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Abstract

In static framework, many hedging strategies can be settled following the various hedge ratios that have been developed in the literature. However, it is difficult to choose among them the best the appropriate strategy according the to preference or economic behavior of the decision-maker such as prudence and temperance. This is so even with the hedging effectiveness measure. After introducing a hedging ratio that take into account the prudence and temperance of the decision maker, we propose a ranking based approach to measure the effectiveness using L-moment to classify hedge portfolios, hence hedge ratios, with regard to their performance. Moreover, we deal with the hedging issue in presence of quantity and rollover risks and derive an optimal strategy that depends upon the basis and insurance contract. Such hedging issue includes the relevant risks encountered in practice and we relate how insurance contract, specially designed for production risk could affect the futures hedge. The application on futures prices data at hands shows that taking into account quantity and rollover risks leads to better hedging strategy based on the L-performance effectiveness measure.

Key-words: Risk Management, Futures Markets, Commodities, Risk Aversion

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1 Introduction

Commodity prices rely on the production of their underlying as well as on the factors related to their economy rationales such as calendar seasons or crop years, consumption and policies, supply and demand balance, inventories...

Commodity prices mainly incur risks in both market and production. Indeed, the major part of producer's revenue consists of their crop and any adverse price move will affect their incomes. On one hand, the globalization of commodity markets provides financial derivatives like futures, forwards or options to hedge against these risks by shifting them to investors that are looking speculation opportunities. On the other hand, commodities can be also stored to avoid disruptions coming from shortage that generates cost of carry due to quality deterioration along with storage period. In agricultural markets, a way to avoid these carry costs is to enter in financial markets with derivatives which values are determined, in some way by the prices of physical goods. Thus, the holding of commodities in inventories for facing eventual scarcity episodes in the future contributes to the rationale of the relationship between spot price and the futures price. Arguably, the impact of price variability on the real economy is greatest in the commodity economy. Indeed, the variations of commodity prices relate to every economic entity; from individuals, to organizations, to the economy. So, the risk management in the commodity economy is of great importance. Individuals need to manage these risks in order to cover their incomes, firms to protect their bottom lines and competitiveness, and the economy to protect its macroeconomic stability. Specifically, agricultural commodities are of concern since they are natural resources that are consumed as basic necessities for human diet. They are also used in a number of other applications as well. For instance, corn is used in everything from artificial sweeteners to fuel sources to papers and containers.

Futures markets are risk management and also price discovery institutions.1 In futures markets, the competing expectations of market participants interact to form the "price discovery mechanism" that will reflect a broad range of information about upcoming market conditions. Specifically, futures are mainly used as hedging instruments against the exposition in cash positions, but since they do not equate to direct exposure of actual commodity prices; they are bets on the expected future spot prices. For instance, a wheat producer who plants a crop is betting that the price of wheat will not drop so low that he would have been better off not to have planted the crop at all. This bet is inherent to the farming business, but the farmer may prefer not to make it. Hence, he can hedge the bet by selling a wheat futures contract. Apart from price risk management, there are a lot of positive externalities associated with hedging. Recall that commodity trading takes place with standardization in sizes as well as in qualities in order to improve efficiency for their extractions, distributions and consumption processes. Then, the hedging and price discovery functions of futures markets enhance the efficiency of production, storage and marketing operations. Hedging also ensures continuity of cash flows in that it insulates the producer from volatile price movements, and will thereby guarantee uninterrupted and stable revenue streams by bringing some certainty in the production process; that is certainty in production planning at a guaranteed minimum prices by using commodity futures to hedge.

The establishment of public commodity markets in the 19^{th} century has improved standardization, transparency and efficiency, as well as hedging for physical good prices. Futures markets allow to transfer price risk from hedgers to speculators (Keynes [27]) and they are used in short position of the long position that constitute the physical good. For instance, such a hedger is a farmer who plants a wheat crop and will incur the risk of losing money if the price of wheat falls before harvest or if part of the production perishes because of bad weather. When futures markets exists, the farmer can reduce the risk of loosing the revenue of his wheat crop by taking a short in futures market. Then, he is guaranteed to receive at maturity of futures contract, a predetermined price. Meanwhile, an appropriate and reliable strategy for optimal position in futures contracts that should sufficiently reduce price and quantity risks as much as possible has to be decided before planting.

This decision making problem is also equivalent to derive a portfolio strategy with which the hedger holds, from planting time to harvest time, a non-traded asset with a significant portion of his wealth in production income. A large body of literature on futures markets has investigated the problem of hedging strategies using various techniques to derive the optimal hedge portfolio. Most of these techniques either minimize a risk function or maximize an expected utility function of wealth. The main difference between the two approach is that minimizing a risk function results in pure hedge while maximizing will include a speculative component according to aversion to risk. In static framework, decision making problem relies on two points in time for objective function optimization. The risk functions that are minimized include variance, semi-variance for downside risk which is generalized to lower partial moments (Chen et al. [9], Sadefo Kamdem[17] and Lien and Tse [34]). Examples of utility functions in expected utility maximization approach are mean-variance, mean-Gini (Shalit and Yitzhaki [40]). As illustration for agricultural markets, Rolfo [39] had used mean variance technique to derive optimal hedge ratio under price risk and output risk for exporter countries. Besides, other methods that rely on stochastic dominance concept have been investigated.

