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Abstract

We analyze a dynamic environmental agreement between two regions. We assume that the agreement is jointly profitable, because the effort associated with emission reductions is overcompensated by a cleaner environment in the future. The two regions are asymmetric in two respects: their value of a cleaner environment is different, and they are responsible for the initial environmental problem in different ways. Because the benefits of a cleaner environment cannot be transferred, we propose a mechanism on how to share the efforts of lowering current emissions, satisfying two main properties. The first property is a benefits pay principle: the greater one region's relative benefit from cooperation, the greater must be its relative contribution. The second property is, a polluter pay principle: a region's relative contribution increases with its responsibility. Moreover, the sharing scheme must be time consistent. At any intermediate time, no country can do better by deviating from cooperation.

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1 Introduction

In this paper we propose a distribution scheme which specifies how to share the effort that the cooperative parts in an environmental agreement need to undertake in order to ameliorate an environmental problem. The sharing scheme is designed for a finite time environmental agreement and satisfies three desirable properties: time consistency, a benefits pay principle, and a polluter pay principle.

We start from the assumption that a cooperative agreement to reduce emissions for a limited period in order to fight a common environmental problem is jointly beneficial for two regions. In particular, we analyze a stock pollution problem as, for example, two neighboring countries that share a lake polluted from wastewater discharges. An agreement to reduce discharges by the two countries across a given period might improve the water quality in the future. The two regions bear costs in terms of lower emissions and the subsequent losses in production, within the cooperative period. The benefits from the agreement can be placed at the end of the cooperative period in the form of lower damage from a less polluted environment. And more importantly, we assume that the two countries value the environment differently and hence obtain different gains from cooperation. For example, one might be interested in fishing activities, while the other might assign the lake a recreational value.[1]

[1] The benefits from a cleaner environment under cooperation within the cooperative period are neglected for two reasons. First, since we are dealing with a stock pollution problem, the accumulated emissions within the cooperative period will have lasting effects. We focus on these effects, which can be especially important in the case of non-linear effects on ecosystems or irreversibilities. Second, adding the instantaneous damage from pollution would complicate the exposition without adding more interesting insights to the main fact that we want to capture, that cooperation implies current sacrifices in exchange for a better future environment; and that the two regions differ on the damage they bear from the environmental problem.

Typically, different gains from cooperation are not the only source of asymmetry. In stock pollution problems it is commonly the case that the regions involved are responsible for the current state of the environmental problem in different ways. There is a wide range of literature on global warming which studies the different responsibilities of countries. This discrepancy between the distribution of climate-change damages and responsibilities from past emissions is, for example, highlighted in De Villemeur and Leroux (2011). Several works analyze to what extent industrialized countries are more responsible than developing countries for the current carbon dioxide concentration in the atmosphere. From 1850 to 2010, Ward and Mahowald (2014) estimate that the responsibility for a rise in temperature assigned to annex I countries can be around 58%, (42% for non-annex I). Similarly, from 1850 to 2005, using the Community Earth System Model (CESM), Wei *et al.* (2016) estimate the responsibility for climatic change of developed countries to be between 53%-61%, and for developing countries to be approximately 39%-47%. Similarly, according to Zhang et al (2008), between 1850 and 2004 the G8 countries accounted for 61% of GHG emissions.

The decomposition overtime of the cooperative payoff from an environmental agreement when players are asymmetric, is analyzed, for example, in Fanokoa *et al.* (2011) for a game of pollution control, or in Cabo*et al.* (2006) who also considers trade. Our proposal shares with these works the asymmetry in the benefits stemming from cooperation. Nonetheless, two characteristics separate our analysis from the existing literature. One is the asymmetry in the responsibility for past emissions. The other is the impossibility to redistribute the benefits from cooperation, as they come at the end of cooperation in the form of a cleaner environment. Motivated by these two facts, our approach partially deviates from the standard approach to distribute payoffs in a cooperative dynamic game. As explained in Jørgensen and Zaccour (2001), Zaccour (2008), and Yeung and Petrosyan (2018), and the references therein, a dynamic distribution scheme seeks to distribute the gains from cooperation, between players and across time, following a particular solution concept such as the Shapley value, the egalitarian rule, or the Nash bargaining solution (NBS). We differ from this approach in two respects:

first, we do not distribute gains from cooperation but instead distribute the effort that the agreement imposes on the cooperating agents; and second, we do not borrow a solution concept from the literature, but define a general distribution scheme which satisfies three desirable properties. Nevertheless, the proposed sharing rule is equivalent to an asymmetric NBS.

First, the agreement must be time consistent or internally stable, implying that at any moment within the cooperative period each region prefers to maintain cooperation rather than to deviate from the agreement and play non-cooperatively henceforth. The question of how to distribute the cooperative effort when the benefits emerge once cooperation halts was analyzed in Cabo and Tidball (2018), who proposed a time-consistent imputation distribution procedure (IDP). Based on this procedure, we define here an IDP which satisfies two additional properties in addition to time consistency.

Second, given the asymmetry in the damage caused by pollution, we consider a benefits pay principle a desirable property. Thus, at any time the relative effort that a region contributes to the agreement must be positively correlated to the relative benefit it gets from a less polluted environment.

Third, since regions are differently responsible for past emissions, we believe that the dynamic distribution scheme must satisfy a responsibility or polluter pay principle. This principle which attempts to assign responsibility to a particular region for their past emissions has been discussed by Singer (2004). According to this principle, at any time within the cooperative period the relative effort that a region contributes to the agreement (from this instant on) must be positively correlated to its relative responsibility for all past emissions. That is, the relative effort should be positively correlated to the responsibility from the current state of the environment. Responsibility is defined as the percentage of past emissions a region is responsible for, minus the relative damage it bears from pollution.

The pollution damage indirectly defines the benefits from cooperation. In fact, in the particular case of a damage function multiplicatively separable in a region-specific parameter and a function of pollution, the relative damage from pollution exactly matches the relative benefit from the agreement. Under this hypothesis, the greater the benefit, the greater the damage and hence the lower the responsibility. Thus, the total effect of a higher relative benefit from cooperation on contribution is a positive direct effect (from the BPP) plus a negative indirect effect from a lower responsibility (from the PPP). We characterize the condition under which the net effect is positive and a strong-BPP applies. This multiplicatively separable damage function is considered in a numerical example which illustrates the proposed IDP. This example corroborates the theoretical results obtained.

In Section 2 we present the model which describes cooperation and define the three desired axioms for our distribution procedure. We introduce the IDP and prove that it satisfies the three required properties. The procedure is applied to a numerical example in Section 3. The conclusions are presented in Section 4.

2 The model

We consider two different regions, 1 and 2, whose productive activities require the emission of a flow of pollutants, $E^i(\tau)$. Any other input or technology accumulation process is ignored, and hence the flow of benefits from production is fully determined by current emissions $w^i(E^i(\tau))$, with $(w^i)'(E^i(\tau)) > 0$ and $(w^i)''(E^i(\tau)) < 0$. The emissions in both regions give rise to a pollution stock according to the dynamics equation:[2]

$$\dot{P}(\tau) = E^1(\tau) + E^2(\tau) - \delta P(\tau), \quad P(0) = P_0, \tag{1}$$

with δ the degree of assimilative capacity of the environment, and P_0 the initial pollution stock.

We analyze a cooperative agreement within a finite period $[0, T]$, which would overcome the tragedy of the commons, inducing a reduction in the flow of emissions (and in current benefits), in exchange for a greater joint gain from a cleaner environment. The optimal emissions under the cooperative game come as the solution to the optimization

[2] A superscript in a given variable refers to the specific region. Variables without superscript indicate global quantities for the two regions jointly considered.

problem:

$$\max_{E^i,\, i\in\{1,2\}} \sum_{i=1}^{2} \left\{ \int_0^T w^i(E^i(\tau))e^{-\rho\tau}d\tau - D^i(P(T))e^{-\rho T} \right\}, \tag{2}$$

subject to the pollution stock dynamics in (1). The valuation that each region assigns to the environment is collected in the scrap value $-D^i(P(T))$. The more the pollution stock grows within the period $[0,T]$, the stronger the damage born by region i at time T is: $(D^i)'(P(T)) > 0$. Note that we are dealing with a stock pollution problem, and hence, the accumulated emissions within the planning horizon will generate lasting effects from T onwards.[3] The optimal cooperative emissions and pollution stock at time $t \in [0,T]$ are denoted by $E^i_{\mathrm{C}}(t)$ and $P_{\mathrm{C}}(t)$, respectively.

