# Agricultural Production Decision using Jumps and Seasonal Volatility in commodities prices dynamics

Zoulkiflou Moumouni, Jules Sadefo-Kamdem

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Zoulkiflou Moumouni
MRE EA 7491
Université Montpellier

Jules Sadefo Kamdem *
MRE EA 7491
Université de Montpellier

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**Abstract**

We use agricultural commodities futures prices to investigate decision making in production when futures prices are governed by jump-diffusion process with seasonal volatility. We derive a preference independent production rule for firms that face both demand and production uncertainty. We compare this rule to the one when only Brownian motion represents the source of risk with constant volatility. Our analysis suggests that for most crops, the jump-diffusion model is sufficiently accurate to guide production decision.

**Keywords:** Production decision, agricultural commodities; futures markets; jump-diffusion; incomplete markets; options pricing, seasonal volatility.

**JEL Codes:** C12; C58; D52; G13; Q11; Q14

## Introduction

The production of agricultural commodities depends on factors such as weather, demand fluctuations, and other state variables which are uncertain but relevant for producer. Futures prices from organized market can provide useful information for production, storage, and processing decisions (Black [9]). The value of a farmer's crops, once planted is affected by the same state variables as the futures prices of the same commodities. These state variables include mainly weather or demand fluctuation. Marcus and Modest [25] have found that the analysis of optimal decision in commodity production is simplified by option pricing framework in continuous time. They focus on an agricultural producer who faces production uncertainty and for whom active futures markets already. Particularly, the value of crop is a claim to producer's revenue like an option written on equilibrium price as underlying. The equilibrium price as is function of futures price and support system for a producer, since option pricing technique is flexible enough to incorporate. But, although futures contracts on agricultural commodities are useful to hedge against uncertainties, they may be more

*Corresponding author: Université de Montpellier - UFR d'Ãconomie - Site de Richter, Avenue Raymond Dugrand CS 79606, 34960 Montpellier Cedex 2 France, email: jules.sadefo-kamdem@umontpellier.fr, jsadefo@gmail.com

risky than expected.

Futures prices are sensitive to unexpected changes due to drastic weather changes or extreme events, which can cause harvest destruction and other risk factors such as negative shift of supply in commodity markets, excessive speculation of institutional investors or decline in the growth rate of food production etc. For instance, when stock markets drown, institutional investors use commodities as safe-haven assets to secure their funds; in that case futures prices may even exhibit spikes. Marcus and Modest [25] use Brownian motion in price dynamic to represent risk factor. But several authors show that market prices are not normally distributed due to stylized facts such as skewness, fat tails, breaks in time series, (see for example Hilliard and Reis [20], Koekebakker and Lien [22], Mandelbrot [26]).

Then, the analysis of Marcus and Modest [25] is limited in two points about price modeling: first, futures and index prices are governed solely by a geometric Brownian motion that does not account for the risk of price jumps. Second, authors follow Black's [9] approach and consider a constant volatility parameter. This approach is not realistic since seasonal, time-varying or/and stochastic volatility in commodity prices has been proven to be essential when dealing with option pricing (Back et al. [5]).

In this paper, we build on Marcus and Modest [25] model, extending it in two respects (stages): we first apply jumps detecting tests recently introduced in the literature on observed prices of selected futures and commodity index and empirically analyze the relevance of seasonal volatility in futures prices of agricultural commodities. We use statistical tests, introduced by AÃ¯t-Sahalia and Jacod [2, 3] to effectively detect the presence of jumps in discretely observed prices. We argue that taking into account seasonal volatility for futures prices is pertinent in futures price dynamics for option pricing purposes.

Secondly, we extend the model of Marcus and Modest [25] by taking into account stylized fact including jump components in price dynamics. This initiative arises from the first analysis of jump detection. We consider jump components in price dynamics, as well as seasonality volatility for futures price. In doing so, we expect to better handle the systematic risk due to market prices. But, when market prices are modeled as LÃ©vy processes, like jump-diffusion processes, the technique of option pricing becomes slightly different. Indeed, the market with jumps is incomplete in the the Harrison and Pliska [18] sense. Hence, the classical portfolio replication is no more possible, since jump risk can not be easily hedged. We obtain a partial integro-differential equation (PIDE) for option premium that we solve with numerical methods.

The remainder of the paper is organised as follows. Section 1 presents market settings and investment conditions. In Section 2 we argue that geometric Brownian motion may not sufficiently represent risk factor. We highlight statistical properties of futures contract of one year maturity and commodity index prices. This consists in testing the presence of jumps and showing how seasonality is relevant in volatility. Section 3 presents the model and the crop valuation formula. We compute numerical results with the model with jump and compare them with the one without jump in Section 4. Section 5 concludes.

## 1 Market setting

We consider the same rule of production decision for an agricultural producer as in Marcus and Modest [25]. The production decision of an agricultural producer is subject to uncertainty. The market of agricultural commodities is modeled as a stochastic supply and demand system. Firms or farmers are assumed to be perfectly competitive price takers in a well-developed futures market where riskless borrowing and lending are possible at a constant interest rate $r$. The production setting is assumed to be point input-point output where the crop is planted at time zero and harvested and sold at time $T$. No interim production decisions are allowed. However, capital markets are assumed to be open at all times so that portfolios can be continuously rebalanced. Crops are normal goods in the sense that demand quantities decline when the market price $P_t$ raises at time $t$. The demand depends on wealth or income via a stochastic shift parameter $S_t$, correlated with the activity of the general economy. We assume that the shift parameter $S_t$ is measured by a commodity index. It is likely that demand shifts are correlated with the activity of the general economy and then they are treated as if they were correlated with a commodity index. By assuming a functional form of constant elasticity, market demand for the good, denoted by $Q_T$, at the time of the harvest $T$, is then given by

$$Q_T = S_T^{\gamma} P_T^{-\varepsilon}, \qquad \gamma > 0,\ \varepsilon > 0 \tag{1}$$

where $P_t$ and $S_t$ are random variables and $\gamma$ and $\varepsilon$ are respectively wealth and price elasticities of demand. The demand function is isoelastic, that means that the price elasticity of demand does not depend on the market price or the shift parameter. In fact, the shift parameter represents all state variables that intervene in the demand variation of the good at a given price. Shifts may come from any shock[1] in the economy that leads to higher consumption due to wealth shock, changes in saving behavior etc.

The supply of the commodity, available at time $T$, is modeled as the initial stock of the good plus the output from the harvest. The output from the harvest, hence the value of the crop, depends on expectations about the shift parameter $S_T$ and the market price $P_T$ at time $T$. Initial stock is set to zero and farmer has little control over the final output, so any yield is primarily determined by exogenous factors (such as wealth).

Furthermore, a price support system is available from the government, which compensates the farmer for the gap between the market and support prices. We consider that the support system does not affect spot or futures prices and that it allows the market to clear itself. In the absence of price support, there is no output after the next harvest or no inventory demand impounded into the market demand curve. The market would clear at the harvest time $T$ at price,

$$P_T = S_T^{\gamma/\varepsilon} Q_T^{-1/\varepsilon}, \tag{2}$$

and the value of the crop in the market at harvest time $T$ is

$$V_T = P_T Q_T = S_T^{\gamma} P_T^{1-\varepsilon}. \tag{3}$$

[1] The analysis can be extended to other demand shocks, for example a shock on price elasticities.

If the support price is available at level $\bar{P}$, then the price of the good is given by

$$P_T = \max\left[S_T^{\gamma/\varepsilon} Q_T^{-1/\varepsilon}\ ,\ \bar{P}\right] \tag{4}$$

and the value of the crop at time $T$ becomes

$$V_T = \max\left[S_T^{\gamma} P_T^{1-\varepsilon}\ ,\ \bar{P} S_T^{\gamma} P_T^{-\varepsilon}\right]. \tag{5}$$

Further, we assume a producer $k$ who holds a market share part $\theta_{k,T}$ of the total output of crop such that $0 \leq \theta_{k,T} \ll 1$, with $\sum_k \theta_{k,T} = 1$. $\theta_{k,t}$ is a random variable at time $t \leq T$ and represents the expectation of market share that a producer $k$ will realize at time $T$. We consider that the share of producer $k$ is not correlated with spot or futures prices or with any systematic factors affecting aggregate demand, such as $S_t$. Hence, there is no correlation between the market share of producers and the aggregate output. Notice that this assumption is reasonable only under perfect competition. Because, only a large producer would perceive a correlation as a simultaneous raise of expected values of both $\theta_k$ and aggregate supply in case of a positive shock of production. Further, no individual producer has effect on the *ex ante* distribution of the returns available for investors.

The perfect competition assumption implies that the decision of an individual producer does not affect the aggregate output or the aggregate value of the crop. Formally, if $V_t$ is the current value of the claim to the stochastic revenue from the aggregate crop (measured in currency), $\theta_{k,T}(I_k)$ the fraction of the crop at time $T$ produced by farmer $k$ and $I_k$ the value of the factor inputs of farmer $k$ (measured in currency), then under perfect competition, each producer perceives that

$$\frac{\partial \theta_k}{\partial I_k} > 0, \qquad \frac{\partial^2 \theta_k}{\partial I_k^2} < 0, \qquad \frac{\partial V}{\partial \theta_k} = 0. \tag{6}$$

The producer decision rule is to increase the size of the crop as long as the increment of the *ex ante* value exceeds the marginal cost of planting. So, the value of any individual farmer's crop at time $T$ is given by

$$V_{k,T} = \theta_{k,T}(I_k) V_T, \tag{7}$$

and each producer solves the following problem at time 0:

$$\max_{I_k} \left\{V_{k,0} - I_k\right\}. \tag{8}$$

In perfectly competitive market, the producer perceives his market share in order to increase with his input level conditional on the input levels of other producers. Hence, the value maximization implies that he should increase the size of the crop as long as the increment of value $\theta_k'(I_k)V$ exceeds the marginal cost of planting.

