if a government can not commit to a policy in the long-run, one can find a policy rule that induces a monopoly to efficiently accumulate the stock of knowledge required for the development of a new clean technology. This policy is made of a tax and subsidy mix. The tax is used to internalize the external cost due to the pollution related to the dirty technology. The subsidy corrects the distorsion that results from the firm anticipation of how its decisions affect the magnitude of the tax rate.

Then, extending the model to the case of a duopoly with knowledge spillovers, I characterize the policy rule that induces efficiency: the subsidy rate may be initially lower under weak commitment than under strong commitment. The reason is that each firm, taking into account that the other firm investment affects the subsidy rate, may be induced to invest at a too high pace initially. A surprising result is that, under weak commitment, the subsidy rate may even be negative to disinduce firms to invest too much in the short run. A general take away of this paper is that abstracting from the issue of the government commitment level yields suboptimal policies. In particular, this may lead one to either underestimate or overestimate subsidy levels allocated to R&D activities.

Of course, this contribution calls for further work. In particular, I do not analyse the case of a closed-loop information structure in the duopoly setting. Such an analysis is very likely to rely on numerical simulations only, due to the presence of two stock variables. I also do not consider the case of a stock of pollution: again, such an extension would require extensive simulation work due to the presence of two stock variables. Such extensions are left for future research work.

A Proof of Lemma 2

I have shown that $\sigma(t)$ must be solution to (75) on the efficient path. Then, substituting (63) and (60) into (75), I get:

$$\dot{\sigma}(t) - r\sigma(t) = \frac{\beta}{2}\left(c_I\frac{\theta^\star}{1+\beta}(K(t) - K_\infty^\star) - c_{x1}\frac{2\gamma - (1+\gamma)(2c_{x0} - c_{x1}K(t))}{1+3\gamma}\right) \tag{89}$$

Now, along the efficient path the stock of knowledge evolves as $K(t) = K_\infty^\star(1 - e^{\theta^\star t})$ so that equation (90) can be written as:

$$\dot{\sigma}(t) - r\sigma(t) = \frac{\beta}{2}\left(-c_I\frac{\theta^\star}{1+\beta}K_\infty^\star e^{\theta^\star t} - c_{x1}\frac{2\gamma - (1+\gamma)(2c_{x0} - c_{x1}K_\infty^\star(1 - e^{\theta^\star t}))}{1+3\gamma}\right) \tag{90}$$

Rearranging the terms:

$$\dot{\sigma}(t) - r\sigma(t) = \frac{\beta}{2}\left(-c_I\frac{\theta^\star}{1+\beta} + \frac{(1+\gamma)c_{x1}^2}{1+3\gamma}\right)K_\infty^\star e^{\theta^\star t} - \frac{\beta c_{x1}}{2}\frac{2\gamma - (1+\gamma)(2c_{x0} - c_{x1}K_\infty^\star)}{1+3\gamma} \tag{91}$$

Standard methods lead to the following solution:

$$\sigma(t) = \frac{\beta}{2}\left(-c_I\frac{\theta^\star}{1+\beta} + \frac{(1+\gamma)c_{x1}^2}{1+3\gamma}\right)\frac{K_\infty^\star}{\theta^\star - r}e^{\theta^\star t} + \frac{\beta c_{x1}}{2r}\frac{2\gamma - (1+\gamma)(2c_{x0} - c_{x1}K_\infty^\star)}{1+3\gamma} \tag{92}$$

The expression of $\sigma(t)$ given in (92) can be simplified. Indeed, using (65), I note that:

$$\frac{(1+\gamma)c_{x1}^2}{1+3\gamma} = \frac{\theta^\star(r - \theta^\star)}{(1+\beta)^2} + \frac{rc_I}{1+\beta} \tag{93}$$

Moreover, using (64), I notice that:

$$\frac{2c_{x1}(\gamma - (1+\gamma)c_{x0})}{1+3\gamma} = \frac{K_\infty^\star}{1+\beta}\left(rc_I - \frac{(1+\beta)(1+\gamma)c_{x1}^2}{1+3\gamma}\right) = -K_\infty^\star\frac{\theta^\star(r - \theta^\star)}{(1+\beta)^2} \tag{94}$$

Finally, substituting (93) and (94) into (92), I get:

$$\sigma(t) = -\frac{\beta K_\infty^\star}{2(1+\beta)}\left(c_I + \frac{\theta^\star}{1+\beta}\right)e^{\theta^\star t} + \frac{\beta K_\infty^\star c_I}{2(1+\beta)} \tag{95}$$