At any intermediate time $t \in [0,T]$ we want to find out what each region gains from the agreement (in the form of a cleaner environment), and what each region contributes to the agreement (in the form of foregone emissions/production benefits). To that aim, the cooperative solution must be compared against the non-cooperative solution from this moment on. Assuming that cooperation has been maintained up until time t, and the two regions play non-cooperatively from this moment on, each region $i \in \{1,2\}$ solves the maximization problem:

$$\max_{E^i} \int_t^T w^i(E^i(\tau))e^{-\rho(\tau-t)}d\tau - D^i(P(T))e^{-\rho(T-t)}, \tag{3}$$

subject to the stock pollution dynamics in (1). The feedback Nash equilibrium of this non-cooperative game starting at time t, will be denoted by $E^i_{\mathrm{N}}(\tau;t)$ for $\tau \in [t,T]$. Correspondingly, $P_{\mathrm{N}}(\tau;t)$ represents the optimal path of the pollution stock in the Nash equilibrium.

As is common in the literature, we assume that once cooperation is halted, the two regions play non-cooperatively henceforth, at least until time T. Alternatively, one could argue that they could, for example, decide to renegotiate an agreement at any time after t and prior to T. Such a possibility is introduced by Sorger (2006) who proposed an immediate renegotiation when the agreement is broken.[4]

[3] Alternatively, we could have also added the instantaneous damage of pollution while cooperation continues, defining the instantaneous payoff as $w^i(E^i(\tau), P(\tau))$.

[4] And yet, this is not the only option; the new agreement could be renegotiated with a delay, or

It is now possible to define how much region i benefits from the cooperative agreement. The benefit from cooperation from time t onwards for region i is defined as the reduction in the environmental damage from a less polluted environment, associated with lower emissions under cooperation, from t to T:

$$B^i(t) = \left[D^i(P_{\mathrm{N}}(T;t)) - D^i(P_{\mathrm{C}}(T))\right] e^{-\rho(T-t)}, \quad t \in [0,T]. \tag{4}$$

In the non-cooperative solution, optimal emissions in one region are computed by taking into account how one region's emissions increase the future pollution stock, and hence, the region's welfare. In the cooperative solution, the negative effect of pollution on the other region's welfare is also taken into account. In consequence, under cooperation the emissions and the pollution stock are kept lower at any time and, specifically, at the final time. Under cooperation, since both regions emit less, a lower pollution stock will imply positive gains from the prospect of a cleaner future environment: $B^i(t) \geq 0$ for all $i \in \{1,2\}$, $t \in [0,T]$. Provided that the benefits from cooperation come at the end of the cooperative period, and are determined by each region's valuation of the state of the environment, they cannot be redistributed. Therefore, only the costs or each region's contribution will be redistributed.

The main objective of this paper is to define an imputation distribution procedure of the payoffs associated with current emissions under cooperation, in order to satisfy some desirable properties. This distribution scheme is a flow of payoffs, $\pi^i(\tau)$, for any region $i \in \{1,2\}$ and at any time $\tau \in [0,T]$, which must first fulfill a feasibility condition at any time:[5]

$$\pi(\tau) = \sum_{i=1}^{2} \pi^i(\tau) = \sum_{i=1}^{2} w_{\mathrm{C}}^i(\tau) = w_{\mathrm{C}}(\tau), \quad \forall \tau \in [0,T]. \tag{5}$$

According to (5), the sharing rule, π, distributes the instantaneous payoff at each time between the two players. Payoffs can not be borrowed from or lent to the future.

The definition of the IDP will determine each region's contribution to the agreement, in the form of foregone emissions/production benefits. To characterize this contribution

signed and broken many times within this period, or its length could be modified to a longer or shorter period, etc.

[5]We use notation $w_{\mathrm{C}}^i(\tau)$ instead of and $w^i(E_{\mathrm{C}}^i(\tau))$ for conciseness.

we first define the payoff that region i gets from time t onwards if either cooperation is maintained from this time until T, or conversely, if cooperation halts and both players play non-cooperatively therein:[6]

$$W^i_\pi(t) = \int_t^T \pi^i(\tau)e^{-\rho(\tau-t)}d\tau - D^i(P_{\mathrm{C}}(T))e^{-\rho(T-t)}, \qquad (6)$$

$$W^i_{\mathrm{N}}(t) = \int_t^T w^i_{\mathrm{N}}(\tau;t)e^{-\rho(\tau-t)}d\tau - D^i(P_{\mathrm{N}}(T;t))e^{-\rho(T-t)}. \qquad (7)$$

We add a subscript π to denote that we are referring to the payoff to go once the distribution procedure is implemented. In the particular case when no redistribution scheme is implemented, each region gets its cooperative payoff without any side-payment. Hence, the payoff to go for region i at time t would read:

$$W^i_{\mathrm{C}}(t) = \int_t^T w^i_{\mathrm{C}}(\tau)e^{-\rho(\tau-t)}d\tau - D^i(P_{\mathrm{C}}(T))e^{-\rho(T-t)}. \qquad (8)$$

It is now possible to define how much region i contributes to the cooperative agreement from time t onwards under the proposed IDP π:

$$C^i_\pi(t) = \int_t^T \left[w^i_{\mathrm{N}}(\tau;t) - \pi(\tau)\right]e^{-\rho(\tau-t)}d\tau, \quad t \in [0,T] \qquad (9)$$

The benefits from cooperation in (4) are independent of the implemented redistribution scheme. Conversely, as observed above, the distribution scheme determines each regions' contribution $C^i_\pi(t)$. Nevertheless, the following proposition proves that the joint contribution is invariant to the chosen IDP.

Proposition 1 *The joint contribution for the two players is the same for any IDP that satisfies Condition (5). In particular, it is equal to the joint contribution in the cooperative case prior to any redistribution scheme.*

$$C_\pi(t) = C(t), \quad \forall t \in [0,T] \quad \textit{if } \pi(t) \textit{ satisfies } (5). \qquad (10)$$

Proof.

$$C_\pi(t) = \sum_{i=1}^{2} C^i_\pi(t) = \int_t^T \sum_{i=1}^{2}\left[w^i_{\mathrm{N}}(\tau;t) - \pi^i(\tau)\right]e^{-\rho(\tau-t)}d\tau =$$
$$\int_t^T \left[w_{\mathrm{N}}(\tau;t) - \pi(\tau)\right]e^{-\rho(\tau-t)}d\tau = \int_t^T \left[w_{\mathrm{N}}(\tau;t) - w_{\mathrm{C}}(\tau)\right]e^{-\rho(\tau-t)}d\tau = C(t).$$

[6] Like the cooperative case, in the non-cooperative case we use notation $w^i_{\mathrm{N}}(\tau;t)$ instead of $w^i(E^i_{\mathrm{N}}(\tau;t))$ for conciseness.

■

Together with feasibility Condition (5) we also make the assumption of global rationality or the Kaldor-Hicks efficiency, without which the agreement would not be signed. Globally, for the two regions, the payoff to go of maintaining cooperation surpasses the payoff to go in the non-cooperative scenario. Moreover, we assume that the agreement is jointly profitable not only initially but at any intermediate time.

Assumption 1 *The global surplus to go linked to the cooperative solution is initially positive as well as at any ulterior time, that is for all $t \in [0, T)$:*

$$S(t) = \sum_{i=1}^{2} \left(W_C^i(t) - W_N^i(t) \right) = B(t) - C(t) > 0.$$

While global rationality is satisfied by this assumption, we need to define an IDP in such a way that three additional desirable properties are fulfilled: individual rationality at any time t, a benefits pay principle and a polluter pay principle. These properties are defined in the following three axioms.

Axiom 1 (Time consistency) *At any intermediate time t, and for each region i, the payoff to go under the distribution scheme π is not lower than the payoff to go in the non-cooperative scenario:*

$$W_\pi^i(t) \geq W_N^i(t), \quad \forall t \in [0, T], \ \forall i \in \{1, 2\}.$$

Axiom 2 (Benefits pay principle-BPP) *All other things being equal, the greater one region's relative gain from cooperation the greater its relative contribution. For any $t \geq 0$,*

$$\frac{\partial \hat{C}_\pi^i(t)}{\partial \hat{B}^i} > 0, \quad \textit{with} \quad \hat{B}^i(t) = \frac{B^i(t)}{B(t)} \ \textit{and} \ \hat{C}_\pi^i(t) = \frac{C_\pi^i(t)}{C(t)}. \tag{11}$$

A hat denotes the relative value of one region with respect to the total.

Moreover, we would like the distribution scheme to also take into account each region's responsibility for past emissions.

Axiom 3 (Responsibility or polluter's pay principal-PPP) *All other things being equal, the greater one region's net responsibility for the damage caused by past emissions,*[7] R^i*, the greater its relative contribution. For any* $t \geq 0$*,*

$$\frac{\partial \hat{C}^i_\pi(t)}{\partial R^i} > 0. \qquad (12)$$

2.1 A time-consistent IDP

We look for a distribution scheme that satisfies Condition (5), according to which the instantaneous joint payoff for the two regions under the IDP equates to the cooperative joint payoff. Thus, the IDP must determine how to share the instantaneous joint cooperative payoff at any time. Furthermore, this sharing rule must also guarantee Axiom 1 (time consistency): at every time t, each player prefers to follow the cooperative behavior rather than the non-cooperative one, as described in the next definition.