The crop valuation is based on market price dynamics. In order to accurately model these dynamics, we analyze data of selected commodity futures of one year maturiry.

## 2 Statistical analysis of commodity prices

Agricultural commodity prices can exhibit jumps due to weather vagaries, decline in food production growth, excessive speculation of institutional investors etc. Price dynamics change more frequently than suggested by geometric Brownian motion and leads to jumps. This implies a different structure of pricing model that should incorporate sudden changes. A number of empirical and theoretical studies show the existence of jumps in price dynamics and their substantial impacts on financial management, from portfolio and risk management to option pricing and hedging (Merton [27]; Bakshi et al. [6]; Bates [8]; and Johannes [21]). Hilliard and Reis [20] use transaction data of commodity futures and futures options to illustrate the considerable differences in modelled by jump-diffusion and geometric Brownian motion option prices. Jumps are extreme events rarely observed in markets, since their occcurance depends on market information (general announcements or speculative bubble). They are associated with company-specific events such as announcements of scheduled earnings or unscheduled news. Johannes [21] discusses impacts of jumps on derivatives securities. Testing for the presence of jumps on futures and stock index prices would have greater implication for the production rule (Lee and Mykland [24]). To effectively detect jumps in price dynamics of our selected commodities prices, we apply statistical tests of AÃ¯t-Sahalia and Jacod [2, 3], since they are based on a more general Itō-semimartingale structure for asset price and volatility. We prefer a more general jump detection tests for possible extention of our model (stochastic volatilities). We may use nonparametric jump test of Lee and Mykland [24] based on realized returns and volatility. But, these authors assume the drift and the diffusion coefficients not to change dramatically over a short time interval and jump processes to be of finite-activity while AÃ¯t-Sahalia and Jacod [2] consider a more general dynamic. Indeed, the intuition behind AÃ¯t-Sahalia and Jacod [2] test statistic is directly link to the continuity (in probaility) of the sample path of the observed process. Next section describes the intuition of AÃ¯t-Sahalia and Jacod [2] jump test.

### 2.1 Jumps detection procedure

In pratice $n$ values of asset price are observed during a time horizon $[0, t]$. Let $\Delta_n$ be the time step between two consecutive observations. Then, $\Delta_n = t/n$ and at period $i\Delta_n$ the log-price value $X_{i\Delta_n}$ is obtained for $i = 0, 1 \ldots$. When $n \to \infty$, $\Delta_n \to 0$. But, in reality $n < \infty$ and $n \to \infty$ is assumed for convergence purposes. To find out whether a process $X = (X_t)_{t\geq 0}$ has discontinuities or not within the time period $[0, t]$, AÃ¯t-Sahalia and Jacod [2] use a statistic of variability measure based on the absolute increment of $X$ which converges under conditions. When convergence holds, it becomes possible to determine if jumps are important enough to be taken into account.

Consider only the observations $X_{i\Delta_n}$ such that $i\Delta_n$ is smaller than or equal to $t$. The goal is to detect sudden important changes in the data that have relative impacts on the observation process. The increment $\Delta_i^n X$ of the process $X$, for two consecutive observed values during $((i-1)\Delta_n, i\Delta_n]$, is used to define a variability statistic. $\Delta_i^n X$ is as follows

$$\Delta_i^n X := X_{i\Delta_n} - X_{(i-1)\Delta_n}, \qquad \text{for} \qquad i = 1, 2, \ldots$$

Let define

$$\widehat{B}(p,\Delta_n)_t = \sum_{i=1}^{\lfloor t/\Delta_n \rfloor} |\Delta_i^n X|^p \,, \tag{9}$$

where $\lfloor x \rfloor$ denotes the integer part of $x \in \mathbb{R}$. AÃ¯t-Sahalia and Jacod [2] prove that $\widehat{B}(p,\Delta_n)_t$ converges in two ways. When $\Delta_n \to 0$,

$$\begin{cases} X \text{ is discontinuous} & \Rightarrow \quad \widehat{B}(p,\Delta_n)_t & \xrightarrow{\mathbb{P}} \quad B(p)_t \\ X \text{ is continuous} & \Rightarrow \quad \frac{\Delta_n^{1-p/2}}{m_p}\widehat{B}(p,\Delta_n)_t & \xrightarrow{\mathbb{P}} \quad A(p)_t \end{cases} \tag{10}$$

where $A(p)_t$ and $B(p)_t$ are defined in Appendix A, and

$$m_p = \mathbb{E}\left[|U|^p\right] = \sqrt{\frac{2^p}{2\pi}}\Gamma\left(\frac{p+1}{2}\right)$$

with $U$ being a standard normal random variable and $\Gamma$ the gamma function. In practice, $m_p$ is estimated by Monte Carlo simulations or hypergeometric function.

Under assumption of $p > 2$, if $X$ has jumps on $[0,t]$, the limit of $\widehat{B}(p,\Delta_n)_t$ does not depend on the sequence $(\Delta_n)_n$ and is strictly positive. On the other hand, when $X$ is continuous on $[0,t]$, then $\widehat{B}(p,\Delta_n)_t$ converges again towards a limit independent of $(\Delta_n)_n$, but only after a normalization that does depend on the sequence $(\Delta_n)_n$. Since the increments containing jumps are much larger than those that do not, their contribution to the summation dominates all other terms. So, if there is a jump in the time interval $((i-1)\Delta_n, i\Delta_n]$, then the magnitude of the increment $\Delta_i^n X$ is large and independent of the sampling interval $\Delta_n$, whereas the magnitude of $\Delta_i^n X$ is small and depends on $\Delta_n$ when there is no jump in that interval.

By introducing a $\Delta_n$-scaling integer $k \geq 2$, a higher power further separates the magnitudes of $|\Delta_i^n X|^p$ in the two expressions of $\widehat{B}(p,k\Delta_n)_t$ and $\widehat{B}(p,\Delta_n)_t$. The nonparametric test statistic defined in AÃ¯t-Sahalia and Jacod [2] quantifies that idea. For $p > 3$,

$$\widehat{S}(p,k,\Delta_n)_t = \frac{\widehat{B}(p,k\Delta_n)_t}{\widehat{B}(p,\Delta_n)_t}. \tag{11}$$

Concretely, testing for the presence of jumps on observed data boils down comparing $\widehat{B}(p,\Delta_n)_t$ on two differents $\Delta_n$-scales. For an integer $k \geq 2$, $\widehat{B}(p,k\Delta_n)_t$ and $\widehat{B}(p,\Delta_n)_t$ are compared by dividing the former by the latter to get the test statistic $\widehat{S}(p,k,\Delta_n)_t$. In the presence of jumps, both $\widehat{B}(p,k\Delta_n)_t$ and $\widehat{B}(p,\Delta_n)_t$ do not depend on $\Delta_n$ and $\widehat{S}(p,k,\Delta_n)_t$ converges to 1. Else, $\widehat{S}(p,k,\Delta_n)_t$ also converges, but depends on the $\Delta_n$-scaling parameter $k$.

For instance, a sum of squared variations of a standard Brownian motion indexed by $t \geq 0$, converges in probability to $t \geq 0$, the commonly called *quadradic variation* of Brownian motion. Then, for $p > 3$, its sum of $p$-power absolute variations also depends on $t$ by means of HÃ¶lder's inequality which also is a function of $\Delta_n$. Thus $\widehat{S}(p,k,\Delta_n)_t$ depends on the scale integer $k$ for Brownian motion. The test statistic then determines a continuity structure for a standard Brownian motion. On the other hand, a variation of Poisson process does not depend on the time interval, only the probability of jump occurrence depends on the interval. Hence, a sum of $p$-power absolute variations of compound Poisson process converges towards the same value for any $k$. That is, the size of the Poisson outcome does not depend on $\Delta_n$ and the probability of a jump occurring in $((i-1)\Delta_n, i\Delta_n]$ goes to zero as $\Delta_n \to 0$. In this case $\widehat{S}(p,k,\Delta_n)_t$ tends to 1 when $\Delta_n \to 0$. In contrast to Brownian motion where some movement always takes place (say continuity and the probability of movement is constant as $\Delta_n \to 0$), the size of the movement tends to zero as $\Delta_n \to 0$.

As a result, $\widehat{S}(p,k,\Delta_n)_t$ behaves substantially different when the sample path of $X$ on the time interval $[0,t]$ encompasses jumps from the case where jumps are absent. From AÃ¯t-Sahalia and Jacod [2], we have the following convergence results. For $p > 3$ and $k \geq 2$, when $\Delta_n \to 0$,

$$\begin{cases} \widehat{S}(p,k,\Delta_n)_t & \longrightarrow & 1 & \text{if there are jumps,} \\ \widehat{S}(p,k,\Delta_n)_t & \longrightarrow & k^{p/2-1} & \text{if there are no jumps.} \end{cases} \tag{12}$$

For example, when $p = 6$ and $k = 2$, the test statistic $\widehat{S}(4,2,\Delta_n)_t$ will converge to 1 in the presence of jumps, while with the same values of $p$ and $k$, it will converge to 4 in the absence of jump.

From the two convergences in (12), central limit theorem is applied to $\widehat{S}(p,k,\Delta_n)_t$ and two different convergence towards normal distribution with appropriately variances given in Appendix A. Indeed, either the null hypothesis of no jump or the null hypothesis of the presence of jumps can be tested. We describe the different rejection regions with their boundary under each null hypothesis in Appendix A.

### 2.2 Seasonality analysis

A commonly accepted definition of seasonality seems not to exist. However, a decision to use seasonally unadjusted data can be justified by a prior suspicion that one's model is at least reliable for thinking about seasonal fluctuations, (Thomas J. Sargent in the foreword to The Econometric Analysis of Seasonal Data, Ghysels and Osborn, 2001).