Hedging strategies strongly depend upon their hedge ratio approach and the criteria to choose an appropriate strategy may be misleading. According to Chen et al. [10], there is no single optimal hedge ratio that is distinctly superior to the others unless an appropriate criterion is defined. A well known performance measure for hedge ratio is by Ederington [14]. Sharpe-type measure of Howard and D'Antonio [23] and certainty equivalent measure. All of these hedging effectiveness measures are misleading in that they are downward biased which leads to under-reported hedging strategy. Using L-moment tool, we provide hedging performance measure that allows to rank different hedging strategies. Furthermore, if the investment horizon is too long, the futures contract matures earlier and the hedger will incur the risk of loosing money by rolling a futures contract to a new one. Indeed, depending on the prevailing market situation (backwardation or contango), rollover process may yield to losses. Noting that the risk incurred by rollover process belongs to basis risk, we analyze an optimal hedging strategy for price and quantity risks that takes into account the rollover process. We have found that the papers from Gardner [20] and Baesel and Grant [3] are nevertheless prominent on rollover issue. Gardner [20] has dealt with rollover hedging when long term futures market is missing with fixed quantity and Baesel and Grant [3] derived optimal sequential strategy for quantity risk. We show that, the combination of the two approaches results into optimal hedging strategy for price uncertainty as well as quantity risk in term of basis risk. Finally, using the L-performance measure shows the superiority of this strategy over other strategies.

This paper is organized as follows. The first section states the issue of hedging and the motivations. The second one describes the existing approaches to derive hedge ratios with their shortcomings. The investigation of the hedging effectiveness measures follows in the third section and will presents a way to select the hedging strategy by ranking the hedge portfolios performance. In section four, optimal hedging strategy is derived, in term of basis, risk for quantity risk and when futures market is missing. The last section is devoted to applications on observed futures prices followed by a conclusion.

2 Related works on hedging with Futures Markets

A financial hedge consists in specific position of an investment that should reduce, as much as possible, the risk incurred by another existing investment. Specially, hedging with futures contracts consists in reducing the spot price risk at the expense of potential reward. In commodity market, the futures contract is usually the simplest hedging instrument. The first motive for hedging with futures in commodity markets comes from the need for optimal balance between risk and return that reduces spot price risk as well as other relative risk such production and storage risks. Indeed, for storable commodities, inventories allows to absorb shocks in the supply and demand balance.

Many papers in the financial literature have investigated the use of futures markets to offset uncertainties pertaining commodity trading activities. The research stream on futures hedging has started with *Price Insurance Theory* and has analyzed the hedger to avoid loss due to any price move related to positions in futures markets. As economic rationale for hedging, Keynes [27], Hicks [21] and Kaldor [25] argued that the hedgers shift the risk to speculators by paying a premium. So did Working [45] for risk insurance[1], but had firstly advocated on earning returns theory where a sort of arbitrage consists to enter the market only when the hedger perceives a promising opportunity for profit. That is to say, a decision for hedging could also include speculation purpose and does not have to be limited on pure risk hedging only.

Later on, *Portfolio Theory* approach of Markowitz has been applied to futures hedging in order to investigate the hedger's risk-return trade-off. Thus, the hedge portfolio considers in priority the asset to be hedged, the non-traded position, and the futures contract as hedging instrument. This statement can be found in Rolfo [39] for static framework and Ho [22] for continuous time framework among other. The hedger is then maximizing the expected utility of his wealth. For active markets, the hedging can include other traded assets to derive optimal hedging strategy like in Adler and Detemple [1]. However, Pennings and Leuthold [37] noticed that William[2] had

[1]"The reason for hedgers to have their orders executed expeditiously is to reduce the interval in which their inventories are left uncovered, exposed to the risk of price change", Pennings and Leuthold [37]

[2]Williams J. C., The economic function of futures markets, Cambridge University Press, 1986.

stressed the difficulty for the portfolio approach to diversify the risk in production, transport and processing (commodity availability) that inventories absorb and which motivates the use of futures contracts. In addition, the portfolio theory in hedging assumes the initial position of inventories to be unhedged is extremely sensitive to the predetermined position. Aside, pure insurance and portfolio approaches to hedging, there is *Loan Markets Theory* and *Liquidity Theory*.[3] Kamara [26] argued that the three theories contribute, in explaining why producers hedge: "the hedger's position in futures is motivated partially by the desire to stabilize income and partially by the desire to increase the expected profits". Meanwhile, it is clear that all the approaches are all based on optimization techniques.

Among the various optimization techniques applied to derive the hedge ratios in static framework, minimum-variance is used as benchmark in comparing different approaches. But, minimum-variance approach equally penalizes both upside and downside deviation of returns from the mean. For instance, an agricultural producer that wants to hedge his business is much more worried by the downside shock from a target level of revenues than the upside deviation. Hence, minimum-variance hedge ratio may lead to suboptimal hedging recommendations. The mean-variance approach is consistent with expected utility theory if all uncertainty factors satisfy the location-scale[4] condition. To overcome this shortcoming, alternative approaches consistent with stochastic dominance concept have been developed. They include mean-extended-Gini (MEG), lower partial moment (LPM), Value-at-Risk (VaR) and conditional Value-at-Risk (CVaR) (see [15], [16] for more details on VaR). Sequential approaches have been also investigated to take into account period and horizon effects on hedge ratios. For instance, Cecchetti et al. [8], Chen et al. [10] and Lien and Luo [32] derived hedge ratio in multi-period analysis, Baillie and Myers [4] based their analysis on conditional distribution approach (ARCH: autoregressive conditional heteroskedastic and GARCH: generalized ARCH) and Fernandez [18]; Conlon and Cotter [11] applied wavelet decomposition to derive hedge ratio according to hedging horizon.