Definition 2 *An IDP* π*, with the corresponding payoff to go* $W^i_\pi(t)$ *in (6), is time consistent if the following condition is satisfied at any time* $t \in [0,T]$*:*

$$W^i_\pi(t) = W^i_N(t) + \phi^i(t)S(t) \qquad \forall t \in [0,T], \qquad (13)$$

with $\phi^i(t)$ *a differentiable function satisfying:*

$$\phi^i(t) \in [0,1], \quad \phi^i(t) + \phi^{-i}(t) = 1, \quad \forall t \in [0,T],\ i \in \{1,2\}. \qquad (14)$$

Under Assumption 1 of a positive surplus to go t, $W^i_\pi(t) \geq W^i_{\mathrm{N}}(t)$ for any time $\forall t \in [0,T]$ and for any region $i \in \{1,2\}$. This definition does not uniquely characterize a time-consistent IDP. A different IDP arises for each differentiable function $\phi^i(t)$ that satisfies Condition (14). Regardless of the chosen function $\phi^i(t)$, the joint payoff to go under the IDP equates to the cooperative payoff to go at any time t: $W_\pi(t) = \sum_i W^i_\pi(t) = \sum_i W^i_{\mathrm{C}}(t) = W_{\mathrm{C}}(t)$, $\forall t \in [0,T]$. Moreover, computing the derivative with respect to t in both sides of this equation one finds that Condition (5) holds at every time.

[7] We will present one definition of net responsibility in Subsection 2.2. However, alternative definitions are possible.

Remark 3 *An alternative way to write Condition (13) is*

$$\phi^i(t) = \frac{W^i_\pi(t) - W^i_N(t)}{B(t) - C(t)} = \frac{B^i(t) - C^i_\pi(t)}{B(t) - C(t)} = \frac{S^i(t)}{S(t)}, \quad \forall t \in [0,T],\ \forall i \in \{d,r\}. \tag{15}$$

Thus, $\phi^i(t)$ is the ratio of region i's surplus to go over the total surplus to go. Hence, it can be interpreted as a measure of the relative net gains for player i, if cooperation is maintained under the IDP π from any time $t \in [0,T]$ onwards.

According to the previous remark, once a differentiable function $\phi^i(t)$ that satisfies Conditions in (14) is chosen, then the net gains that each region obtains from the agreement are fully determined. Therefore, given the function $\phi^i(t)$ we can univocally characterize the flow of current benefits under the IDP.

Proposition 4 *Let $\phi^i(t)$ be a differentiable function satisfying (14). Given this function, there is a unique IDP that satisfies Conditions (5) and (13):*

$$\pi^i(t) = w^i_C(t) + \phi^i(t)IVC(t) - IVC^i(t) - \dot{\phi}^i(t)S(t), \tag{16}$$

with $IVC^i(t)$, the instantaneous value of cooperation at time t, for region i:

$$\begin{aligned} &IVC^i(t) = w^i_C(t) - w^i_N(t;t) + \\ &\int_t^T \dot{w}^i_N(\tau;t)e^{-\rho(\tau-t)}d\tau + \left(SV^i\right)'(P_N(T;t))\dot{P}_N(T;t)e^{-\rho(T-t)}. \end{aligned} \tag{17}$$

and $IVC(t) = \sum_{i=1}^2 IVC^i(t)$.

Proof. See Appendix. ■

At time t, if cooperation has been maintained until then, and if cooperation does not halt at this time, region i would typically get a lower instantaneous payoff linked to lower emissions: $w^i_C(t) - w^i_N(t;t) < 0$, which defines the instantaneous effort to cooperate for region i at time t. However, because cooperative regions emit less at time t, this would allow for higher future emissions for country i if the deviation from cooperation takes place at the instant immediately afterwards (the integral term in Expression (17)). Moreover, lower instantaneous emissions at time t also induce a lower pollution stock at time T and therefore lower environmental damage born by this region from T onwards (last term in expression (17)).

According to the definition of the $IVC^i(t)$, the IDP in (16) grants the i−th player his instantaneous cooperative payoff, plus the gap between the share $\phi^i(t)$ of the joint instantaneous value of cooperation and the i−th player's instantaneous value of cooperation. Thus, if the i−th IVC is larger than its share of the total IVC, this region would transfer part of its instantaneous payoff to its opponent. Furthermore, the i−th instantaneous payoff is reduced at the speed in which his share of the total surplus to go increases.

Moreover, the IDP in (16) satisfies the feasibility Condition (5), which implies that the instantaneous payoff that the IDP distributes between the two players matches the total instantaneous cooperative payoff at every time t. Therefore, $\pi^i(t) - w^i_{\mathrm{C}}(t)$ defines the instantaneous side-payment to region i (if positive), or from region i (if negative).

Any differentiable function $\phi^i(t)$, lying between 0 and 1 and which satisfies the conditions in (14), guarantees time-consistency. Among these functions, we look for the specification(s) which also implies the satisfaction of axioms 2 and 3.

2.2 An IDP satisfying BPP and PPP: axioms 2 and 3

We define region i's net responsibility (or responsibility for shortness) as the damage that this region's accumulated past emissions have caused region $-i$, minus the damage that region $-i$'s past emissions have caused region i. This definition of responsibility takes into account the three mayor factors highlighted in Hayner and Weisbach (2016): who causes the problem and to what extent; what is the size of the harm caused; and to what extent each region has been impacted. Equivalently, responsibility can be defined as the total damaged caused by region i which is not born by this region. Define by r^i all past emissions from region i divided by all past emissions, i.e. the percentage of the initial pollution stock country i is responsible for. Defining the damage born by region i from all previous emissions as the damage associated with the initial pollution stock, $D^i(P_0)$, this region's responsibility would read:

$$R^i = r^i \hat{D}^{-i}(P_0) - r^{-i}\hat{D}^i(P_0), \quad \text{with} \quad \hat{D}^i(P_0) = \frac{D^i(P_0)}{\sum_{i=1}^2 D^i(P_0)}. \tag{18}$$

Or alternatively,

$$R^i = r^i - \widehat{D}^i(P_0). \tag{19}$$

Expression (18) and Expression (19) are equivalent and it is immediately obvious that $R^i = -R^{-i}$. In general, the two regions are both responsible for past emissions although probably at different scales. We will say that region i is responsible (or more responsible) if $R^i > 0$ and not responsible (or less responsible if $R^i < 0$).

Given this definition of responsibility, the next lemma proposes a specification for $\phi^i(t)$, as a function of $\hat{B}^i(t)$ and R^i, for which the IDP π fulfills the desired axioms 2 and 3.

Proposition 5 *Define $\phi^i(t)$ as:*[8]

$$\phi^i(t) = \begin{cases} 0 & \text{if } \; R^i > 0 \;\wedge\; \alpha \geq \alpha_{\max}(t), \\ \hat{B}^i(t) - \alpha \dfrac{C(t)}{S(t)} R^i & \text{if } \; \alpha \in [0, \alpha_{\max}(t)). \\ 1 & \text{if } \; R^i < 0 \;\wedge\; \alpha \geq \alpha_{\max}(t), \end{cases} \tag{20}$$

with

$$\alpha_{\max}(t) = \frac{\hat{B}^j(t)}{R^j} \frac{S(t)}{C(t)}, \quad j = \arg\max_{i \in \{1,2\}} \{R^i\}. \tag{21}$$

Then, the relative contribution of region i reads:

$$\hat{C}^i_\pi(t) = \begin{cases} \hat{B}^i(t) \frac{B(t)}{C(t)} & \text{if } \; R^i > 0 \wedge \alpha \geq \alpha_{\max}(t), \\ \hat{B}^i(t) + \alpha R^i & \text{if } \; \alpha \in [0, \alpha_{\max}(t)), \\ \hat{B}^i(t) \frac{B(t)}{C(t)} - \frac{S(t)}{C(t)} & \text{if } \; R^i < 0 \wedge \alpha \in (0, \alpha_{\max}(t)). \end{cases} \tag{22}$$

And the IDP, π, defined in (16), satisfies axioms 2 and 3 whenever $\alpha \in (0, \alpha_{\max}(t))$.

Proof. The relative contribution in (22) comes from (15) and (20). The interior expression for the relative contribution straightforwardly satisfies axioms 2 and 3. ■

According to (20), the relative net gains of region i are greater the higher this region's relative benefit from the cooperative agreement, $\hat{B}^i(t)$. Furthermore, $\phi^i(t)$ shrinks (resp.