Or, in feedback form:

$$\sigma(t) = -\frac{\beta}{2(1+\beta)}\left[\frac{\theta^\star}{1+\beta}K_\infty^\star + \left(c_I + \frac{\theta^\star}{1+\beta}\right)K(t)\right] \tag{96}$$

B Proof of Proposition 2

Plugging the expressions of $I^\star(K)$ and $x^\star(K)$ given in (63) and (60) into (86), I get a polynomial expression of degree one in K. Then, I collect terms involving K and terms not involving K and

get the following system of equations:

$$
\begin{cases}
\left(\theta^\star/2 - r\right)\sigma_1 = \frac{\beta}{2}\left(\frac{c_I \theta^\star}{1+\beta} - \frac{(1+\gamma)c_{x1}^2}{1+3\gamma}\right) + \frac{(1+\beta)\tau_1}{2}\frac{(1+\gamma)c_{x1}}{1+3\gamma} \\[4mm]
-\frac{\theta^\star K_\infty^\star \sigma_1}{2} - r\sigma_0 = -\frac{\beta}{2}\left(\frac{c_I \theta^\star}{1+\beta}K_\infty^\star + c_{x1}\frac{2\gamma - 2(1+\gamma)c_{x0}}{1+3\gamma}\right) + \frac{(1+\beta)\tau_1}{2}\frac{2\gamma - 2(1+\gamma)c_{x0}}{1+3\gamma}
\end{cases}
\tag{97}
$$

Straightforward algebra yields:

$$
\begin{cases}
\left(\theta^\star/2 - r\right)\sigma_1 = \frac{\beta}{2}\left(\frac{c_I \theta^\star}{1+\beta} - \frac{(1+\gamma)c_{x1}^2}{1+3\gamma}\right) + \frac{(1+\beta)\tau_1}{2}\frac{(1+\gamma)c_{x1}}{1+3\gamma} \\[4mm]
-r\sigma_1 K_\infty^\star - r\sigma_0 = -\frac{\beta}{2}\left(\frac{(1+\gamma)c_{x1}^2}{1+3\gamma}K_\infty^\star + c_{x1}\frac{2\gamma - 2(1+\gamma)c_{x0}}{1+3\gamma}\right) + \frac{(1+\beta)\tau_1}{2}\left(\frac{2\gamma - 2(1+\gamma)c_{x0}}{1+3\gamma} + \frac{(1+\gamma)c_{x1}}{1+3\gamma}K_\infty^\star\right)
\end{cases}
\tag{98}
$$

Simplifying, I obtain:

$$
\begin{cases}
\sigma_1 = \frac{r-\theta^\star}{r-\theta^\star/2}\frac{\beta}{2(1+\beta)}\left(c_I + \frac{\theta^\star}{1+\beta}\right) - \frac{1}{r-\theta^\star/2}\frac{(1+\beta)\tau_1}{2}\frac{(1+\gamma)c_{x1}}{1+3\gamma} \\[4mm]
-r\sigma_1 K_\infty^\star - r\sigma_0 = -\frac{\beta}{2(1+\beta)}rc_I K_\infty^\star + \frac{(1+\beta)\tau_1}{2}\left(\frac{2\gamma - 2(1+\gamma)c_{x0}}{1+3\gamma} + \frac{(1+\gamma)c_{x1}}{1+3\gamma}K_\infty^\star\right)
\end{cases}
\tag{99}
$$

Rearranging the terms:

$$
\begin{cases}
\sigma_1 = \frac{\beta}{2(1+\beta)}\left(c_I + \frac{\theta^\star}{1+\beta}\right) - \frac{(1+\beta)}{2}\frac{\tau_1}{r}\frac{(1+\gamma)c_{x1}}{1+3\gamma} - \frac{\theta^\star/2}{r-\theta^\star/2}\left(\frac{\beta}{2(1+\beta)}\left(c_I + \frac{\theta^\star}{1+\beta}\right) + \frac{(1+\beta)}{2}\frac{\tau_1}{r}\frac{(1+\gamma)c_{x1}}{1+3\gamma}\right) \\[4mm]
\sigma_0 = -\sigma_1 K_\infty^\star + \frac{\beta}{2(1+\beta)}c_I K_\infty^\star - \frac{(1+\beta)}{2}\frac{\tau_1}{r}\left(\frac{2\gamma - 2(1+\gamma)c_{x0}}{1+3\gamma} + \frac{(1+\gamma)c_{x1}}{1+3\gamma}K_\infty^\star\right)
\end{cases}
\tag{100}
$$

Finally, rearranging the terms, I obtain expression (88). $\qquad\square$

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