Seasonality is roughly defined as the intra-year variation that is repeated constantly or in an evolving fashion from year to year (moving seasonality). If the increase in the seasonal factors from year to year is too large, then the seasonal factors will introduce distortion into the model. Mean and/or variance may vary over time, hence time-constant seasonal mean and/or variance may be inappropriate. Franses [16] shows, on UK stock price index and US Composite Leading Indicators index, seasonal fluctuations when the means and/or variances vary over a period.

Back et al. [5] study how volatility seasonality affects option prices in commodity market and point out a definition of seasonality given by Svend Hylleberg in his book[2]

> *"... the systematic, although not necessarily regular, intra-year movement caused by the changes of the weather, the calendar, and timing of decisions, directly or indirectly through the production and consumption decisions made by agents of the economy. These decisions are influenced by*

[2] This definition can be found in Svend Hylleberg, *Modeling Seasonality*, Oxford University Press, 1992.

*endowments, the expectations and preferences of the agents, and the production techniques available in the economy."*

Agricultural commodity markets are incumbent on weather cycles or calendar because of harvest period and the perishability of agricultural goods (see Back et al. [5]). Agricultural commodities are different than classical financial assets such as equities since the commodity is grown and harvested in a seasonal fashion. Their price is higher just before harvest and lower after, implying volatility clustering with regard to the different periods. The perishability, thus the inventories are essential when new harvest is not ready. Because harvested crop is consumed throughout the year, new information about supply and demand affects all new crop contracts. Thus, the cost-of-carry[3] relationship, that link futures and spot prices, implies that a seasonal volatility trend should be observed in all delivery months. In nutshell, these events are likely to occur at the same period in a year, inducing a seasonality pattern in price volatility.

We analyze seasonality in agricultural commodity volatilities by computing the historical estimated volatilities on grouped daily returns. We separately group the daily returns by months for each year. The standard deviation of the daily returns is then computed for each observation month and annualized to make the results easier to interpret. In Section 2.3.3, we discuss the seasonal volatility of the selected commodity futures and figure 6 shows the volatility patterns.

Besides, we test the presence seasonality in monthly volatilities by comparing the sum of squared errors between a model with trend and a model with trend and seasonality component. The test, known as Fisher seasonality test, is based on test statistic that is compared to theoretical of Fisher-Snedecor table. There is seasonality when test statistic value is greater than the theoretical value. Table 3 presents seasonality test on monthly volatilities of selected commodity futures.

## 2.3 Data analysis

### 2.3.1 Empirical properties and stylized facts

Model selection and specification are generally driven by empirical facts of the time series at hand. We compute standard statistics on daily returns of agricultural futures prices. We select agricultural commodity in order to illustrate stylized facts that are known for financial time series.

The data set consists of daily prices of one year futures contract of agricultural commodities of US market and of the Commodity Research Bureau Commodity Index (CRBCI) price. Commodity Research Bureau or CRB is an analysis and research company on commodity futures markets which is the benchmark of commodity markets in U.S. trade. All data span from the beginning of January 2000 to the end of December 2012 and they are extracted from Stooq website. Particularly, we use one year futures contract of corn, soybean, rough rice, coffee and cocoa prices. We

[3]Cost-of-carry is the cost of holding a position on futures market. For most investments, the cost-of-carry is the risk-free interest rate that could be earned instead of holding the position. But, it also includes storage costs for commodities or any income you missed by holding the position. So, cost-of-carry = interest + storage cost - income earned.

continuously compound daily returns $r_i$ on each serie of observed prices between two consecutive days as

$$r_i = X_{i\Delta_n} - X_{(i-1)\Delta_n}; \qquad i = 2, 3, \ldots$$

In doing so, we ignore the roll effect by assuming that closed to maturity position of a contract is replaced by another contract of the same maturity. Hence, the returns are considered continuously observed.

Table 1 provides descriptive statistics and standard test statistics from data. All agricultural futures and the commodity index have positive annualized average return, which indicates an investment in these asset would perform over the period from 2000 to 2012, without risk risk factor. The corn futures contract has the highest average return (9.5%) of the entire sample while the lowest one is realized by coffee futures contract (1.8%). The risk, as measured by the annualized standard deviation, ranges from 26.41% for soybean futures to 34.14% for coffee futures. A quick look on volalitily values refines an investment analysis in these commodity futures contracts. For instance, over the considered period, coffee futures presents the highest volatility and is more risky.

Table 1: Descriptive statistics

| Products | $n$ | $\mu$ | $\sigma$ | $sk$ | $ku$ | $lev$ | $JB$ | $Qsq$ | $ADF$ |
|---|---|---|---|---|---|---|---|---|---|
| Corn | 3266 | 0.095 | 0.3002 | 0.133 | 5.179 | -0.022 | 656 | 29.70 | -24.05 |
| Soybean | 3264 | 0.085 | 0.2642 | -0.615 | 7.711 | -0.025 | 3224 | 31.86 | -22.75 |
| RoughRice | 3257 | 0.078 | 0.2836 | 0.612 | 13.07 | -0.032 | 13973 | 40.83 | -23.44 |
| Coffee | 3244 | 0.018 | 0.3414 | 0.190 | 6.617 | 0.053 | 1787 | 34.25 | -23.56 |
| Cocoa | 3244 | 0.080 | 0.3260 | -0.213 | 5.101 | 0.011 | 621 | 25.42 | -23.01 |
| CRBCI | 3262 | 0.048 | 0.1837 | -0.314 | 5.614 | -0.069 | 982 | 2640.7 | -24.84 |

$N$ is size of observed returns by futures contract. $\mu$ is the average return and $\sigma$ is the volatility of return of thirteen years.
$\mu$ and $\sigma$ are annualized respectively by $\bar{n}$ and $\sqrt{\bar{n}}$ with $\bar{n} = n/13$ being the average number of days per year. Recall data span over thirteen years.
$sk$ denotes the skewness of the return series, $sk = 0$ for normal distribution.
$ku$ denotes the kurtosis of the return series, $ku = 3$ for normal distribution.
$lev$ is the unconditional correlation between the squared return at date $t$ and the return at date $t-1$. Negative values for $lev$ indicate that large volatility tends to follow upon negative returns.
$JB$ is the Jarque-Bera statistic for testing normality. The test statistic is asymptotically $\chi^2$ distributed with 2 degrees of freedom. The relevant critical value at the 1% level is 9.518.
$Qsq$ is the Ljung-Box portmanteau test for the null hypothesis of no autocorrelation in the squared returns up to order 20. The test statistic is asymptotically $\chi^2$ distributed with 12 degrees of freedom. The relevant critical value at the 1% level is 37.566.
$ADF$ is the Augmented Dickey-Fuller test for the null hypothesis of unit root in the returns up to order 5. The test statistic is asymptotically Student distributed. The relevant critical value at the 1% level from McKinon table is -2.56.

The skewness ($sk$) is non zero for all the selected commodities as well as the commodity index returns. So, large positive returns occur more often than large negative returns for positive skewness and vice versa for negative skewness. The kurtosis ($ku$) exceeds the normal value of 3 for all the assets. This implies that the presence of high peaks and/or heavy tails. Skewness and kurtosis, together reflect that large outlying observations occur more often than are expected under the assumption of normality. The Jarque-Bera test statistic, as reported in column $JB$, confirms the departure from normality for all return series at significance level of 1%. Mandelbrot [26] argues that

many commodity asset returns follow a leptokurtic distribution. Further, the figure 2 of Appendix B displays QQ-plots of returns against the quantiles of the standard normal distribution for price time series. If the asset returns were normally distributed, then the QQ-plot would approximate the straight line representing the standard normal distribution. Here, QQ-plots show that the historical quantiles in the tails of the distribution are significantly larger than the normal distribution. Note how large the $y$-axis scale of rough rice QQ-plot is than the one others', which confirms the high kurtosis and the high Jarque-Bera test statistic. High kurtosis induces the presence of infrequent observations that are explained by stochastic volatility or the presence of jumps, or both.
All these facts are in line with fat tails observed in financial asset returns. Therefore, modeling by Brownian motion provides at best only a rough approximation for these commodity prices. Besides, in Figure 3, the kernel density plots ovelaying histogram charts of return series strengthen the fact that observed prices are not normally distributed. The distribution of log returns on prices is unimodal, so, skewness and kurtosis are easy to interpret. The tail is longer and fatter on one side than on the other (refer to $x$-axis) reflecting non zero skewness.

The augmented Dickey-Fuller test indicates if data are stationary and it is also a simple way to check whether there is a mean-reverting behavior following Daniel [14]. Testing for mean reversion is equivalent to testing for stationarity, because the coefficient of a stationary AR(1) process is less than 1.

Another important property for modeling the asset price is the independence of asset returns. The assumption of independence, in statistical terms, means there is no autocorrelation in historical returns. For all the commodities, Figure 4 diplays the autocorrelation functions (ACF) and figure 5 the partial autocorrelation function (PACF) in Appendix B. We observe no significant lags in the historical returns, which means that the independence assumption is acceptable for the returns of all the price series. However, the ACF and the PACF of returns do not reveal substantial information, but the ACF of squared returns exhibits significant correlations up to an extended lag length. The Box-Ljung statistics, reported in column $Qsq$ of Table 1 confirms a significant correlation for the squared returns at significance level 1% for all series. Hence, the relative small returns of quiet periods alternate with relatively volatile onr where the variations of prices are rather large. The phenomenon of persistently changing of volatility over time is pointed out by Mandelbrot [26] who relates it as volatility clustering.

The so called leverage effect in stock market has the reverse effect in commodity market. Leverage effect arises when stock prices drop, panic kicks into the markets and volatilities raise. In many commodities this effect is reversed. The inverse leverage effect means volatilities raise when commodity prices increase. When commodity prices go up, it is generally bad for the economy and panic sets in and it implies volatility increase. Here, neither leverage effect nor its reverse is significant.

As fat-tailed distribution may imply the presence of jumps, we check in the next section whether there are jumps in the series of observed prices with AÃ¯t-Sahalia and Jacod [2] jump detection test.