[8] In fact, the interior expression applies for $\alpha \in [0, \alpha_{\max}(t))$, whatever the value of R^i; but also for $R^i = 0$ whatever the value of α. We do not write down this latter case for brevity. This particular case will be commented on the following subsection.

widens) with region i's net responsibility, if positive (resp. negative). In this definition, the responsibility is valued by the effort or contribution which is required to achieve a unit of surplus. Parameter α measures the weight given to responsibility with respect to the relative benefit. If this weight is too large, $\alpha > \alpha_{\max}(t)$, then the region which is more responsible for past emissions gets no share of the total surplus to go, which is fully assigned to the region less responsible. However, if the weight given to responsibility does not fully offset the importance of the relative benefit, then both agents get a positive share of the surplus to go from time t (i.e., an interior expression with $\phi^i(t) \in (0,1)$).

In the interior case, when $\alpha \in [0, \alpha_{\max}(t))$, the relative contribution of region i in (22) is determined by its relative gains from cooperation increased by a fraction α of its net responsibility for past emissions (decreased by this fraction if the region has a negative net responsibility). Thus, the relative contribution is positively correlated with the relative benefit and with the responsibility, and therefore it fulfills Axiom 2 as well as Axiom 3 when $\alpha > 0$.

If the weigh given to responsibility is very large, $\alpha > \alpha_{\max}(t)$ and region i is responsible, then its relative contribution is given by a ratio defined by its relative benefit, scaled up by the benefit per unit of contribution, $B(t)/C(t) > 1$. Conversely, if region i is not responsible, then this ratio is reduced in the total surplus to go per unit of contribution,

$$C_\pi^i(t) = \begin{cases} B^i(t) & \text{if} \quad R^i > 0 \wedge \alpha \geq \alpha_{\max}(t), \\ B^i(t)\dfrac{C(t)}{B(t)} + \alpha R^i C(t) & \text{if} \quad \alpha \in [0, \alpha_{\max}(t)), \\ B^i(t) - S(t) & \text{if} \quad R^i < 0 \wedge \alpha \geq \alpha_{\max}(t). \end{cases} \tag{23}$$

In absolute terms, Expression (23) shows that, for the interior case, at time t, each region contributes an equal share, $C(t)/B(t)$ of its benefit from cooperation. Furthermore, if the region is responsible for past emissions, its contribution rises by a share αR^i of the total contribution (which is subtracted to the non-responsible region). If the weight given to responsibility is too large, $\alpha > \alpha_{\max}$, and if region i is responsible, the contribution of this region from any given time t onwards equals its benefits from cooperation from this moment onwards. This is the maximum amount that this region can contribute compatible with time consistency. Conversely, if the region is not responsible, then its contribution is given by its benefits decreased by the total surplus from this time onwards.

To summarize, the IDP $\pi^i(t)$ defined in Proposition 4, with the proposed specification for $\phi^i(t)$ in (20) is time consistent. Furthermore, whenever $\alpha \in (0, \alpha_{\max}(t))$, it also satisfies the benefits pay principle and the polluter pay principle.

2.3 Additional facts about the proposed IDP, and particular cases

This subsection first explores the links between our proposal and the Nash bargaining solution. It later comments on the characteristics of the proposed IDP for two particular cases: when the responsibility axiom is ignored, and when the damage function is multiplicatively separable.

2.3.1 Comparison of the proposed IDP and the NBS

Proposition 6 *For the proposed IDP in (16), (17) and (20), if the region who is net responsible for past emissions benefits less from the agreement, then it never holds true that* $\phi^i(t) = 1/2$. *The egalitarian rule never arises.*

Proof. If region i is net responsible and benefits less from the agreement ($R^i > 0$ and $\hat{B}^i(t) < 1/2$), then from (20) it is immediately obvious that $\phi^i(t) \in [0, 1/2)$ for any $\alpha \geq 0$. Likewise, if region i is less responsible and benefits more from the agreement ($R^i < 0$ and $\hat{B}^i(t) > 1/2$), then $\phi^i(t) \in (1/2, 1]$ for any $\alpha \geq 0$. ■

Proposition 6 rules out the egalitarian rule. Interestingly, the payoffs to go from our proposed IDP, when players agree to share the surplus to go according to the expression $\phi^i(t)$ in (20), would equivalently arise from the asymmetric Nash bargaining solution in which $\phi^i(t)$ defines region i's bargaining power.

Proposition 7 *The payoffs to go at time* t, $W^i_\pi(t)$, $i \in \{1, 2\}$, *which satisfy Condition (13) are equivalent to the payoffs stemming from an asymmetric Nash bargaining solution* *of the form:*

$$\max_{W_\pi^i(t), i\in\{1,2\}} \prod_{i=1}^{2} \left(W_\pi^i(t) - W_N^i(t)\right)^{\phi^i(t)} \tag{24}$$

$$s.t.: \sum_{i=1}^{2} W_\pi^i(t) = W_C(t) \quad and \quad (14). \tag{25}$$

Proof. Let us remove the time argument. Problem (24)-(25) can be rewritten as:

$$\max_{W_\pi^1} \left(W_\pi^1 - W_\text{N}^1\right)^{\phi^1} \left(W_\text{C} - W_\pi^1 - W_\text{N}^2\right)^{1-\phi^1}.$$

Taking the derivative wrt W_π^1, and if the non-cooperative solution ($W_\pi^i = W_\text{N}^i$) is discarded, the FOC for this problem reads:

$$\left(\frac{W_\text{C} - W_\pi^1 - W_\text{N}^2}{W_\pi^1 - W_\text{N}^1}\right)^{1-\phi^1} \left\{\phi^1 - (1-\phi^1)\frac{W_\pi^1 - W_\text{N}^1}{W_\text{C} - W_\pi^1 - W_\text{N}^2}\right\} = 0.$$

And by equating the second term in brackets to zero, it immediately follows that $W_\pi^1 = W_\text{N}^1 + \phi^1 S(t)$. And therefore $W_\pi^2 = W_\text{N}^2 + \phi^2 S(t)$. ■

2.3.2 Symmetric agents, or no concern for responsibility

In the standard case in which players are symmetric, or the responsibility principle is ignored, the following remark can be obtained.

Remark 8 *From (22) it is straightforward to conclude that, if $\alpha = 0$, or if $R^i = R^{-i} = 0$ (i.e. $r^i = \hat{D}^i(P_0)$), then:*

$$\hat{C}_\pi^i(t) = \hat{B}^i(t), \qquad C_\pi^i(t) = B^i(t)\frac{C(t)}{B(t)}.$$

If all the weight is given to the benefits pay principle ($\alpha = 0$), or both regions are equally responsible ($R^i = 0$), then the relative contribution matches the relative benefit from cooperation for both regions. Each region pays the same percentage of its benefits from cooperation. A distribution procedure which strives for an exact equivalence between relative gains and relative contributions would be uniquely characterized by (16) with function $\phi^i(t) = \hat{B}^i(t)$, that is, disregarding the PPP or responsibility axiom, $\alpha = 0$.

2.3.3 A multiplicatively separable damage function

This item analyzes the particular case in which the damage is multiplicatively separable in a region-specific parameter d^i, and a function of pollution, $D(P)$, identical for the two regions: $D^i(P) = d^i D(P)$. Several particular facts are true under this specification:

i) The relative damage from pollution for region i does not depend on the level reached by the stock of pollution at T. Likewise, the relative gains obtained by this region (if it remains in the agreement from t until T) is independent of the time t.

$$\hat{D}^i(P) = \frac{d^i}{\sum_{j=1}^2 d^j}; \quad \hat{B}^i(t) = \frac{D^i(P_{\mathrm{N}}(T;t)) - D^i(P_{\mathrm{C}}(T))}{\sum_{j=1}^2 (D^j(P_{\mathrm{N}}(T;t)) - D^j(P_{\mathrm{C}}(T)))} = \frac{d^i}{\sum_{j=1}^2 d^j}.$$

In fact, the two time independent expressions coincide, $\hat{B}^i(t) = \hat{D}^i(P) = \hat{D}^i = d^i / \sum_{j=1}^2 d^j$.

ii) In consequence, the net responsibility of region i reads:

$$R^i = r^i - \hat{D}^i(P_0) = r^i - \hat{B}^i(t). \tag{26}$$

Interestingly, in this case the responsibility shows a one-to-one negative relation with the relative benefit from the agreement.

iii) From (21) the upper bound for α in order to have an interior relative contribution in (22) is:

$$\alpha_{\max}(t) = \frac{\hat{B}^j}{R^j}\frac{S(t)}{C(t)} = \frac{d^j}{r^j d^{-j} - r^{-j} d^j}\frac{S(t)}{C(t)}, \quad j = \arg\max_{i \in \{1,2\}} \{R^i\}. \tag{27}$$

iv) From (20), the expression $\phi^i(t)$ easily follows under this specification of the damage function. For the interior case, $\alpha \in [0, \alpha_{\max}(t))$:

$$\phi^i(t) = \hat{B}^i \left(1 + \alpha \frac{C(t)}{S(t)}\right) - \alpha \frac{C(t)}{S(t)} r^i \tag{28}$$