3 Some hedging approaches in static framework

Consider a time period defined on the basis of initial date, $t = 0$, and final date, $t = T$. At time $t = T$, the hedger wants to sell his production of unknown quantity Q_T, at prevailing spot price S_T. Assume that there is a futures market for him to reduce the risk of losing part of total revenue his will incur. In this section, we also assume the futures contract lives the investment horizon $[0, T]$. Denote by F_0 the futures price at time $t = 0$ and by F_T the futures price at T. If the investment horizon is longer than T, the futures contract should be rolled over the next period and this analysis is addressed in section 4. The issue is to find the investment strategy that will reduce, as much as possible, both spot price and quantity risks. That is to decide, at initial date, the position, x, in futures market. The portfolio then modifies from $S_T Q_T$ to hedge

[3]Other motivations for hedging with futures markets relate to loan markets theory and to liquidity theory. Loan markets theory refers to hedging operation by getting the accessibility for a period of time while liquidity theory is the provision that organized markets facilitate.

[4]Normal distribution is typically assumed but will not be realistic in practice.

portfolio as follows

$$\mathcal{W}_T = S_T Q_T - x\Delta F_T, \tag{3.1}$$

where ΔF_T stands for $F_T - F_0$. Equivalently, the hedge portfolio return is given by

$$R_h = \frac{S_T Q_T R_s - x F_T R_f}{S_T Q_T} = R_s - hR_f, \tag{3.2}$$

where R_s and R_f are respectively the spot and futures returns with

$$R_s = \frac{S_T - S_0}{S_0} \qquad \text{and} \qquad R_f = \frac{F_T - F_0}{F_0} \tag{3.3}$$

and h is the hedge ratio defined by

$$h = \frac{xF_T}{S_T Q_T}. \tag{3.4}$$

Hedging strategy consists in finding the proportion, h, of futures contract. There are various approaches to derive the hedging ratio, h, that rely on producer's preference according to either risk psychology or risk ordering. The psychology risk stream relates to expected utility that involves coefficients like aversion, prudence and temperance while risk ordering stream relies on stochastic dominance concept. In the context of agricultural farmer, we recall the various ratio according to these two streams.

3.1 Hedge ratios based on risk psychology

The basic hedge ratio is the value of h that minimizes the variance of the a portfolio with return R_h and it had been introduced by McKinnon (1967) for a commodity producer. The minimum-variance hedge ratio, that we denote $h^*_{min\text{V}}$, gives the proportion position in futures contracst that will make the hedge portfolio variance as small as possible to reduce the risk incurred in spot price return at maturity T. The minimum-variance hedge ratio is given as follows:

$$h_{\min \text{V}} = \frac{Cov\left[R_s, R_f\right]}{Cov[R_f, R_f]} = \frac{Cov\left[R_s, R_f\right]}{V[R_f]} \tag{3.5}$$

This hedge ratio is pure hedge and it does not account for the portfolio expected return that allows for speculative component in the same time. The shortcoming of minimum-variance hedge ration is that, in case of multiple contracts, it is pronounced on low volatility contracts at the expense of exploiting correlation properties (Stoyanov [42]). Therefore, the mean-variance is generally seen as the extension (see for example Anderson and Danthine [2], Duffie [13]) of minimum-variance strategy that takes into account the hedger's preference in terms of portfolio return and risk aversion γ.

The mean-variance is also referred to as quadratic utility when returns distribution is normally distributed. The optimal hedging value that maximizes the expected quadratic utility function is obtained as follows:

$$h^*_{\text{QUE}} = \frac{Cov\left[R_s, R_f\right]}{V[R_f]} - \frac{E[R_f]}{\gamma V[R_f]}. \tag{3.6}$$

where h^*_{QUE} is composed of the pure hedge component, $h^*_{min\text{V}}$, in equation (**??**) and a speculative component. Mean-variance hedge ratio allows the the risk-averse producer can hedge his income variability on the futures markets by buying or selling futures.

The amount of futures for speculation is determined by risk aversion and futures price variability. The speculative component position then converges towards zero with infinite risk aversion ($\gamma \to \infty$) or if the futures price process is martingale ($E[F_T] = F_0$). That is the case where the hedger is extremely reluctant to take risks or does not expect any additional return. Pure speculative strategy holds whenever the spot and the futures are uncorrelated in case of no insurance motive. Futures trading by producers results from a mixture of hedging and speculative motives.

Similarly, using Arrow-Pratt approximation of risk premium by second-order Taylor-expansion, the hedger's preference as quadratic utility function makes the mean-variance hedge ratio equivalent to the case where the portfolio returns are normally distributed. Subsequently, when the returns are not normally distributed, the mean-variance hedge ratio will be suboptimal and the quadratic utility then becomes unrealistic. In practice, normality assumption fails because fat tails distribution.

Proposition 3.1. An extension of the Arrow-Pratt approximation to fourth-order Taylor expansion leads to coefficients of prudence and temperance that are respectively associated to third and fourth moments.[5] The corresponding expected utility maximization program is given by

$$\max_h \left\{ E[R_h] - \frac{\gamma}{2}\, V[R_h] + \frac{\chi}{6}\, M_3[R_h] - \frac{\psi}{24}\, M_4[R_h] \right\}, \tag{3.7}$$

where M_3 and M_4 are respectively third (skewness) and fourth centered moments (kurtosis) and the coefficients for psychology risk χ and ψ, express respectively the taste for asymmetry (prudence) and aversion to fat tails (temperance). Instead using skewness and kurtosis, the maximization program becomes

$$\max_h \left\{ E[R_h] - \frac{\zeta}{2}\, V[R_h] + \varphi\, s_3(R_h) - \frac{\xi}{2}\, s_4^2(R_h) \right\}, \tag{3.8}$$

with s_3 and s_4 being skewness and kurtosis operator respectively with modified set of coefficients for risk psychology. The hedge ratio solution of the above program is as follows

$$h^{\text{MVSK}} = \frac{E[R_h] + \varphi s_3(R_h)}{\zeta V[R_h] - \xi s_4^2(R_h)}. \tag{3.9}$$

Remark 3.2. The optimization problem in program (3.7) is usually solved numerically.