### 2.3.2 Testing for presence of jumps in agricultural futures prices

Following the jump detection test of AÃ¯t-Sahalia and Jacod [2] described earlier in Section 2.1, we compute the test statistic $\widehat{S}(p,k,\Delta_n)_t$ by setting $p=6$ and $k=2$ for all futures prices. We derive critical values $c^c_{n,t}$ and $c^j_{n,t}$ respectively, for the null hypothesis of absence of jumps and the null hypothesis of presence of jumps at level 5% and 10%. The null hypothesis of absence of jumps is rejected at significance level 5% for

Table 2: Jump detection tests

| Product | $\widehat{S}(p,k,\Delta_n)_t$ | Absence of jumps | | | Presence of jumps | | |
|---|---|---|---|---|---|---|---|
| | | $c^c_{n,t}$ (5%) | $c^c_{n,t}$ (10%) | $p_c$ | $c^j_{n,t}$ (5%) | $c^j_{n,t}$ (10%) | $p_j$ |
| Corn | 3.063 | 3.076 | 3.280 | 0.048 | 19.28 | 15.24 | 0.43 |
| Soybean | 1.267 | 2.952 | 3.184 | <1e-5 | 23.67 | 18.66 | 0.49 |
| Rough rice | 1.047 | 2.790 | 3.057 | <1e-4 | 36.77 | 28.87 | 0.50 |
| Coffee | 1.230 | 3.136 | 3.326 | <1e-7 | 29.89 | 23.51 | 0.49 |
| Cocoa | 3.322 | 3.205 | 3.380 | 0.081 | 15.20 | 12.06 | 0.39 |
| CRBCI | 2.921 | 2.990 | 3.213 | 0.039 | 14.39 | 11.43 | 0.41 |

The decision rule for details for null hypothesis of no jump is $\widehat{S}(p,k,\Delta_n)_t < c^c_{n,t}$ and for null hypothesis of presence of jumps is $\widehat{S}(p,k,\Delta_n)_t > c^j_{n,t}$, (see Appendix A). The probability that the null hypothesis is true is $p$-value; $p_c$ is the $p$-value of null hypothesis of absence of jumps and $p_j$ is the $p$-value of null hypothesis of presence of jumps.

all the observed prices, excepted for cocoa futures price, for which the same hypothesis is rejected at level 10%. As the null hypothesis of presence of jumps is not rejected for cocoa futures price at significance levels 5% and 10%, we assume that the cocoa futures jumps within the period 2000 to 2012. However, the test statistic converges towards 1 for soybean, rough rice and roffee futures prices as shown by the result of convergence in case of absence of jumps in (12). When the test statistic $\widehat{S}(p,k,\Delta_n)_t$ converges towards to one, $p$-value is low. For other futures the test statistic is different from 1. This could be caused the the daily prices that we use instead of using high frequency data as advocate AÃ¯t-Sahalia and Jacod [2]. Another reason for non convergence towards to right value could be small data size. Note that the null hypothesis of presence of jumps is not rejeted for all price series.

One the presence of jumps is suggested, what kind of jump process is the right to consider for the price dynamics? In fact, jumps are generally characterized by small jumps and large jumps. AÃ¯t-Sahalia and Jacod [3] develop a statistical procedure to discriminate between the finite and infinite activity of jumps in a semimartingale discretely observed. The test is based on an arbitrary cutoff level to distinguish between small and large jumps. There is always a finite number of big jumps. The question is then, whether there is a finite or infinite number of small jumps. In our case, we assume a finite number of jumps and we consider a compound Poisson process to represent jump component for all price series. The compound Poisson process is the sum of a number of jump sizes. The number of jump sizes follows a Poisson process with its intensity being the average number per unit of time. Jump sizes are independent from the number of jumps and they are independent and identically distributed random variables. A compound Poisson process is a of finite activity, because its random number of jumps is finite[4]. Adding jump component to the model considered by

[4]Compound Poisson process is piecewise constant LÃ©vy process and its LÃ©vy measure $\ell(dx)$

Marcus and Modest [25], the new price model tries to capture excess skewness and kurtosis.

### 2.3.3 Seasonality testing

Figure 6 represents the monthly volatilities of the commodity futures prices. Volatilities vary according to same month of years and their increases and decreases alternate periodically. The pattern of monthly volatilities over a determined period (not the same period for all commodity futures) repeats itself along the thirteen years, subject to varying levels. The periodic variations are more pronounced in some years than in others. This fact may be due to sudden changes which cause irregular variation of volatilities. The July 2000 volatility of coffee futures shows an extreme variation. This fact could be attributed to the presence of jumps.

The test statistic of Fisher seasonality test is reported in Table 3 for monthly volatilities of commodity futures. These values suggest seasonal volatility as shown by the graphics of figure 6.

Table 3: Seasonality test on monthly volatilities

| | Corn | Soybean | Rough rice | Coffee | Cocoa |
|---|---|---|---|---|---|
| Test statistic | 7.958 | 10.243 | 25.001 | 18.491 | 28.549 |

The test statistic is compared to the theoretical value of 1.81 which is the 5% quantile of Fisher distribution with degree of freedom 11 and 143. The degree of freedom 143 is the number of months in thirteen years minus the number of independent parameters (which is 11, because as there are twelve months in a year, then 12 parameters which are linked to each other) under the model with trend and seasonality and minus the number of independent parameters (which is 2) under the model with trend only.

Figure 6 also exhibits a deformation of seasonality in volatility pattern for all the commodity futures in December 2007 or just after this period. This highlight the subprime crisis where speculators have used commodities as safe-haven because of their non correlation with stock markets that had drowned.

## 3 Model and crop valuation

Marcus and Modest [25] show the equivalence of describing market equilibrium in terms of the futures price and of the commodity, denoted by $F_t$, and $S_t$. Futures contracts are settled daily and have zero value. Further, when the contract matures

$$F_T = P_T \tag{13}$$

The current value $V_t$ of the claim maturing at time $T$ on stock of the commodity is then function of $F_t$, $S_t$, $\theta_{k,t}$ and $t$, which together summarize all relevant information concerning the crop valuation. We denote the current value of the prospective stock of the commodity by $V_t = V(F_t, S_t, t)$.

is given by the multiplication of the intensity $\lambda$ and the jump size probability density $f(dx)$: $\lambda f(dx)$. Compound Poisson process is of finite activity because its LÃ©vy measure is finite $\int_{\mathbb{R}} \ell(dx) = \lambda < \infty$.

Formally, we consider a vector $(F_t, S_t, \theta_t)'$ of stochastic processes at time $t$ where $F_t$ denotes the futures price, $S_t$ the commodity index price and $\theta_t$ the share of a producer $k$. The futures price is assumed to follow mean-reverting jump-diffusion. Mean-reversion is motivated by the interaction of the supply and of the demand[5]. It is suggested by Augmented Dickey-Fuller test results shown in Table 1 above. The stock index follows a jump-diffusion process and the share of a producer a mean zero diffusion.

Schwartz and Smith [31] and Geman and Nguyen [17] observe that the market price of risk can only be estimated with very low precision from derivatives data. We assumed there exists a risk neutral probability measure $\mathbb{Q}$ under which price dynamics are considered.

$$\begin{aligned} \frac{dF_t}{dF_{t-}} &= \alpha(m - \ln F_t)dt + \sigma_1 e^{\varphi(t)} dW_{1,t} + (Y_{1,t} - 1)\, dN_{1,t}, \\ \frac{dS_t}{dS_{t-}} &= (\mu)dt + \sigma_2 dW_{2,t} + (Y_{2,t} - 1)dN_{2,t} \end{aligned} \tag{14}$$

with $\alpha > 0$ the rate at which the futures return reverts towards its long-term mean $m$, $\mu$ is the rate of the return on shift demand parameter measured by the commodity index. The volatility of the futures price $F_t$ is characterized by $\sigma_1 > 0$ and by function $\varphi(t)$, that describes the seasonal behavior of futures return volatility, $\sigma_2 > 0$ is the volatility of commodity index return. $W_{1,t}$ and $W_{2,t}$ are two correlated standard Brownian motions associated to price processes $F$ and $S$ respectively, with constant correlation $\rho$. $N_{1,t}$ and $N_{2,t}$, are independent Poisson processes with intensities $\lambda_1$ and $\lambda_2$ respectively, and $Y_{1,t}$ and $Y_{2,t}$ are respectively the jump sizes of $F_t$ and $S_t$ following a log normal distribution. $t-$ is the instant immediately before time $t$ where there is jump.

Back et al. [5] argue that the function $\varphi(t)$ impacts the value of an option by affecting the volatilities of underlying assets. We set $\varphi(t)$ as in Back et al. [5] by

$$\varphi(t) = \psi \sin\left(2\pi(t + \omega)\right) \tag{15}$$

where we impose $\psi \geq 0$ and $\omega \in [-0.5, 0.5]$ to ensure their uniquenss.

Expectations about market share of a producer change only as new (unexpected) information becomes available. Hence, under assumption of rational expectations in the sense that expected value of change in $\theta$ is zero, we assume $\theta$ to be described by a simple diffusion process with zero drift.

$$\frac{d\theta_{k,t}}{\theta_{k,t}} = \sigma_3 dW_{3,t} \tag{16}$$

where $\sigma_3$ is the diffusion parameter associated to the standard Brownian $W_3$. $W_3$ is independent of both $W_1$ and $W_2$ since the market share of a producer is assumed not to be correlated with the futures and the commodity index prices.

[5] Mean-reversion in commodity market models the correlation between the convenience yield and spot prices, spot price level dependent time-varying basis risk and negative relation between interest rates and prices. The convenience yield is all effects evolving from the ownership of the physical commodity compared to the ownership of a futures contract. See Schwartz [30] on the mean-reverting behavior of commodity prices.