Although the relative damage and the relative benefit are time independent, the contribution required to attain one unit of surplus, $C(t)/S(t)$, does not remain constant, and neither does $\phi^i(t)$. If the effort per unit of surplus increases across time, then the region which is net responsible will see its relative net benefit reduced across time (at the expense of the region which in not responsible).[9] Opposite reasoning would apply if the effort per unit of surplus decreases across time. If, conversely, the contribution per unit of surplus decays with time, then the region responsible will experience an increment in its net benefits.

v) The relative contribution from a given time t onwards is constant in the interior case, where $\alpha \in [0, \alpha_{\max}(t))$.

$$\hat{C}_{\pi}^{i}(t) = \hat{C}_{\pi}^{i} = r^{i} + (1-\alpha)\hat{B}^{i}, \quad \forall t \in (0, T). \tag{29}$$

More interestingly, a higher relative benefit for region i has a twofold effect on this region's relative contribution. A higher relative benefit from the agreement, $\hat{B}^{i}$, directly induces a larger relative contribution, $\hat{C}_{\pi}^{i}$, as stated by the BPP. However, under this particular damage function, from (26), it also implies lower responsibility, which due to the PPP calls for a lower relative contribution. This defines an indirect negative effect of relative benefits on relative contributions. The direct positive effect of relative benefits on relative contributions outweighs the indirect negative effect if $\alpha < 1$. Thus, a strong version of Axiom 2 (strong-BPP) is satisfied:

$$\frac{d\hat{C}_{\pi}^{i}(t)}{d\hat{B}^{i}} > 0, \quad \forall \alpha \in [0, \min(\alpha_{\max}(t), 1)). \tag{30}$$

3 Numerical example

In this section, we describe how the IDP presented in the previous section can be applied to a specific example. Consider two regions who share a polluted environment. Region 1's responsibility exceeds its share of the burden from current pollution and hence $R^{1} = -R^{2} > 0$. Moreover, Region 2 is more severely hit by the environmental problem, or equivalently, it will bear more damage if no agreement on emissions reduction is implemented, $\hat{B}^{2}(t) > \hat{B}^{1}(t)$.

[9] $\phi^{i}(t)$ is negatively related to $C(t)/S(t)$ if $r^{i} > \hat{B}^{i}$ (region i is responsible); and conversely, it is positively related if $r^{i} < \hat{B}^{i}$ (region i is not responsible).

An example could be the global warming problem. The world is divided in two regions; Region 1 encompasses industrialized countries and economies in transition, or annex I countries; and Region 2 encompasses the other non-annex I countries. Region 1 corresponds to those countries more responsible for past emissions, $R^1 > 0$. Correspondingly, the possible losses from the rise in temperatures associated with global warming are higher in Region 2 (warmer countries). Consequently, Region 2 has more to gain if the agreement is signed and maintained: $\hat{B}^1(t) < \hat{D}^2(t)$, for any $t \in [0, T)$.[10]

For tractability, we assume that the instantaneous benefits linked to current emissions are described by a linear quadratic function. The damage from the pollution stock at time T is assumed to be quadratic, and hence of the multiplicatively separable type described in the previous section.

$$w(E^i(\tau)) = a^i E^i(\tau) - \frac{(E^i)^2(\tau)}{2}, \quad D^i(P(T)) = d^i P^2(T). \tag{31}$$

A sketch on how to compute the cooperative and the non-cooperative solutions can be found in the Appendix. We cannot obtain an analytical solution for the non-cooperative case. Hence we rely on numerical simulations considering the following parameter values:

$$r^1 = 0.72, \ (r^2 = 0.28), \ d^1 = 0.1 < d^2 = 0.12, \ a = 1, \ \delta = 0.1, \ P_0 = 1, \ \rho = 0.03. \tag{32}$$

We make the assumption that $a^2 = a^1 = a$ to clearly state that the two countries only differ in their responsibility for past emissions and their valuation of a cleaner environment. Thus, the marginal gains for additional emissions or the cost of abatement are identical in both regions.[11] We further assume that $d^2 > d^1 > 0$, implying that region 2 will be more strongly hit by the environmental problem. At the same time, we consider that the ratio of past emissions is relatively larger for Region 1.

[10] Characterizing the production payoffs associated with current emissions, the future losses from a 2 or 3 degrees rise in temperature, or the responsibility for past emissions from different countries, is an extraordinarily challenging task that we do not undertake. Instead, we illustrate the proposed IDP for a toy model and using some of the imperfect figures offered in the literature. For a meta-analysis on the global impact of Climate Change, see Nordhaus and Moffat (2017)

[11] This assumption can be easily removed, introducing asymmetry in production technologies, $a^1 \neq a^2$. All the parameter values in (32) are chosen for illustration purposes.

Note first that these parameters are compatible with the assumption that Region 1 is more responsible, while it benefits less from the agreement. Because the damage function in (31) is multiplicatively separable, as previously commented, region i's relative benefit from cooperation from time t onwards is constant across time, and indeed is equal to the relative damage at the beginning of the agreement: $\hat{B}^i(t) = \hat{B}^i = \hat{D}^i(P) = d^i / \sum_{j=1}^2 d^j$, with $\hat{B}^1 = 0.\widehat{45} < \hat{B}^2 = 0.\widehat{54}$. Region 1 has a positive net responsibility, $R^1 = 0.1655 > 0$, while conversely, for Region 2, $R^2 = -R^1 < 0$.

Our numerical illustration highlights that, depending on the parameter values, the cooperative agreement either is already time consistent, or an appropriate scheme must be implemented to guarantee time consistency. For the parameters in (32), as Figure 1 (left) shows, the payoffs to go under cooperation surpass the non-cooperative payoffs to go for each region and at any time $t \in [0, T]$. Hence, cooperation is time consistent prior to any redistribution scheme. However, for different parameters, for example rising d_2 up to 0.15, Region 1 would deviate from cooperation unless an appropriate IDP is defined, as shown in Figure 1 (right).[12]

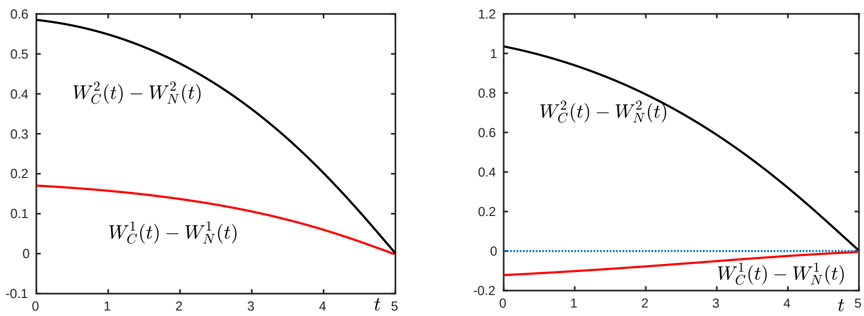

Figure 1: $W_{\mathrm{C}}^i(t) - W_{\mathrm{N}}^i(t)$ for $d_2 = 0.12$ (left) and $d_2 = 0.15$ (right).

Next we analyze to what extent the weight given to responsibility, α, influences each region's share of the surplus to go, each region's contribution, and each region's payoff under non-cooperation, as well as in the cooperative game with and without the proposed redistribution scheme. We present the results for different values of α running from 0 to 1. As shown in Expression (22) the responsibility Axiom 3 applies for $\alpha > 0$.

[12] We maintain parameters in (32) to better illustrate the effect of α on our results, which is one of the main interests of this section.

Likewise, $\alpha < 1$ is a necessary condition for the strong-BPP in (30).

First, notice that, in this example the effort (in terms of total contribution) per unit of surplus, $C(t)/S(t)$, increases across time; or equivalently, the benefits per unit of contribution, $B(t)/C(t)$, decreases across time.[13] As this ratio increases, the share of the total surplus to go allocated to the more-responsible region, $\phi^1(t)$, shrinks; while the share allocated to the less-responsible region, $\phi^2(t)$, enlarges. This is shown in Figure 2 (right) which depicts the evolution of $\phi^i(t,\alpha)$ as time runs from 0 to 5 for different values of α (in this illustrative section we add an α argument to those functions which depend on this parameter).

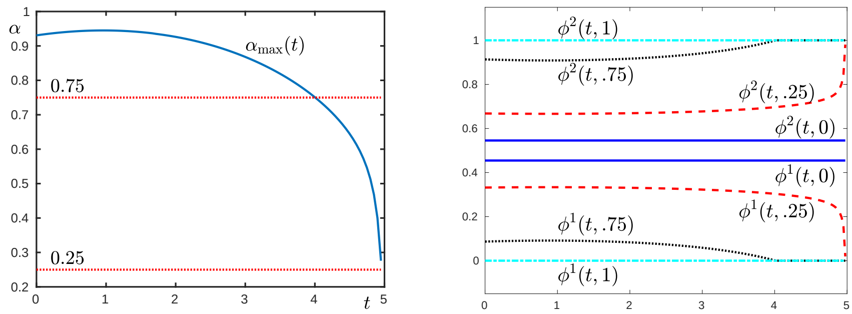

Figure 2: $\alpha_{\max}(t)$ (left); and $\phi(t,\alpha)$ for $\alpha = 0, 0.25, 0.75, 1$ (right).