For the proof of the precede results is similar some results of Le Courtois and Walter [28] that is concern by the portfolio assets allocation in finance. This has similarity with the mean-variance hedge ratio, h^*_{MV}, in equation (3.6). The hedge ratio h^{MVSK} includes asymmetry and fat tails influences on respectively the mean and the variance

[5]Alternative method based on higher moments to derive the hedge ratio without Taylor expansion has been developed in from Brooks et al. [7]

of the hedge portfolio. Indeed, skewness and kurtosis together capture risk distribution of the hedge portfolio in that skewness indicates difference between profits and losses and kurtosis the occurrence of extreme events.

Other utility function can be used to derive hedge ratio in static framework. For instance, Rolfo [39] had also considered logarithm preference and suggested futures contract trading as hedging instrument for variability in both the price and the production of its output. Meanwhile, alternative way to deal with hedging problem is to consider risk ordering concept.

3.2 Hedge ratio based on risk ordering

The risk ordering approach for hedging strategies corresponds to hedge ratios that are consistent with stochastic dominance concept[6] known to capture the properties of a distribution. Particularly, these hedge ratios rank different hedge portfolios according to preference (with only limited information about the utility function of a particular consumer) with no constraint on taste and aversion or particular distribution. They include mean-extended-Gini, lower partial moment approach (Chen et al. [10], Lien and Tse [34]) as well as famous risk measures in finance such as Value-at-Risk and Conditional Value-at-Risk. The purpose is then to minimize a risk specific measure.

Let's R_1 and R_2 be two random variables defined on probability space, $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, Prob)$ with their respective cumulative distribution functions,

$$G_1(x) = Prob(R_1 \leqslant x) \quad \text{and} \quad G_2(x) = Prob(R_2 \leqslant x).$$

Recall that R_1 dominates R_2 by the first-order (respectively by the second-order) stochastic dominance, SD1 (respectively, SD2) if and only if all investors that prefer more to less (respectively are risk-averse) would prefer R_1 to R_2. We denote the SD1 relation by $R_1 \succeq_{\text{SD1}} R_2$ and the SD2 relation by $R_1 \succeq_{\text{SD2}} R_2$. Formally, R_1 is said to first-order stochastically dominate R_2, if

$$G_1(x) \leqslant G_2(x), \, \forall x.$$

R_1 is said to second-order stochastically dominate R_2, if

$$\int_{\infty}^{x} G_1(x)\mathrm{d}x \leqslant \int_{\infty}^{x} G_2(x)\mathrm{d}x, \, \forall x.$$

The first-order stochastic dominance relation corresponds to all choices made by investors with monotonic expected utility function while the second-order stochastic dominance relation is all choices made by risk-averse expected-utility investors. We simply write $R_1 \succeq_{\text{SD1}} R_2$ and $R_1 \succeq_{\text{SD2}} R_2$ whenever R_1 dominates R_2 according to SD1 and SD2 respectively. Besides, the first-order stochastic dominance relation implies the second-order stochastic dominance relation,

$$R_1 \succeq_{\text{SD1}} R_2 \quad \Longrightarrow \quad R_1 \succeq_{\text{SD2}} R_2.$$

[6] The rationales for the stochastic dominance are well documented in Rothschild M. & Stiglitz J. E., "Increasing risk I. A definition". *Journal of Economic Theory*, 2, 225-243; 1970.

3.2.1 Mean-extended-Gini hedge ratio

The MEG coefficient is a non-negative, non-decreasing and bounded function of a risk parameter $1 \leqslant \delta < \infty$. Following Shalit and Yitzhaki [40], it can be applied to a hedge portfolio returns, R_h,

$$\Gamma_h(\delta) = \int_a^b (1 - G_h(R_h))\, \mathrm{d}R_h - \int_a^b (1 - G(R_h))^\delta\, \mathrm{d}R_h, \tag{3.10}$$

where a, b with $(a \leqslant b)$ are real numbers and G is the cumulative probability distribution of the portfolio return R_h. The parameter δ plays the role of risk aversion as the extend-Gini coefficient can be viewed as risk premium that should be subtracted from the expected value of portfolio. Hence, when $\delta = 1$ the investor is risk neutral and $\Gamma_h(0) = 0$; for a risk-seeker, $0 \leqslant \delta < 1$ and when $\delta > 1$, the investor is risk-averse.

Consider two portfolios, say R_1 and R_2, with their respective returns distribution G_1 and G_2. Let's $(\epsilon_n)_{n \in N^*}$ be the sequence defined as follows

$$\epsilon_n = \int_a^b (1 - G_1(x))^n\, \mathrm{d}x - \int_a^b (1 - G_2(x))^n\, \mathrm{d}x. \tag{3.11}$$

Yitzhaki and Schechtman [46] have proved that if $\epsilon_n \geqslant 0$, $R_1 \succeq_{\text{SD1}} R_2$ and $R_1 \succeq_{\text{SD2}} R_2$. Consequently, mean-extended-Gini coefficient $\Gamma_\delta(R_h)$ is the risk measure and can be minimized to achieve an optimal hedging strategy[7], h^*_{MEG}. However, as the mean-extended-Gini coefficient in equation (3.10) is difficult to evaluate in practice since there is no explicit analytic formula, Shalit and Yitzhaki [40] have suggested the following expression

$$\Gamma_h(\delta) = -\delta\, Cov\left(R_h\,,\, (1 - G(R_h))^{\delta - 1}\right) \tag{3.12}$$

that leads to the optimal hedge ratio (Shalit [41]) given by

$$h^*_{\text{MEG}} = \frac{Cov\left(R_s\,,\, (1 - G(R_h))^{\delta-1}\right)}{Cov\left(R_f\,,\, (1 - G(R_h))^{\delta-1}\right)}. \tag{3.13}$$

Therefore, the mean-extended-Gini hedge ratio can be estimated under assumption of probability distribution.