### 3.1 Crop valuation

The value $V_{k,t}$ of claim to a producer's revenue is function of $\theta_t$, $F_t$, $S_t$ and $t$ : $V(\theta_{k,t}, F_t, S_t, t)$. It is the discounted value of expected revenues $V(\theta_{k,T}, F_T, S_T, T)$ in a risk-neutral world[6]

$$V(\theta_{k,t}, F_t, S_t, t) = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}\left[V(\theta_{k,T}, F_T, S_T, T)|\,\mathcal{F}_t\right] \tag{17}$$

where $e^{-r(T-t)}$ is the discount factor with the risk-free interest rate $r$ and $\mathcal{F}_t$ is the information set available at time $t$.

The final value of $V_{k,T}$ at maturity $T$ depends on whether there is government support or not,

$$V_T = \begin{cases} S_T^{\gamma} F_T^{1-\varepsilon} & \text{if no support} \\ \max\left[S_T^{\gamma} F_T^{1-\varepsilon}, \bar{P} S_T^{\gamma} F_T^{-\varepsilon}\right] & \text{if support.} \end{cases} \tag{18}$$

where $\theta_k$ has been dropped to simplicify notation.

When there is no support, the crop value is straightforward from (18)

$$V_t = e^{(g-r)\tau} S_t^{\gamma} F_t^{1-\varepsilon} \tag{19}$$

with

$$\tau = T - t$$
$$g = \gamma r + \frac{1}{2}\varepsilon(\varepsilon - 1)\sigma_1^2 e^{2\varphi(t)} + \frac{1}{2}\gamma(\gamma - 1)\sigma_2^2 + \gamma(1-\varepsilon)\rho\sigma_1\sigma_2 e^{\varphi(t)}.$$

When agricultural support system is available, equations (5) and (13) give the final condition of

$$V_T = \max\left[S_T^{\gamma} F_T^{1-\varepsilon}\ ,\ \bar{P} S_T^{\gamma} F_T^{-\varepsilon}\right]. \tag{20}$$

So, an increase in support level $\bar{P}$ increases the value of the claim, as does the wealth parameter $S_t$. Therefore, the value of $V_t$ behaves like an European put option price with random strike.

Let $X_{1,t} = \ln F_t$ denotes the logarithm of the commodity futures price and $X_{2,t} = \ln S_t$ be the logarithm of the commodity index price. We derive a partial integro differential equation (PIDE) for $V_t = V(F_t, S_t, t)$ following Shreve [32] with final condition

$$V_T = \max\left[S_T^{\gamma} F_T^{1-\varepsilon}\ ,\ \bar{P} S_T^{\gamma} F_T^{-\varepsilon}\right] \tag{21}$$

in the following form

$$\begin{aligned} 0 = \frac{\partial V_t}{\partial t} &+ \sum_{i=1}^{2}\left(\eta_i - \frac{1}{2}\upsilon_i^2 - \lambda_i\kappa_i\right)\frac{\partial V_t}{\partial x_i} + \sum_{i=1}^{2}\frac{1}{2}\upsilon_i^2\frac{\partial^2 V_t}{\partial x_i^2} \\ &+ \rho\upsilon_1\upsilon_2\frac{\partial^2 V_t}{\partial x_1 \partial x_2} + \sum_{i=1}^{2}\lambda_i\int_{-\infty}^{\infty}\left[V(X_{i,t} + Y_{i,t}) - V_t\right] f_{Y_i}(y_i) dy_i \end{aligned} \tag{22}$$

[6] Risk-neutral world summarizes probability of future outcome adjusted for risk, which is then used to compute expected asset value. The benefit of risk-neutral pricing approach is that once the risk-neutral probability are calculated, it can be used to price the asset based on its expected payoff. Theoretical risk-neutral probability differ from actual real world probability; if the latter were used, expected value of a security would need to be adjusted for its individual risk profile.

where $f_{Y_i}$ is the probability density function of the jump size $Y_i$ and

$$\eta_1 = \alpha(m - X_{1,t}), \qquad \upsilon_1 = \sigma_1 e^{\varphi(t)},$$
$$\eta_2 = \mu, \qquad \upsilon_2 = \sigma_2.$$

The crop value $V_t$ depends on price dynamics modeled as jump-diffusion processes. Closed form solution is no more possible because it is possible for hedging strategy involving only continuous model as in Black [9]. When the underlying price dynamic contains jump component, the hedging strategy as generalized in Harrison Pliska [18] is no more possible and jump risk could be not hedged. So, we solve (22) numerical with Monte Carlo method. Monte Carlo simulations provide a convenient framework that approximate both continuous and jump-diffusion models of price dynamics. Another way to solve a PIDE is by Fourier transform via the characteristic function, (see Carr and Manda [10]).

The basic idea behind of Monte Carlo simulations is for approximating the expectation of a function of a random variable. Option pricing with Monte Carlo method is to calculate the expected value of a quantity which is a function of the solution to a stochastic differential equation. It is based on the distribution of terminal asset prices, determined by the process governing the future price movements. The calculation generates a serie of asset price trajectories and the terminal asset prices from the trajectories are used to estimate the option price. The option price with Monte Carlo simulation is often used as benchmark because the method is consistent in approximating expectation of function of random variable[7]. Since this is independent of the number of dimensions, the Monte Carlo method does not suffer from the "curse of dimensionality" that affects other numerical techniques. However, Monte Carlo simulations are comparatively slow for pricing options on a single asset, but it can easily be extended to multidimensions.

### 3.2 Optimal production rules

The producer chooses the size of the crop that maximize the value of his harvest. From relation (8), it follows that the optimal choice of input $I_k$ for producer $k$ is implicitly obtained by the first-order condition for the value maximization

$$\frac{\partial \theta_k(I_k)}{\partial I_k} V = 1. \tag{23}$$

So, when individual production decisions are optimal, one extra unit of investment at the margin leads to one unit increase in the ex ante value of crop for every producer $k$. Indeed, using (7) we have

$$\begin{aligned} dV_{k,T} &= V_T d\theta_{k,T} + \theta_{k,T} dV_T \\ &= V_T \frac{\partial \theta_k(I_k)}{\partial I_k} dI_k \quad \text{with} \quad dV_T = 0. \end{aligned} \tag{24}$$
$$\text{For} \quad dI_k = 1, \quad \text{we have} \quad dV_{k,T} = 1.$$

[7]Sam Savage has said about Monte Carlo simulation : *What is the last thing you do before you climb on a ladder? You shake it, and that is Monte Carlo simulation.*

Then, $dV_{k,T}$ is proportional to $dI_k$, and it follows that if any producer $k$ overestimate $V$, it will be incurring too high a marginal cost and will be overinvesting in the the crop. Note that the marginal cost of one unit of output is

$$\frac{1}{\theta'_{k,T}(I_k)Q_T}.$$

Let $V^{ref}(F_t, S_t, t)$ be the estimate value of crop when output are non-stochastic. In this case futures price and commodity index are locally perfectly correlated, $\rho = 1$ and we have

$$V^{ref}(F_t, S_t, t) = e^{(a-r)\tau} F_t^{1-\varepsilon} S_t^{\gamma} \tag{25}$$

with

$$a = \gamma r + \frac{\gamma\sigma_2^2}{2}\left(\frac{\gamma}{\varepsilon} - 1\right)$$

which can also be written as

$$V^{ref}(F_t, S_t, t) = e^{-r\tau} Q_T F_t. \tag{26}$$

The optimal production rule, when output is non-stochastic, is to set the marginal cost of one unit of production equal to the discounted futures price

$$F_t e^{-r\tau} = \frac{1}{\theta'_{k,T}(I_k)Q}. \tag{27}$$

Now, let denote by $V^M(F_t, S_t, t)$, the estimate value of crop when output is stochastic, where $M$ represents the price model. $M = mm$ is the model of Marcus and Modest [25] when price dynamics follow constant drift and volatility, $M = mr$ when futures price dynamics follow mean-reverting with constant volatility and $M = j$ when price dynamics follow jump-diffusion as (14).

The application of (27) when output is stochastic will lead producers to an inadequate level of resources and expression (24) shows that the optimal level of marginal cost is proportional to the farmer's perception of the *ex ante* value of the crop. Therefore, by comparing the estimates of $V(F_t, S_t, t)$ when output is nonstochastic to when output is stochastic, one may evaluate the percentage difference in marginal costs chosen by a farmer who uses the rule in (27) instead of the correct rule. The percentage error in marginal cost is a measure[8] of resource misallocation

$$e^M = \frac{V^{ref}(F_t, S_t, t) - V^M(F_t, S_t, t)}{V^M(F_t, S_t, t)}, \quad M = mm\,, mr,\, j. \tag{28}$$

Section 4.2 discuss the percentage error in marginal cost for different model of price dynamics.

[8]However, that measure would require several inputs, such as farmers production functions, which are not integral to the rest of the analysis. It does not necessarily indicate the magnitude of welfare loss, which would more properly be measured by lost producer plus consumer surplus.

## 4 Numerical applications

Numerical applications compare the decision rule under non-stochastic and stochastic output subject to assumption of price dynamics. Marcus and Modest [25] assume Brownian motion to represent the source of risk. We assume mean-reverting and mean-reverting jump-diffusion for risk factor as well as seasonality in futures price volatility. We derive the same decision rule as in Marcus and Modest [25] by considering these stylized facts. As price dynamics include jump components, hedging against systematic risk does not have closed form solution. We use Monte Carlo method to valuate the crops as put option. The valuation from Monte Carlo method serves as reference price.

The crop value depends on model parameters. Fundamental parameter knowledge, based on economic analysis of the factors, is perhaps the most meaningful channel for model calibration. Nevertheless, data based methods are often needed to complement and validate a particular model. Maximum likelihood method is a way to deal with model parameter estimates from data at hand. However, we do not estimate price elasticities with maximum likelihood method.