For (27), and since $C(t)/S(t)$ increases across time, then $\alpha_{\max}(t)$ is a decreasing function of time as shown in Figure 2 (left).

If no weight is given to responsibility, $\alpha = 0$, $\phi^i(t)$ equals the constant $\hat{B}^i \in \{0.4\widehat{5}, 0.5\widehat{4}\}$. If little weight is given to responsibility, $\alpha = 0.25$, $\phi^i(t)$ is the interior solution in (20) at any time and therefore, $\phi^1(t)$ decreases and $\phi^2(t)$ increases across time as $C(t)/S(t)$ falls down. However, if $\alpha = 0.75$ it surpasses $\alpha_{\max}(t)$ before the end of the cooperative period (about $t = 4$). At this time $\phi^1(t)$ and $\phi^2(t)$ become 0 and 1 respectively, and remain at these values henceforth. In this last subinterval, the entire surplus to go from cooperation goes to Region 2, and hence, Region 1 is indifferent towards cooperating or defecting. Finally, if $\alpha = 1$ it surpasses $\alpha_{\max}(t)$ from the very beginning and $\phi^1(t,1) = 0$ at any time $t \in [0,5]$. The responsibility principle is so strong that the IDP allocates all

[13]The inverse relation between $C(t)/S(t)$ and $B(t)/C(t)$ becomes immediately clear since $S(t) = B(t) - C(t)$.

the surplus to the less-responsible Region 2 right from the beginning of cooperation.

As shown in Figure 2 (right), the greater the weight given to responsibility, α, the lower the share of the total surplus to go for the more-responsible Region 1 and the greater the share to the less-responsible Region 2. For Region 1, since $R^1 > 0$ then $\phi^1(t,\alpha) \leq \phi^1(t,0) = \hat{D}^1 = 0.\widehat{45} < 1/2$. Similarly, for Region 2, $R^2 < 0$ and then $\phi^2(t,\alpha) \geq \phi^2(t,0) = \hat{D}^2 = 0.\widehat{54} > 1/2$. Thus, regardless of the value of α, region 1 gets less than 1/2 of the surplus and Region 2 gets more than 1/2 of the surplus. This illustrates the result in Proposition 6 according to which the egalitarian rule cannot arise under this setting, provided that one region is more responsible for and, at the same time, less strongly hit by the environmental problem.

The effect of a greater weight given to responsibility can be equally observed in each region's relative contribution, $\hat{C}^i_\pi(t,\alpha)$. As α increases, the relative contribution of the more-responsible region also rises, while the contributions of the less-responsible region decays (see Figure 3). The relative contribution is constant across time in the interior, $\alpha \in [0, \alpha_{\max}(t))$. However, for $\alpha \geq \alpha_{\max}(t)$ the contribution of the more-responsible Region 1 is defined by a share $B(t)/C(t)$ of its relative benefit $\hat{B}^1$. Because in the example the benefit per unit of contribution decreases across time, so too does the relative contribution of Region 1. And correspondingly, the relative contribution of the less-responsible Region 2 increases.

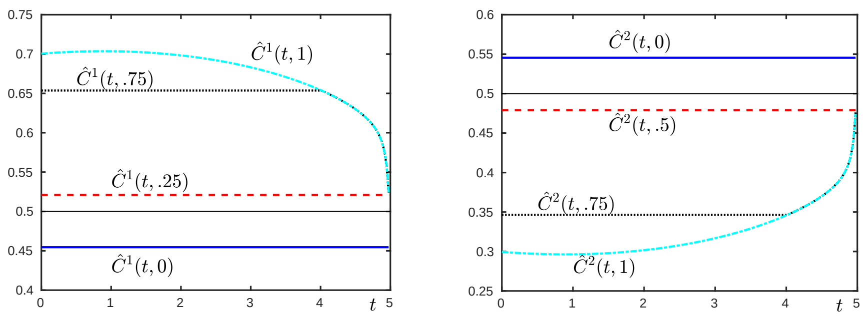

Figure 3: $\hat{C}^i_\pi(t,\alpha)$ for $\alpha = 0, 0.25, 0.75, 1$.

Finally, we compare the payoffs in the non-cooperative case with the cooperative payoffs before and after the implementation of the IDP π. This comparison can be observed in Figure 4, which depicts the payoffs to go from any given time t onwards, in the non cooperative case, $W^i_{\mathrm{N}}(t)$, and in the cooperative case without any redistribution scheme, $W^i_{\mathrm{C}}(t)$, or once the IDP is implemented, $W^i_{\pi}(t,\alpha)$ (in this latter case we distinguish two extremes: $\alpha = 0$ and $\alpha = 1$). The cooperative payoff to go exceeds the non-cooperative payoff to go for each region and at any time, $W^i_{\mathrm{C}}(t) > W^i_{\mathrm{N}}(t)$, for any $i \in \{1,2\}$ and any $t \in [0,T)$. If the IDP is implemented disregarding the responsibility, $\alpha = 0$, the payoff increases for the more-responsible region and decreases for the less responsible region. If we move to the other extreme compatible with the strong-BPP in this example, $\alpha = 1$, the responsible region sees its payoff reduced under non-cooperation. By contrast, the non-responsible region gets a higher payoff to go than before the implementation of the IDP. At any time t, the gap $W^i_{\pi}(t,\alpha) - W^i_{\mathrm{C}}(t,\alpha)$ defines the side-payment to go (the total side-payment from this time onwards) that region i would get from region $-i$. If negative, the side-payment would conversely flow from i to $-i$. Thus a side-payment flows towards the more responsible region for small α and vice versa. Figure 4 clearly states that, for any intermediate α between 0 and 1, the IDP makes the agreement time consistent.

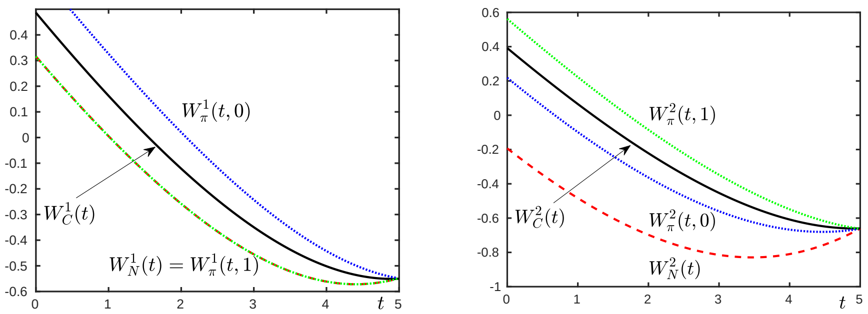

Figure 4: $W^i_{\mathrm{C}}(t)$, $W^i_{\mathrm{N}}(t)$, $W^i_{\pi}(t,\alpha)$ for $\alpha = 0, 1$.

At any time t, the payoffs in Figure 4 encompass the discounted flow of benefits from emissions from this time on, $w^i(E^i(\tau))$, $\tau \in [t,T]$, as well as the damage from pollution placed at T, $D^i(P(T))$. A similar analysis can be made to compare the instantaneous benefits at a specific instant of time t. The effect of α on instantaneous payoffs can be observed in Figure 5. The instantaneous payoff is the greatest with no cooperation (the red-dashed line), when the two regions make no effort to reduce emissions. Cooperation comes with the associated cost of lower emissions across the whole cooperative period, which defines each region's contribution. Cooperation is assumed profitable implying a strong reduction in environmental damage from T onwards which, jointly for the two regions, overcomes the aggregate contribution, $S(t) > 0$. The instantaneous cooperative payoff, without any side-payment, is depicted by the solid black line. An IDP that disregards responsibility, $\alpha = 0$, would imply a lower effort for Region 1 and a higher effort for Region 2, than a cooperative agreement without any redistribution scheme. At each time t, when $\alpha = 0$, the gap $\pi^1(t,0) - w^1_{\mathrm{C}}(t)$ defines an instantaneous payoff transfers from Region 2 to Region 1. By contrast, when the responsibility principle is strong $\alpha = 1$, then the situation is reversed. Region 1 has to increase its effort while Region 2 reduces its contribution. The instantaneous transfer $\pi^2(t,1) - w^2_{\mathrm{C}}(t)$ flows from Region 1 to Region 2.