3.2.2 Hedging with lower partial moment

Lower partial moment belongs to class of downside risk measures. Downside risks only focus on the losses and then considers the worse case scenarios from a target level of revenues. The lower partial moment is characterized by two parameters, the target level return, c, that determines the shortfalls and the power, n, of the shortfalls. The lower partial moment of the hedge portfolio returns, $x = R_h$, is defined by

$$\text{LPM}_n(c, x) = \int_{-\infty}^{c} (c - x)^n\, dG(x), \qquad n \in N, \tag{3.14}$$

[7] The hedger can also obtain an efficient set based on each value of δ. The efficient set is progressively reduced when the hedger performs the mean-extended-Gini analysis for different values of δ and retains only the intersection of the efficient sets.

where the cases $n < 1$ and $n > 1$ characterize, respectively a risk seeking investor and implies risk averse investor (Fishburn [19][8]). Note that semi-variance is a special case of lower partial moment approach, with $c = 0$ and $n = 2$, $\ell_2(2, \cdot)$.

Furthermore, the lower partial moment satisfies the first and second order stochastic dominance relations and can be used as risk measure. Bawa [5] showed that n^{th} order lower partial moment is consistent with stochastic dominance of the $(n+1)^{th}$ order. Lien and Tse [34] had observed that, when $n > 1$, the n^{th} order lower partial moment is given by

$$\ell_n(c, R_h) = E\left\{[\max(0, c - R_h)]^n\right\}. \tag{3.15}$$

The first order condition with right to the hedge ratio is

$$-n\, E\left\{[\max(0, c - R_h)]^{n-1}\, R_f\right\} = 0,$$

with the second order condition always satisfied (positive).

3.2.3 Hedge ratio based on VaR and CVaR

From Ogryzak and Ruszczyński [36] Value-at-Risk and conditional Value-at-Risk (hence, from now on VaR and CVaR) satisfy SD1 and SD2 properties respectively. VaR and CVaR belong to the class of downside risk measures when dealing with hedging. They measure the "potential losses" associated with a risky position on a predefined horizon, at a given risk level $\alpha \in (0, 1)$. Specially, VaR indicates the potential loss of amount at probability $1 - \alpha$ for a strategy over a specified time horizon, while the CVaR, as an extension of VaR, gives the total amount of a given loss event. Formally, the VaR at probability level α for the hedge portfolio returns R_h is defined as

$$VaR_\alpha(R_h) = \inf\left\{x \in R, prob(R_h > x) \leq 1 - \alpha\right\}, \tag{3.16}$$

and the corresponding CVaR is as follows

$$CVaR_\alpha(R_h) = E\left[-R_h | - R_h > VaR_\alpha(R_h)\right]. \tag{3.17}$$

Thus, given an amount, VaR stresses how often a portfolio could loose and CVaR will indicate the potential loss beyond a given amount. Meanwhile, VaR lacks the sub-additivity property, which is fundamental for portfolio diversification and will provide no information about the portfolio losses corresponding to period of predefined risk. The CVaR is sub-additive and accounts for tail risk. Hence, it allows portfolios optimization as shown in Rockafellar and Uryasev [38]. Besides, CVaR overcomes lack of sub-additivity and indifference to tail losses, but will require a large size data for consistent estimation even more sensitive to estimation errors than VaR.

The optimization problems[9] for VaR and CVaR are given by

$$\begin{aligned} h^*_{VaR} &= arg \min_{h \in R} VaR_\alpha(R_h) \\ h^*_{CVaR} &= arg \min_{h \in R} CVaR_\alpha(R_h) \end{aligned} \tag{3.18}$$

[8]The Fishburn risk measure has the same form but allows for a non integer, positive power function.

[9]These problems require to assume that VaR_α and $CVaR_\alpha$ are continuously differentiable in h and that the distributions of the spot return R_s and futures return R_f have positive density.

However, one knows that VaR and CVaR measures depend on the distribution of the hedge portfolio returns which would lead hedging strategy extremely dependent on predetermined risk level α and distribution. Overall, each hedge ratio leads to a specific hedging strategies that depends upon the approach used. That is to say with the same data, the hedging strategy to apply, among the above described, remains elusive. A criteria to distinguish them is hedging effectiveness that evaluates the hedging performance.

3.3 Hedging performance

Hedging performance is a measure of hedging effectiveness that serves as criterion to compare the consistency in both estimation and post samples of different hedge ratios. There are three main measures of effectiveness in financial literature that relate to futures hedging: Ederington [14] measure, Howard and D'Antonio [23] Sharpe-type measure and the certain equivalent measure. Ederington [14] has first defined effectiveness measure to indicate the reduction effect provided by the futures contract in term of the percentage reduction of the hedge portfolio variance over the spot asset variance,

$$\mathbf{HE}_{\text{ED}} = 1 - \frac{V[R_h]}{V[R_s]}. \tag{3.19}$$

According to the measure in equation (3.19), a hedge ratio is deemed better than another if it leads to a smaller variance of the hedge portfolio.

The Howard and D'Antonio [23] measure takes into account both expected return and volatility of hedge portfolio,

$$\mathbf{HE}_{\text{SH}} = \frac{E[R_h] - r}{\sigma_h} - \frac{E[R_s] - r}{\sigma_s}, \tag{3.20}$$

where r is the risk-free interest rate and σ_h and σ_s are respectively the return volatilities of hedge portfolio and the spot.