### 4.1 Parameter estimates for price dynamics

We first consider two types of continuous models for futures prices: constant drift and volatility diffusion process as in Marcus and Modest [25] and mean-reverting model with constant volatility. The commodity index price always follows constant drift and volatility diffusion process in continuous futures price dynamic. Secondly, we use discontinuous models of (14). Tables 4 and 5 display maximum likelihood estimation of parameters of respectively continuous and discontinuous models using observed prices. The standard error of each parameter estimate is reported in square brackets just below the parameter.

Table 4: Parameter estimates of continuous model

| | Parameters | Corn | Soybean | R. Rice | Coffee | Cocoa | CRBCI |
|---|---|---|---|---|---|---|---|
| Simple diffusion | $\mu$ | 0.1411 | 0.1203 | 0.1169 | 0.0763 | 0.1328 | 0.0656 |
| | | [0.0045] | [0.0058] | [0.0166] | [0.0163] | [0.0096] | [0.0084] |
| | $\sigma$ | 0.3002 | 0.2642 | 0.2837 | 0.3414 | 0.3257 | 0.1842 |
| | | [0.0084] | [0.0101] | [0.0116] | [0.0171] | [0.0158] | [0.0085] |
| Mean-reverting diffusion | $\alpha$ | 0.0161 | 0.0125 | 0.0269 | 0.0014 | 0.0093 | |
| | | [0.0141] | [0.0092] | [0.0270] | [0.0798] | [0.0070] | |
| | $m$ | 0.1016 | 0.1693 | 0.2700 | 0.0992 | 0.1647 | |
| | | [3.8630] | [3.7373] | [2.0942] | [6.5905] | [5.5788] | |
| | $\sigma$ | 0.3002 | 0.2642 | 0.2836 | 0.3415 | 0.3257 | |
| | | [0.0054] | [0.0060] | [0.0086] | [0.0071] | [0.0058] | |

This table reports parameter estimates for constant drift and volatility in the upper part and mean-reverting with constant volatility model in the lower part.

Volatility estimates with maximum likelihood method in Table 4 are the same as historical volatilities in Table 1, while the average return is different the long-run term. Positive long-run shows the up trend as Figure 1 exhibits it.

Table 5 reports parameter estimates of the discontinuous model as, specified in equations (14).

Table 5: Parameter estimates of jump-diffusion model

| Parameters | Corn | Soybean | R. Rice | Coffee | Cocoa | CRBCI |
|---|---|---|---|---|---|---|
| $\mu$ | | | | | | 0.0368 |
| | | | | | | [0.0119] |
| $\alpha$ | 0.0162 | 0.0123 | 0.0156 | 0.0014 | 0.0094 | |
| | [0.0238] | [0.0101] | [0.0530] | [0.2222] | [0.0069] | |
| $m$ | 0.0159 | 0.0146 | 0.1331 | 0.1754 | 0.1291 | |
| | [4.1709] | [4.3184] | [1.7094] | [7.0804] | [5.5088] | |
| $\sigma$ | 0.2997 | 0.2641 | 0.2484 | 0.3412 | 0.3257 | 0.1841 |
| | [0.0054] | [0.0060] | [0.0043] | [0.0071] | [0.0058] | [0.0035] |
| $\psi$ | 0.0099 | 0.0125 | 0.2108 | 0.0077 | 0.0077 | |
| | [0.0002] | [0.0003] | [0.0049] | [0.0002] | [0.0001] | |
| $\omega$ | 0.0107 | 0.0102 | -0.0203 | 0.0054 | 0.0139 | |
| | [0.0002] | [0.0002] | [0.0004] | [0.0001] | [0.0002] | |
| $\lambda$ | 0.4246 | 0.2137 | 3.5690 | 0.1974 | 0.1361 | 0.3187 |
| | [0.0593] | [0.0203] | [0.0141] | [0.4445] | [0.0063] | [0.1964] |
| $\kappa$ | 0.0004 | 0.0014 | 0.0094 | 0.0009 | 0.0014 | 0.0009 |
| | [0.0026] | [0.0009] | [0.0110] | [0.0295] | [0.0012] | [0.0008] |
| $s$ | 0.0182 | 0.0096 | 0.0930 | 0.0078 | 0.0100 | 0.0092 |
| | [0.0019] | [0.0004] | [0.0139] | [0.0028] | [0.0002] | [0.0009] |

Volatility estimates are almost the same even in presence of jumps in price dynamics, excepted for rough rice futures. Rough rice futures has the highest kurtosis and the highest Jarque-Bera test statistic, from Table 1, which means that its fat tails are likely due by more upward jumps than downward one (its skewness is positive). A higher $\lambda$ implies more jumps in data on average and the sense of jump depends on the signs of skewness and $\kappa$. So, as expected rough rice futures varies more frequently than other commodity futures; its jumps size volatility is the largest. The speed of reversion is also stable compared to parameters of mean-reversion model without jump component. The long-term mean is quiet instable for all commodity futures. Notice that the parameter estimates by maximum likelihood method is model dependent or sensitive to computation techniques like initialization.

We set wealth elasticity at $\gamma = 0.50$ as in Marcus and Modest [25], and estimate price elasticities of considered commodity futures using the trade volume of year 2012.

Table 6: Price elasticity

| Commodity | $\rho$ | $\gamma$ |
|---|---|---|
| Corn | 0.589 | 0.238 |
| Soybean | 0.637 | 0.493 |
| Rough righ | 0.739 | 1.055 |
| Coffee | 0.615 | 0.455 |
| Coffee | 0.502 | 0.798 |

The crop value is larger (*i*) with increases in the futures price for $0 < \varepsilon < 1$, (*ii*) with decreases in the futures price for $\varepsilon > 1$, $\partial V_{k,t}(\cdot)/\partial F_t = (1-\varepsilon)(V_t/F_t)$, and (*iii*) with increases in wealth, $\partial V_{k,t}(\cdot)/\partial S_t = \gamma(V_t/S_t) > 0$. For instance, a decrease in rough rice futures price involves a larger crop value for more investment, whereas it is relatively the opposite for other commodities.

## 4.2 Comparison of optimal production rules

We consider three models for the crop value when output is stochastic that is compared to the crop value when output is nonstochastic. The following tables present the percentage error in marginal cost under different models for the considered commodities. The crop value $V(F_t, S_t, t)$ is estimated by Monte Carlo simulations when output is stochastic, even for Marcus and Modest [25]. We simulate one million price trajectories using parameter estimates in the previous section. We use the December, 31st 2012 prices as initial values for both futures and commodity index.

### 4.2.1 Valuation in absence of agricultural support

In absence of price support, the value of crop is given by relation (19) subject to the price dynamics when output is stochastic. Table 7 reports numerical results of the difference in marginal costs. Our results suggest that marginal cost equal the futures price (25), leads to produce too little whereas in Marcus and Modest [25] the results are different. Our results also show that the crop value is underestimated with Brownian motion price dynamics than with jump-diffusion price dynamics. The magnitude of percentage errors are higher when prices follow jump-diffusion than when prices follow continuous models. So, representing the source of risk by only Brownian motion do not take into account all the risk factors.

Table 7: Comparisons of production rules without price support

| Commodity | $e^{mm}$ | $e^{mr}$ | $e^{j}$ |
|---|---|---|---|
| Corn | -0.0522 | -0.0475 | -0.0579 |
| Soybean | -0.0368 | -0.0329 | -0.0464 |
| Rough rice | -0.0115 | -0.0121 | -0.0318 |
| Coffee | -0.0348 | -0.0243 | -0.0396 |
| Cocoa | -0.0242 | -0.0204 | -0.0363 |

### 4.2.2 Valuation in presence of agricultural support

In the presence of agricultural price supports, for non-stochastic output, farmers would not be expected to use the rule that the discounted futures price equals marginal cost if $F_t < \bar{P}$, because the effective price must be at least $\bar{P}$. Thus, we assume the farmer sets the discounted value of $\max(F_t, \bar{P})$ equal to the futures price, and

$$V_t^{ref} = e^{r\tau} \times \max(F_t, \bar{P}). \tag{29}$$

When there is price support, Marcus and Modest [25] find a closed form formula when output is stochastic. But, we use Monte Carlo method to compute the crop value when output is stochastic whichever the price dynamics follow to make the results easy to interpret.

The following Tables 8, 9 and 8 present estimates of the percentage difference in marginal cost that would result from using the simple in (29) instead of the optimal rule derived for stochastic output under price supports. We choose three possible ratios of support price to futures price to illustrate the effect of stochastic output.

Finally, if the government were to guarantee farmers a minimum price equal to the current futures price (which can be interpreted as the risk-adjusted expected spot price), then the percentage error would be negative, except for corn futures. Again, the crop value under Brownian motion underestimates risk factor.

To summarize, either in the absence or in the presence of price support, our results indicate that the simple rule is not a good approximation to the market reality. Only the corn producer with price support produces stands out from the considered agricultural products.

Table 8: Comparison of production rule at support less than futures price

| | $\bar{P}/F = 0.75$ | | |
|---|---|---|---|
| Commodity | $e^{mm}$ | $e^{mr}$ | $e^{j}$ |
| Corn | 0.0731 | 0.0955 | 0.0952 |
| Soybean | -0.0744 | -0.0714 | -0.0714 |
| Rough rice | -0.0964 | -0.0964 | -0.0977 |
| Coffee | -0.0377 | -0.0364 | -0.0590 |
| Cocoa | -0.0984 | -0.0979 | -0.0979 |

Table 9: Comparison of production rule at support equal futures price

| | $\bar{P}/F = 1$ | | |
|---|---|---|---|
| Commodity | $e^{mm}$ | $e^{mr}$ | $e^{j}$ |
| Corn | 0.0731 | -0.0984 | 0.0970 |
| Soybean | -0.0744 | -0.0714 | -0.0713 |
| Rough rice | -0.0964 | -0.0964 | -0.0976 |
| Coffee | -0.0377 | -0.0362 | -0.0567 |
| Cocoa | -0.0984 | -0.0979 | -0.0978 |

Table 10: Comparison of production rule at support greater than futures price

| | $\bar{P}/F = 1.25$ | | |
|---|---|---|---|
| Commodity | $e^{mm}$ | $e^{mr}$ | $e^{j}$ |
| Corn | 0.1318 | 0.0954 | 0.1318 |
| Soybean | -0.0679 | -0.0715 | -0.0680 |
| Rough rice | -0.0977 | -0.0964 | -0.0977 |
| Coffee | -0.0538 | -0.0364 | -0.0364 |
| Cocoa | -0.0978 | -0.0979 | -0.0978 |

## 5 Conclusion

We consider, in this paper, the production decision of a firm facing both demand and output uncertainty as in Marcus and Modest [25], whom use Brownian motion to describe uncertainty faced by an agricultural producer. We extend their model by including jump component in price dynamics and derive the same preference-free production rules which do depend on the existence of well-developed futures markets. Particularly, including jump involves crop value to better consider sysmatic risk that does not have a closed form valuation.