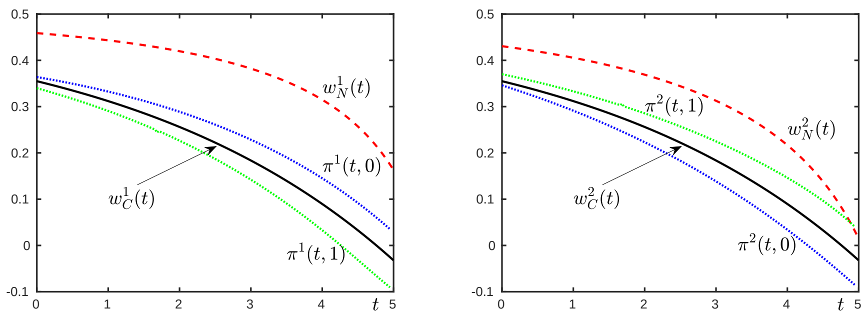

Figure 5: $w^i_{\mathrm{C}}(t)$, $w^i_{\mathrm{N}}(t)$, $\pi^i(t,0)$, $\pi^i(t,1)$.

Finally, in Figure 6, we illustrate the strong-BPP defined in Expression (30). For $\alpha = 0.75$ we compute the relative contribution of Region 1 for $d^1 = 0.1$ and for $d^1 = 0.11$. A rise in d^1 implies an increment in Region 1's relative benefit from $\hat{B}^1 = 0.\widehat{45}$ to 0.4783. This implies a reduction in this region's responsibility, from $R^1 = 0.2655$ to 0.2417. A higher d^1 also increases $\alpha_{\max}(t)$ as displayed in Figure 6 (left) by the upward shifts from the blue-solid line to the red-dashed line. Whenever α remains below $\alpha_{\max}(t)$ for $d^1 = 0.1$ and $d^1 = 0.11$ (the relative contribution is given in (29)), the total effect of a rise in d^1 is a constant increment[14] from 0.6536 to 0.6596. In the example, we observe that the strong-BPP holds true not only for the interior case, but also after the instant at which α surpasses $\alpha_{\max}(t)$ and the solution is no longer at the interior ($\phi^1(t) = 0$ and

[14]This increment corresponds to $(1-\alpha)\Delta\hat{B}^1 = 0.25(0.4783 - 0.\widehat{45}) = 0.006$.

$\phi^2(t) = 1)$.

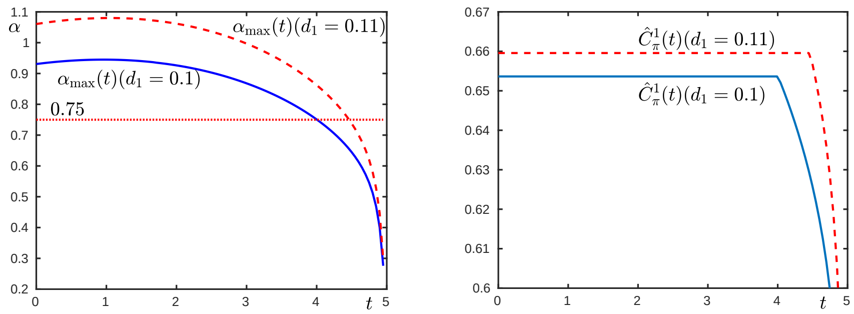

Figure 6: $\alpha_{\max}(t)$ for $d_1 = 0.1, 0.11$ (left); and $\hat{C}^i_\pi(t, 0.75)$ for $d_1 = 0.1, 0.11$ (right).

4 Conclusions

We analyze a finite time cooperative agreement between two regions in order to carry out abatement activities in a stock pollution problem. We propose an imputation distribution procedure which defines how to share the efforts in the form of emission reductions. This IDP is designed considering an asymmetric setting. The two parts differently value the environment and, in consequence, they are differently benefited by the agreement. Moreover, they are differently responsible for the accumulated emissions prior to the signature of the cooperative agreement.

At any time while the two parts cooperate, the proposed sharing mechanism splits the total ongoing surplus from cooperation between the two regions. This sharing rule guarantees the time-consistency of the cooperative solution: each region prefers to remain in the agreement rather than to deviate to a non-cooperative mode of play from this time onwards. This allocation rule is defined so that two additional desirable properties are satisfied. First, a benefits pay principle: *ceteris paribus,* the more one region benefits from the agreement, the higher its relative contribution. Second, a polluter pay principle: *ceteris paribus,* the more responsible a region is for past emissions, the greater its relative contribution.

The proposed sharing scheme is not based on a specific cooperative solution concept (as is common in the literature). We rather seek a distribution scheme which jointly satisfies time consistency, the BPP, and the PPP, and we come up with a family of sharing rules which fulfill these properties. Thus, the IDP is undetermined, and depends on the weight given to each region's relative benefit from cooperation versus its responsibility for past emissions. Whatever this weight, we prove that the egalitarian rule or the symmetric NBS never fulfills the three desired properties. Interestingly, our proposed IDP is equivalent to an asymmetric NBS where the bargaining power was defined by the same function which in our distribution scheme divides the total surplus to go between the two regions.

In the particular case when the damage function is multiplicative in a region-specific parameter and is a common function of the pollution stock, it easily follows an equivalence between one region's relative damage from pollution and its relative benefit from cooperation. Thus, a higher relative benefit from the agreement corresponds to a higher relative damage from pollution and, in consequence, a lower net responsibility. Under these circumstances we observe that a higher relative benefit directly induces a higher relative contribution (from the BPP). Moreover, because it also means lower responsibility, it indirectly induces a lower contribution (from the PPP). If the sharing rule does not weigh responsibility too high then the direct positive effect outweighs the indirect negative one. In consequence, the total effect of a higher relative benefit on the relative contribution is positive, which defines a strong BPP.

The proposed IDP is applied for a particular example, considering quadratic damage from pollution and a linear-quadratic flow of profits associated with emissions. This numerical example serves to illustrate previous findings. In particular, it analyzes the role played by the weight given to responsibility. As this weight gets bigger, the share of the surplus assigned to the more-responsible region shrinks and even vanishes. Correspondingly, its relative contribution increases. This weight determines whether a side-payment flows from the less to the more-responsible region (if small) or vice versa (if large). The numerical example also illustrates that if there is a rise in one region's relative damage from pollution (i.e. its relative benefit from the agreement) then there is also a rise in its relative contribution. This is true only if the direct positive effect is considered

(BPP), and also if the associated reduction in responsibility is taken into consideration (strong-BPP).

For further research, it is important to notice that the proposed sharing rule is not unique for two reasons. First, we have presented a particular functional form to characterize how to share the surplus to go between the two regions. And second, this family of sharing rules is parameterized by the undetermined weight given to responsibility. More research should be done in order to reduce this multiplicity of sharing rules. We also think that another interesting line of research would be to define responsibility as a function of time.

Appendix

Proof proposition 4

Computing the time derivatives in (6) and (13) we get:[15]

$$\dot{W}^i_\pi = -\pi^i + \rho W^i_\pi, \quad \dot{W}^i_\pi = \dot{W}^i_N + \dot{\phi}^i S + \phi^i \dot{S}.$$

And computing the time derivatives in (8) and (7)

$$\dot{W}^i_{\text{C}} = -w^i_{\text{C}} + \rho W^i_{\text{C}},$$

$$\dot{W}^i_{\text{N}} = -w^i_{\text{N}} + \rho W^i_{\text{N}} + \int_t^T \dot{w}^i_{\text{N}}(\tau;t)e^{-\rho(\tau-t)}d\tau + \left(SV^i\right)' (P_{\text{N}}(T;t))\dot{P}_{\text{N}}(T;t)e^{-\rho(T-t)}.$$

We call

$$I^i_{\text{N}}(t) = \int_t^T \dot{w}^i_{\text{N}}(\tau;t)e^{-\rho(\tau-t)}d\tau + \left(SV^i\right)' (P_{\text{N}}(T;t))\dot{P}_{\text{N}}(T;t)e^{-\rho(T-t)}.$$

Using these equations we get

$$-\pi^i + \rho W^i_\pi = -w^i_{\text{N}} + \rho W^i_{\text{N}} + I^i_{\text{N}} + \dot{\phi}^i S + \phi^i \dot{S} = -w^i_{\text{N}} + \rho(W^i_\pi - \phi^i S) + I^i_{\text{N}} + \dot{\phi}^i S + \phi^i \dot{S},$$

then using that $\dot{S} = \dot{W}^i_{\text{C}} + \dot{W}^{-i}_{\text{C}} - \dot{W}^i_{\text{N}} - \dot{W}^{-i}_{\text{N}}$,

$$\pi^i = w^i_{\text{N}} + \rho\phi^i S - I^i_{\text{N}} - \dot{\phi}^i S - \phi^i \dot{S} - \phi_i[\rho S + w^i_{\text{N}} - w^i_{\text{C}} - I^i_{\text{N}} + w^{-i}_{\text{N}} - w^{-i}_{\text{C}} - I^{-i}_{\text{N}}]$$

Calling

$$IVC^i(t) = w^i_{\text{C}}(t) - w^i_{\text{N}}(t) + \int_t^T \dot{w}^i_{\text{N}}(\tau;t)e^{-\rho(\tau-t)}d\tau + \left(SV^i\right)' (P_{\text{N}}(T;t))\dot{P}_{\text{N}}(T;t)e^{-\rho(T-t)},$$

and $IVC(t) = \sum_{i=1}^2 IVC^i(t)$, we obtain the result. Moreover:

$$\pi^i + \pi^{-i} = w^i_{\text{C}} + w^{-i}_{\text{C}} - IVC - (\dot{\phi}^i + \dot{\phi}^{-i})S + (\phi^i + \phi^{-i})IVC = w^i_{\text{C}} + w^{-i}_{\text{C}}.$$

[15] An upper dot refers to the derivative wrt t. For shortness and clarity we remove the time arguments in this proof when no confusion can arise.