The third criteria of hedging effectiveness is based on the certainty equivalent measure and is defined such that position in futures contract equates its same expected utility, as follows

$$E[u(R_s + e)] = E[u(R_s - hR_f)],$$

where u is an increasing and concave utility function and e is the certainty equivalent.

Remark 3.3. Recall that, in all the above three cases, hedging effectiveness is based on the estimated hedge ratio. Lien [30, 31] has shown that all these measures are unreliable because they are downward biased leading to under-reported hedging strategy. Specifically, the Ederington measure is likely to perform only with minimum-variance hedge ratio. In the case of portfolio non normality (as results of spot and futures returns' distribution asymmetry and fat tails), the Sharpe-type hedging effectiveness will fail to onsider relevant properties of portfolio.

To overcome this limits, we propose a ranking based measure of hedging effectiveness by applying the L-performance defined with regard to L-moment approach. The advantage of using L-moment relies on their consistency in estimation. The L-performance measure ranks different hedge portfolios regardless the methodology of the hedge ratio.

Darolles et al. [12] have used L-performance criterion to rank hedge funds on different portfolio strategies just as Sharpe-ratio ranking. Following the precede literature [12], let's denote by $\mathbf{HE}_{\text{LP}}$ the effective L-performance and by $L^h_{q,p}$ and $L^s_{q,p}$ be L-performance of respectively hedge portfolio and spot asset for given q and p. The L-performance effectiveness is defined by

$$\mathbf{HE}_{\text{LP}} = L^h_{q,p} - L^s_{q,p} \tag{3.21}$$

where the L-performances $L^h_{q,p}$ and $L^s_{q,p}$ are presented in Appendix B. To estimate a L-performance, consider a sample of independent and identically distributed returns r_i, $i = 1, \ldots, N$ with their order statistics: $r_{1:N} \leq \ldots \leq r_{N:N}$. The estimator of L-performance is a ratio of the two linear combinations of order statistics given by

$$\widehat{L}_{q,p,N} = \frac{\sum_{i=1}^{N} r_{i:N} P_{1,p}\left(\frac{i}{N}\right)}{\sum_{i=1}^{N} r_{i:N} P_{2,q,p}\left(\frac{i}{N}\right)}, \tag{3.22}$$

for L-performance defined[10] on $u \in (0,1)$, $p \in N$ and $0 \leq q \leq p-1$ and polynomials $P_{1,p}$ and $P_{2,q,p}$ described as follows

$$P_{1,p}(u) = \frac{(2p+1)!}{p!} u^p (1-u)^p.$$

$$P_{2,q,p}(u) = \frac{(2p+1)!}{q!(2p-q)!} \left[u^{2p-q}(1-u)^q - u^q(1-u)^{2p-q}\right]$$

The L-performance estimator is consistent and asymptotically normal, under standard regularity conditions, Darolles et al. [12].

In the aims of comparing the hedge ratios as well as the hedging

3.4 Limits of existing hedge ratios

So far, the issue of hedging commodity revenue with futures market has considered the same date for the maturity and delivery. In reality, these dates may differ mainly when there is no futures contract with long maturity.

Remark 3.4. One could think about the producer that has to set a hedging strategy for his activity against either adverse price and yield variation over the planting season with available futures contracts which mature earlier than the delivery date, say T. In such a situation, the above described will not be effective to dates mismatch between the position futures market and cash position. Typically, when the production period exceeds maturity date of the active futures contract, the producer will usually initiate a rollover strategy.

[10] Notice that, in practice, it is insightful to consider several different pairs of parameters q, p to obtain alternative rankings of portfolios with respect to $\widehat{L}_{q,p,N}$.

The rollover strategy consists in closing out the position in the nearby futures few days prior to its maturity date and taking another position in a contract with longer maturity. This involves closing out one futures contract just few days prior to its delivery date, and then taking position in a new futures contracts with a longer delivery date. one can think of the farmer that makes a planting decision at time zero and will harvest the crop at time T, but the maturity dates of the available futures contracts as hedging instruments in the market, hold before harvest time. Note that, the rollover strategy is subject to additional basis risk. Gardner [20] had designed sequential rollovers as marketing strategy to hedge against this basis risk in this situation. He suggested this marketing strategy as a way to efficiently hedge against the additional basis risk.

However, his strategy includes constant outputs neglecting the production risk that matters for storable commodities. In fact, his strategy assumes constant quantity, as implicitly considered in expression (3.2), does not take into account the quantity risk that always pertain when trading storable commodities. For an agricultural producer, the risk of not to get the expected output always exists since his production is subject to weather conditions.

More generally, in agricultural markets, price and output uncertainties are interrelated in that prices react inversely to large variations of output (Conroy and Rendleman[1983]). The combination of these two risks rises the problem of the appropriate position in hedging instruments, specially for the futures contracts with longer maturity that should also account for the additional basis risk in the rollover process. But, for the rollover strategy, production risk is also relevant in inter-crop period for stock and the coming crop year on uncertainties related to weather conditions that could lead to imperfect hedge. Hence, hedging in rollover strategy need to be extended to tackle the inter-crop season to further guarantee revenue over long time exposure.

Additional hedging instruments are insurance products products designed for agricultural producers that have the opportunities to purchase an insurance contract.

The following section considers rollover strategy as in Gardner [20] and include stochastic output. Following Baesel and Grant [3] approach to hedge quantity risk at the rolling time.