## A Jump detection test procedure

We summarize here the jump detection test designed by AÃ¯t-Sahalia and Jacod [2, 3].

Let $X = (X_t)_{t\in[0,T]}$ be the log-price process defined on a fix filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F})_{t\geq 0}, \mathbb{P})$, $\mathcal{F}$ can be seen as information filtration for market participants. Formally, $X$ is assumed to have more general form of Itō-semimartingale which has a nice representation in terms of a Wiener process and a Poisson random measure. That is

$$\begin{aligned} X_t = X_0 + \int_0^t b_s ds + \int_0^t \sigma_s dW_s + \int_0^t \int_{\mathbb{R}} \kappa \circ \delta(s,x)(\mu - \nu)(ds, dx) \\ + \int_0^t \int_{\mathbb{R}} \kappa' \circ \delta(s,x)\mu(ds,dx) \end{aligned} \tag{30}$$

where $W$ denotes a $(\mathcal{F}_t)$-standard Wiener process and $\mu$ is a $(\mathcal{F}_t)$-Poisson random measure on $(0,\infty) \times \mathbb{R}$ with intensity measure $\nu(dt, dx) = dt \otimes \lambda(dx)$, and $\lambda$ is a $\sigma$-finite or infinite measure. The processes $b = (b_t)_{t\in[0,T]}$ and $\sigma = (\sigma_t)_{t\in[0,T]}$ are optional. Moreover, $\kappa$ is a continuous function with compact support and $\kappa'(x) = x$ on a neighborhood of 0, and $\kappa'(x) = x - \kappa(x)$. The characteristics of $X$ are defined as follow

$$A_t = \int_0^t b_s ds, \qquad \langle M \rangle_t = \int_0^t \sigma_s^2 ds, \qquad \nu(dt, dx) = dt F_t(dx)$$

and are absolutely continuous with respect to the Lebesgue measure and $F_t(D)$ is optional for all Borel subsets $D$ of $\mathbb{R}$. $\langle M \rangle$ is the quadratic variation of the continuous local martingale. The volatility $\sigma_t$ could have the same form as $X$, but with another Wiener independent of the one $(W, \mu)$.

When $X$ jumps, it is expressed by

$$\Delta X_t = X_t - X_{t-} \qquad \text{with} \qquad X_{t-} = \lim_{u\uparrow t} X_u.$$

Notice that $t-$ is the instant immediately before time $t$, so $\Delta X_t$ is different from an increment of $X$ which is expressed as $X_t - X_s$ for $s \leq t$. Futhermore, detecting jumps for one-dimensional process is not a restriction since even if it were multimensional, a jump would necessarily be a jump for at least one of its components.

Following Barndorff-Nielsen and Shepard [7], who argue that multipower absolute variations can separate the continuous part of the quadratic variation, AÃ¯t-Sahalia and Jacod [2] base the jump detection procedure on processes that measure separately the continuous and jump components of $X$. The variability of the continuous part and the discontinuous part are defined respectivily for any positive number $p$ as follow

$$A(p)_t = \int_0^t |\sigma_s|^p ds \qquad \text{and} \qquad B(p)_t = \sum_{s\leq t} |\Delta X_s|^p \tag{31}$$

with $\sigma_t$ the instantaneous volatility parameter of the asset price dynamic. The higher the value of $p$, the more seperable are $A(p)_t$ and $B(p)_t$ (AÃ¯t-Sahalia and Jacod [2]). From $n$ values of the price process observed within the time horizon $[0,t]$ with each value at $i\Delta_n$, where $\Delta_n = t/n$ and $i = 1, 2, \ldots$, one has to decide whether $X$ has discontinuities or not the time period $[0,t]$. To find out whether discontiniuities are relevant for the observed data within the time period $[0,t]$, AÃ¯t-Sahalia and Jacod [2] propose a statistic of variability measure based on the absolute increment of $X$. The statistic converges under conditions and gives raise to the target decision. The increment of $X$ within two periods is expressed, for $i = 0, 1, \ldots$,

$$\Delta_i^n X := X_{i\Delta_n} - X_{(i-1)\Delta_n}. \tag{32}$$

Suppose that $X$ is observed at each time step $i\Delta_n$ for all $i = 0, 1, \ldots$ such that only observation times $i\Delta_n$, $i = 1, 2, \ldots$ smaller than or equal to $t$ are considered. So, when $n \to \infty$, then $\Delta_n \to 0$. Notice that in the reality $n < \infty$, but $n \to \infty$ is assumed for convergence convenience. Deciding, for discretely observed data, whether there are jumps or not is to detect sudden changes in data. In statistical term, it means, on the basis of the observations $X_{i\Delta_n}$ over to the time interval $[0,t]$, in which of the following two complementary sets the path that is discretely observed falls

$$\begin{cases} \Omega_t^j = \{\omega : s \mapsto X_s(\omega) \text{ is discontinuous on } [0,t]\}, \\ \Omega_t^c = \{\omega : s \mapsto X_s(\omega) \text{ is continuous on } [0,t]\} \end{cases}$$

For the given stochastic process in expression (30), it is equivalent to test whether the coefficient $\delta$ is identically 0. We have

$$\Omega_t^j = \{B(p)_t > 0\} \text{ for any } p > 0.$$

Therefore, in any event, everything boils down to estimate, on the basis of the observations, the quantity $B(p)_t$ in (31) which depends on the choise of $p$. Indeed, for $p > 2$, an estimator of $B(p)_t$ is defined by

$$\widehat{B}(p, \Delta_n)_t = \sum_{i=1}^{\lfloor t/\Delta_n \rfloor} |\Delta_i^n X|^p. \tag{33}$$

The following convergences hold from AÃ¯t-Sahalia and Jacod [2]; when $\Delta_n \to 0$,

$$\begin{cases} \text{X is discontinuous} & \Rightarrow & \widehat{B}(p, \Delta_n)_t & \xrightarrow{\mathbb{P}} & B(p)_t \\ \text{X is continuous} & \Rightarrow & \frac{\Delta_n^{1-p/2}}{m_p} \widehat{B}(p, \Delta_n)_t & \xrightarrow{\mathbb{P}} & A(p)_t \end{cases} \tag{34}$$

with

$$m_p = \mathbb{E}\left[|U|^p\right] = \sqrt{\frac{2^p}{2\pi}} \Gamma\left(\frac{p+1}{2}\right)$$

where $U$ is a standard normal random variable and $\Gamma$ the gamma function. In pratice, one can compute $m_p$ by Monte Carlo simulations. More generally we have

$$\begin{aligned} m_{k,p} &= \mathbb{E}\left[|U|^p |U + \sqrt{k-1} V|^p\right] \\ &= \tfrac{2p}{\pi} (k-1)^{p/2} \Gamma\left(\tfrac{p+1}{2}\right)^2 F_{2,1}\left(-\tfrac{p}{2}, \tfrac{p+1}{2}, \tfrac{1}{2}, \tfrac{-1}{k-1}\right) \\ \text{and} & \\ m_{1,\frac{p}{2}} &= m_p \end{aligned}$$

for $U$ and $V$ independent $\mathcal{N}(0,1)$ variables and $F_{2,1}$ is gaussian hypergeometric function.

When $p > 2$, the limit of $\widehat{B}(p, \Delta_n)_t$ does not depend on the sequence $(\Delta_n)_n$ which converges to 0, and it is strictly positive if $X$ has jumps on $[0, t]$. On the other hand, when $X$ is continuous on $[0, t]$, $\widehat{B}(p, \Delta_n)_t$ converges again to a limit independent of $(\Delta_n)_n$, but only after a normalization which depends on $(\Delta_n)_n$.

AÃ¯t-Sahalia and Jacod [2] propose a nonparametric test statistic in the following form for $p > 3$

$$\widehat{S}(p, k, \Delta_n)_t = \frac{\widehat{B}(p, k\Delta_n)_t}{\widehat{B}(p, \Delta_n)_t}. \tag{35}$$

The intuition behind the test statistic $\widehat{S}(p, k, \Delta_n)_t$ is that if there is a jump in the time interval $((i-1)\Delta_n, i\Delta_n]$, then the magnitude of the increment $\Delta_i^n X$ is large and independent of the sampling interval $\Delta_n$, whereas the magnitude of $\Delta_i^n X$ is small and depends on $\Delta_n$ when there is no jump in that interval. A high power of $\Delta_i^n X$ further separates the magnitudes of $|\Delta_i^n X|^p$ in the two expressions of $\widehat{B}(p, k\Delta_n)_t$ and $\widehat{B}(p, \Delta_n)_t$. Since the increments containing jumps are much larger than those that do not, their contribution to the summation dominates all other terms.