Cooperative and non-cooperative solutions to the L-Q differential game in Section 3

The solution to the cooperative problem (2) subject to (1) must satisfy the Hamilton-Jacobi-Bellman equation:

$$\rho V^C(P,t) + \frac{\partial V^C(P,t)}{\partial t} = \max_{E^1,E^2}\left\{w^1(E^1) + w^2(E^2) + \frac{\partial V^C(P,t)}{\partial P}(E^1 + E^2 - \delta P)\right\},$$
$$\text{s.t.: } V(P(T),T) = -(d_1 + d_2)P^2(T).$$

By conjecturing a linear quadratic value function, $V^C(P,t) = v_2^C(t)P^2 + v_1^C(t)P + v_0^C(t)$, and taking into account the functional forms in (31), one gets the following system of 5 Riccati differential equations.

$$\rho v_2^C(t) - \dot{v}_2(t) = -2(\delta - 2v_2^C(t))v_2^C(t),$$
$$\rho v_1^C(t) - \dot{v}_1(t) = -\delta v_1^C(t) + 2(a_1 + a_2 + 2v_1^C(t))v_2^C(t),$$
$$\rho v_0^C(t) - \dot{v}_0(t) = 1/2(a1^2 + a2^2 + 2v_1^C(t)(a1 + a2 + v_1^C(t))),$$
$$\dot{P}(t) = a_1 + a_2 - P(t)\delta + 2v_1^C(t) + 4P(t)v_2^C(t),$$
$$P(0) = P_0, \quad v_2^C(T) = -(d1 + d2), v_1^C(T) = v_0^C(T) = 0.$$

The optimal expressions for $v_2^C(t)$ $v_1^C(t)$, $P^C(t)$ and $E_i^C(t)$ can be analytically computed from this system. We do not present them here for conciseness and because they do not add too much insight. They are available from the authors upon request.

Similarly, the non-cooperative optimization problem (3) subject to (1) must satisfy the Hamilton-Jacobi-Bellman equations:

$$\rho V^{1N}(P,t) + \frac{\partial V^{1N}(P,t)}{\partial t} = \max_{E^1}\left\{w^1(E^1) + \frac{\partial V^{1N}(P,t)}{\partial P}(E^1 + E^2 - \delta P)\right\},$$
$$\rho V^{2N}(P,t) + \frac{\partial V^{2N}(P,t)}{\partial t} = \max_{E^2}\left\{w^2(E^2) + \frac{\partial V^{2N}(P,t)}{\partial P}(E^1 + E^2 - \delta P)\right\},$$
$$\text{s.t.: } V^{1N}(P(T),T) = -d_1P^2(T),\ V^{2N}(P(T),T) = -d_2P^2(T).$$

By again conjecturing linear quadratic value functions, $V^{iN}(P,t) = v_2^{iN}(t)P^2 + v_1^{iN}(t)P + v_0^{iN}(t)$, for region $i \in \{1,2\}$, and taking into account the functional forms in (31), the following system of 7 Riccati differential equations must hold.

$$
\begin{aligned}
&\rho v_2^{1N}(t) - \dot{v}_2^{1N}(t) = 2v_2^{1N}(t)(-\delta + v_2^{1N}(t) + 2v_2^{2N}(t)),\\
&\rho v_2^{2N}(t) - \dot{v}_2^{2N}(t) = 2v_2^{2N}(t)(-\delta + 2v_2^{1N}(t) + v_2^{2N}(t)),\\
&\rho v_1^{1N}(t) - \dot{v}_1^{1N}(t) = 2v_2^{1N}(t)(a_1 + a_2 + v_1^{2N}(t)) + v_1^{1N}(t)(-\delta + 2v_2^{1N}(t) + 2v_2^{2N}(t)),\\
&\rho v_1^{2N}(t) - \dot{v}_1^{2N}(t) = -(\delta - 2v_2^{1N}(t))v_1^{2N}(t) + 2(a_1 + a_2 + v_1^{1N}(t) + v_1^{2N}(t))v_2^{2N}(t),\\
&\rho v_0^{1N}(t) - \dot{v}_0^{1N}(t) = \frac{1}{2}\left(a_1^2 + (v_1^{1N}(t))^2 + 2v_1^{1N}(t)(a_1 + a_2 + v_1^{2N}(t))\right),\\
&\rho v_0^{2N}(t) - \dot{v}_0^{2N}(t) = \frac{1}{2}\left(a_2^2 + 2(a_1 + a_2 + v_1^{1N}(t))v_1^{2N}(t) + (v_1^{2N}(t))^2\right),\\
&\dot{P}(t) = a_1 + a_2 - P(t)\delta + v_1^{1N}(t) + 2Pv_2^{1N}(t) + v_1^{2N}(t) + 2Pv_2^{2N}(t),\\
&\text{s.t.: } P(0) = P_0,\ v_2^{1N}(T) = -d_1,\ v_2^{2N}(T) = -d_2,\ v_1^{1N}(T) = v_1^{2N}(T) = v_0^{1N}(T) = v_0^{2N}(T) = 0.
\end{aligned}
$$

The solution to this system cannot be analytically computed.

References

[1] Cabo, F. and Tidball, M. (2017), Promotion of cooperation when benefits come in the future: A water transfer case. *Resource and Energy Economics*, 47, pp. 56-71

[2] Cabo, F., Escudero, E. and Martín-Herrán, G. (2006), A time-consistent agreement in an interregional differential game on pollution and trade, *International Game Theory Review*, 8, pp. 369-393. https://doi.org/10.1142/S0219198906000977

[3] De Villemeur, E. B. and Leroux, J. (2011), Sharing the cost of Global Warming. *The Scandinavian Journal of Economics*, 113, pp. 758-83. www.jstor.org/stable/41407744.

[4] Fanokoa, P. S., Telahigue, I. and Zaccour, G. (2011), Buying cooperation in an asymmetric environmental differential game. *Journal of Economic Dynamics and Control*, 35, pp. 935-946. DOI: 10.1016/j.jedc.2010.11.008

[5] Jørgensen, S. and Zaccour, G. (2001), Time consistent side payments in a dynamic game of downstream pollution. *Journal of Economic Dynamics and Control*, 25, pp. 1973-1987. https://doi.org/10.1016/S0165-1889(00)00013-0.

[6] Hayner, M. and Weisbach, D. (2016), Two theories of responsibility for past emissions. *Midwest Studies in Philosophy*, 40, pp.96-113. doi:10.1111/misp.12049

[7] Nordhaus, W. D. and Moffat, A. (2017), A Survey of Global Impacts of Climate Change: Replication, Survey Methods, and a Statistical Analysis. *NBER Working Paper*, No. 23646

[8] Singer, P. (2004), One World: The Ethics of Globalization. New Haven, CT: Yale University Press.

[9] Sorger, G. (2006), Recursive bargaining over a productive asset. *Journal of Economic Dynamics and Control*, 30, pp. 2637-2659

[10] Ward, D. S. and Mahowald, N. M. (2014), Contributions of developed and developing countries to global climate forcing and surface temperature change. *Environmental Research Letters*, 9, 074008.

[11] Wei, T., Dong, W., Yan, Q., Chou, J., Yang, Z. and Tian, T. (2016), Developed and developing world contributions to climate system change based on carbon dioxide, methane and nitrous oxide emissions. *Advances in Atmospheric Sciences*, 33, pp. 632-643. https://doi.org/10.1007/s00376-015-5141-4

[12] Yeung, D.W. and Petrosyan, L. (2018), Dynamic Shapley Value and Dynamic Nash Bargaining. Nova Science, NewYork.

[13] Zaccour, G. (2008), Time consistency in cooperative Differential Games: A tutorial. *INFOR Information Systems and Operational Research*, 46, pp. 81-92 doi: 10.3138/infor.46.1.81

[14] Zhang, Z. Q., J. S. Qu, and J. J. Zeng, (2008), A quantitative comparison and analytical study on the assessment indicators of greenhouse gases emissions. *Journal of Geographical Sciences*, 18, p. 387-399. doi: 10.1007/s11442-008-0387-8

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