4 Optimal sequential hedging

In this section we investigate the optimal hedging strategy thet include an insurance policy. The optimal sequential hedging combines the approach of Gardner [20] on rolling over futures contracts and the sequential hedging of Baesel and Grant [3] to derive a hedging strategy that accounts for both the price and production risk using futures and insurance contracts.

4.1 The Strategy

Consider an agricultural producer that plans to sell his crop for the $T > 2$ coming years. Since futures contracts are available with short maturities, using a rollover strategy on their positions, one can lock in price, in the first year, for the T coming years. Multiyear futures contracts, or sequential rollovers as a substitute, make more sense, along Working (1953) lines, as a device for locking in receipts within an T-years period, argued Gardner (1989). In rollover strategy, a producer faces additional risks including pests infestation of the stocks, low revenue for the coming crop years due to production risk like weather conditions, etc. The producer can purchase an insurance contract to further hedge his revenue. On using insurance to reduce these risks, the producer need an optimal policy together with the futures hedging strategy. An insurance policy is described by the couple $(I(\cdot), prem)$ where $I(\cdot)$ and $prem$ are are respectively the risk-neutral indemnity paid to risk-averse producer and insurance premium. We assume; at any time t, the premium to depend on actuarial value of the policy and the indemnity function to be non-negative and less than the insured value (see Mahul and Wright (2003),

$$0 \leq I_t(x) < x, \qquad \forall x \geq 0 \qquad \text{and} \qquad prem = \zeta\, E[I_t(x)] \tag{4.1}$$

with $\zeta(0) = 0$, $\zeta^{'}(x) > 1$ for all $x > 0$ is a deterministic loading factor. An optimal insurance contract for a crop year is the insurance premium and the indemnity function that maximize the producerÂs expected utility of gross revenue under the above mentioned constraints:

$$\max_{I_t(\cdot), prem} E[u(R_t + I_t(\cdot) - prem)] \tag{4.2}$$

with R_t the producer gross revenue at time t. The indemnity function depends on insurance contract and the wealth process is function of both the indemnity and the marketing strategy. Consider a revenue insurance (like Income Protection, IP, or Revenue Assurance, RA) where the producer chooses a proportion of the expected revenue to insure. In such insurance contract, the price at which the crop is valued moves with price changes in the market. Therefore, the producer will receive indemnity equal to the difference between the percentage of the value he has insured and the revenue at end of period, if only if the former is greater than the latter. For simplicity, assume the expected revenue at time t to be the average revenue at time $t-1$, is as follows:

$$E_t[S_t Q_t] := F_{t-1,t} \bar{Q}_{t-1} \tag{4.3}$$

with E_{t-1} being the conditional expectation on information available at $t-1$ and $\bar{Q}_{t-1}$ is the average output over the period $[0, t-1]$. The indemnity schedule $I_t(\cdot)$ is as follows

$$I_t(S_t, Q_t; v_t) = \Big[v_t F_{t-1,t} \bar{Q}_{t-1} - S_t Q_t\Big]^+, \qquad v_t \in (0, 1] \tag{4.4}$$

where the notation $[\cdot]^+$ stands for $\max(0, \cdot)$ function. The producer has to decide the proportion v_t of his expected revenue to choose according to his hedging strategy with future contract.

Consider a producer following the marketing strategy as in Gardner [20], his wealth will rely on the sequential rollover strategy for achieving the whole T-years period hedge. That is, in planting season T crops, each of quantity Q_t for $t \in \{1, 2, 3, \ldots, T-1\}$, is the price of futures contract traded at t for delivery at $t+1$. The producer's wealth at initial time, $t = 0$, is given by

$$\mathcal{W}_0 = F_{0,1} Q_{1,T}, \tag{4.5}$$

with $F_{0,1}$ being the price of futures contract traded at initial time for delivery at the end of the first crop year and $Q_{1,T}$ is the total quantity for first year to T. More generally, we denote the total quantity for period from $t = j$ to $t = T$ by

$$Q_{j,T} = \sum_{t=j}^{T} Q_t, \qquad j \in \{1, 2, 3, \ldots, T\}. \tag{4.6}$$

At inception at the hedging strategy, no indemnity could be received but the decision about v_1 is made for $t = 1$ by paying a premium $prem$. Determining the premium at a time step is a pricing issue and relates to insurance company. So we neglect the term $prem$. Indeed, we assume that the producer has already select his insurance contract and he does know the corresponding premium. We then focus on deciding the optimal futures hedge and proportion of the expected revenue following market outcome. This aims at comparing how insurance contract will affect the hedging strategy in futures market.

At time $t = 1$, a number of T contracts are bought at price $F_{1,1}$. At the same time, a unit of the first crop quantity Q_1 is sold at the spot price S_1 and the remaining contracts $T-1$ of the total quantity $Q_{2,T}$, are rolled over by selling futures for delivery at T. The wealth, $\mathcal{W}_1$, at the end of the first crop year, is

$$\mathcal{W}_1 = \mathcal{W}_0 - sp_1\, Q_{2,T} - cQ_{1,T} + \max\left(b_1 Q_1, v_1 F_{0,1} Q_0 - F_{1,1} Q_1\right) \tag{4.7}$$

where b_1 and sp_1 are respectively the basis and the spread[11] and Q_0 has to be set.