Finally, $\widehat{S}(p, k, \Delta_n)_t$ behaves substantially different when the sample path of $X$ on the time interval $[0, t]$ encompasses jumps from the case where jumps are absent. $\widehat{S}(p, k, \Delta_n)_t$ converges in two ways.
For $p > 3$ and $k \geq 2$, when $\Delta_n \to 0$

$$\begin{cases} \widehat{S}(p, k, \Delta_n)_t & \longrightarrow & 1 & \text{if there are jumps,} \\ \widehat{S}(p, k, \Delta_n)_t & \longrightarrow & k^{p/2-1} & \text{if there are no jumps.} \end{cases} \tag{36}$$

This limiting results hold for any Itō-semimartingale of the form of $X$ with no need to estimate the model parameters, the test is then nonparametric.

A statistical test procedure needs convergence in distribution. AÃ¯t-Sahalia and Jacod [2] provide central limit theorem for (12). $\widehat{S}(p, k, \Delta_n)_t$ converges to normal distribution on both the sets $\Omega_t^j$ and $\Omega_t^c$, but with different parameters respectively.
When $\Delta_n \to 0$, we have

$$\begin{cases} \frac{\widehat{S}(p,k,\Delta_n)_t - 1}{\sqrt{\widehat{V}_{n,t}^j}} & \xrightarrow{\mathcal{L}} & \mathcal{N}(0,1) & \text{holds on} & \Omega_t^j \\ \frac{\widehat{S}(p,k,\Delta_n)_t - k^{p/2-1}}{\sqrt{\widehat{V}_{n,t}^c}} & \xrightarrow{\mathcal{L}} & \mathcal{N}(0,1) & \text{holds on} & \Omega_t^c \end{cases} \tag{37}$$

where $\widehat{V}_{n,t}^j$ and $\widehat{V}_{n,t}^c$ are the variances appropriately defined in each case. As the two convergence results hold, either the null hypothesis of no jumps or the null hypothesis of the presence of jumps can be tested. Hence with a significance level $\alpha \in (0, 1)$ with a $\alpha$-quantile $z_\alpha$ of normal random $U$ such that that $\mathbb{P}(U > z_\alpha) = \alpha$, the critical (rejection) region can be computed.

Under the null hypothesis of no jumps, $\widehat{S}(p, k, \Delta_n)_t$ converges towards $k^{p/2-1} > 1$ for an integer $k \geq 2$ and a real number $p > 3$. The critical (rejection) region is given by

$$C_{n,t}^c = \{\widehat{S}(p, k, \Delta_n)_t < c_{n,t}^c\}$$

Figure 1: One year of agricultural futures price: from january 2000 to december 2012

with the boundary

$$c_{n,t}^c = k^{p/2-1} - z_\alpha \sqrt{\widehat{V}_{n,t}^c}. \tag{38}$$

where

$$\begin{cases} \widehat{V}_{n,t}^c = \dfrac{\Delta_n M(p,k)\widehat{A}(2p,\Delta_n)_t}{\widehat{A}(2p,\Delta_n)_t^2} \\ \widehat{A}(p,\Delta_n)_t = \dfrac{\Delta_n^{1-p/2}}{m_p^2} \displaystyle\sum_{i=1}^{\lfloor t/\Delta_n \rfloor} \left|\Delta_n^i X\right|^p \mathbb{1}_{\{|\Delta_n^i X| \leq \vartheta \Delta_n^{\varpi}\}} \\ M(p,k) = \dfrac{k^{p-2}(1+k)m_{2p} + k^{p-2}(k-1)m_p^2 - 2k^{p/2-1}m_{k,p}}{m_p^2}. \end{cases}$$

Under the null hypothesis of presence of jumps, $\widehat{S}(p,k,\Delta_n)_t$ converges towards 1 for an integer $k \geq 2$ and a real number $p > 3$. The critical region is given by

$$C_{n,t}^j = \{\widehat{S}(p,k,\Delta_n)_t > c_{n,t}^j\}$$

with the boundary

$$c_{n,t}^j = 1 + z_\alpha \sqrt{\widehat{V}_{n,t}^j}. \tag{39}$$

where

$$\begin{cases} \widehat{V}_{n,t}^c = \dfrac{\Delta_n(k-1)p^2\widehat{D}(2p-2,\Delta_n)_t}{2\widehat{B}(p,\Delta_n)_t^2} \\ \widehat{D}(p,\Delta_n)_t = \dfrac{1}{k_n\Delta_n} \displaystyle\sum_{i=1}^{\lfloor t/\Delta_n \rfloor} \left|\Delta_n^i X\right|^p \sum_{j\in I_{n,t}(i)} (\Delta_n^j X)^2 \mathbb{1}_{\{|\Delta_n^j X| \leq \vartheta \Delta_n^{\varpi}\}} \end{cases}$$

where $(k_n)_n$ is a sequence of integers satisfying, for $\Delta_n \to 0$, the criteria $k_n \to \infty$, $k_n\Delta_n \to 0$. One may take $k_n = \left\lfloor K/\Delta_n^{1/2} \right\rfloor$ or $k_n = \left\lfloor K/\Delta_n^{1/4} \right\rfloor$, for a constant $K$ to construct the set $I_{n,t}(i)$ (see Le Courtois and Walter [23], p. 54)

$$I_{n,t}(i) = \{j \in \mathbb{N} : j \neq i : 1 \leq j \leq \lfloor t/\Delta_n \rfloor, |i-j| \leq k_n\}.$$

AÃ¯t-Sahalia and Jacod [2] discuss about the choice of parameters $\vartheta$ and $\varpi$ and advice to set $\varpi$ closed to 0.5 (as $\varpi = 0.47$ or $\varpi = 0.48$) and $\vartheta \in (0,1)$ is chosen between 3 and 5 times the "average" value of $\sigma$. The volatility $\sigma$ of the continuous part of the semimartingale can be consistently estimated, in the presence of jumps, by $\left(\int_0^t \sigma_s^2 ds\right)^{1/2}$.

# B Statistical properties

qqplot.pdf

Figure 2: QQ-plot of return versus Standard Normal

`kdensity.pdf`

Figure 3: Kernel density overlayed with histogram

figAutocorrelation.pdf

Figure 4: Autocorrelation functions

figPAF.pdf

Figure 5: Partial Autocorrelation functions

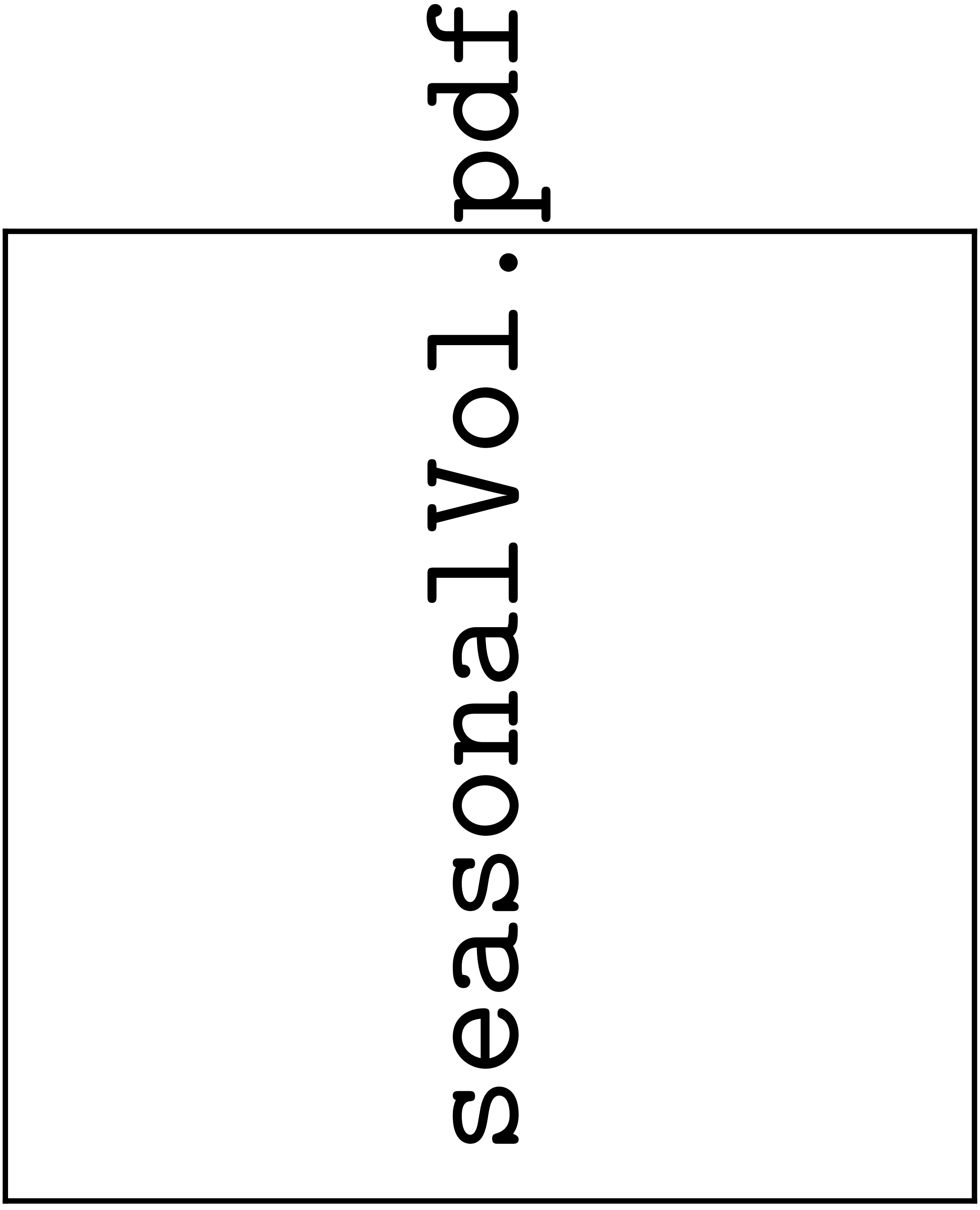


Figure 6: Monthly volatility of futures prices

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