Analogously to the precede logic, at any time $t < T$, there are $T - t - 1$ contracts bought back at price $F_{t,t}$ and Q_t will be sold at spot price S_t with the remaining contracts $T - t$ of total quantity $Q_{t,T}$ rolled over by selling futures for delivery at T. The wealth, $\mathcal{W}_t$, at the the end of the $t^t h$ crop year follows as

$$\mathcal{W}_t = \mathcal{W}_{t-1} - sp_t\, Q_{t+1,T} - c\, Q_{t,T} + \max\left(b_t Q_t, v_t F_{t-1,t} \bar{Q}_{t-1} - F_{t,t} Q_t\right). \tag{4.8}$$

At the final time T, the wealth, $\mathcal{W}_T$, over the T-years period is given by

$$\mathcal{W}_T = \mathcal{W}_{T-1} - c\, Q_T + \max\left(b_T Q_T, v_T F_{T-1,T} \bar{Q}_{T-1} - F_{T,T} Q_T\right). \tag{4.9}$$

Remark 4.1. The sequence of quantities $(Q_t)_{t=1,2,\ldots,T}$ can be determined by using the backward recursive technique. At each step, as soon as the quantity, Q_t, is obtained, the insurance policy v_t can be settled afterward.

The quantities Q_0 and Q_{T_0} can be determined by using the backward recursive technique of stochastic dynamic programming.

4.2 The solution

Consider a producer with quadratic utility where the objective to maximize the expected wealth over the the time period $[t, T]$ subject to wealth variance constraint,

$$E\left[u(\mathcal{W}_{T_0})\right] = E[\mathcal{W}_T] - \frac{\gamma}{2} V[\mathcal{W}_T], \qquad t \in \{1, 2, 3, \ldots, T\}, \tag{4.10}$$

where γ is the risk aversion parameter of the hedger. The larger γ is, the higher is the hedger's aversion to risk.

[11] The basis at time t on futures contract for delivery at $t+1$ is $b_t = S_t - F_{t,t+1}$ and the spread between the period ahead futures and nearby futures prices is $sp_t = F_{t,t+1} - F_{t,t}$.

Remark 4.2. Note that the producer aversion may change from period to period in the rollover process, but since it is assume as a given parameter, herein we will let it constant over the whole period.

Since the max function is not differential along the line $x = y$; $\forall x, y \in R$ and the optimization problem boils down to two cases. That is, the producerÂs wealth, over each period $[t, t+1]$, $t \in \{1, ..., T-1\}$, depends upon

1. either the revenue at end of each period, if it is greater than the proportion of expected revenue,
2. or a proportion of expected revenue, if it is greater than the revenue at end of each period.

So, the wealth at any time does not include the revenue at end of each period and a proportion of expected revenue all together. However insurance contract is purchased by the production whatever his expectation in the market. Particularly, the case (ii) refer addresses a guarantee against production risk when the crop yield is lower than expected. LetÂs start the optimization problem at final time of the hedging horizon T and apply backward recursion to determine the quantities for earlier dates. Using expression (4.9), the quantity Q_T and the wealth $\mathcal{W}_T$ are such that one has:

1. in case 1, $v_T F_{T-1,T} \bar{Q}_{T-1} < S_T Q_T$,

$$E[u(\mathcal{W}_T)] = \mathcal{W}_{T-1} + Q_T\left(E[b_T] - c\right) - \frac{\lambda}{2} Q_T^2 V[b_T] \tag{4.11}$$

with the optimal quantity for futures contract given by

$$Q_T^{*,\phi} = \frac{E[b_T] - c}{\gamma V[b_T]} \tag{4.12}$$

2. in case 2, $v_T F_{T-1,T} \bar{Q}_{T-1} \geq S_T Q_T$,

$$E[u(\mathcal{W}_T)] = \mathcal{W}_{T-1} \;+\; v_T F_{T-1,T} \bar{Q}_{T-1}\, Q_T\left(E[F_{T,T}] - c\right) - \frac{\lambda}{2} Q_T^2 V[F_{T,T}] \tag{4.13}$$

where the optimal quantity in futures contracts and insurance policy are respectively given by

$$Q_T^{*,v} = -\frac{E[F_{T,T}] + c}{\gamma V[F_{T,T}]} \tag{4.14}$$

and

$$v_T^* = 1 - \frac{E[F_{T,T}] Q_T^{*,v}}{F_{T-1,T} \bar{Q}_{T-1}}. \tag{4.15}$$

The proportion, v_T^*, of the expected revenue is given at optimal crop yield according to the expected futures price. Observe that, in the case 2 where revenue insurance payoff is paid to the producer, crop yield will be so low to make the insurance payoff as maximum as possible, (see (4.4). The insurance payoff is a decreasing function of the crop yield until a trigger function (see Mahul and Wright (2003)). Since the worse case scenario would be no crop yield and the producer will have bought the insurance contract at the total expected revenue, the proportion, v_T^*, is given by expression

(4.14)-(4.15). Similarly, to find the optimal hedging strategy at any time t prior to final time T, consider the two cases (i) and (ii) with their corresponding expected utility expressions at time t. For $t < T$, we follow the backward recursion and replace Q_{t+1} by Q^*_{t+1}, determined earlier, in expression (4.8). It gives raise to

1. in case 1, $v_T F_{T-1,T}\bar{Q}_{T-1} < S_T Q_T$,

$$E[u(\mathcal{W}_T)] = \mathcal{W}_{T-1} + Q_t\left(E[b_T] - c\right) - \frac{\lambda}{2}\left\{Q_T^2 V[b_t] + 2Q_t Q^*_{t+1,T} Cov(b_T, sp_T)\right\} \tag{4.16}$$

   with the optimal quantity of futures contract being

$$Q_t^{*,\phi} = \frac{E[b_T] - c}{\gamma V[b_T]} - \frac{Q^*_{t+1,T} Cov(b_t, sp_t)}{\gamma V[b_T]} \tag{4.17}$$

2. in case 2, $v_T F_{T-1,T}\bar{Q}_{T-1} \geq S_T Q_T$,

$$E[u(\mathcal{W}_T)] = \mathcal{W}_{T-1} + Q_t\left(E[F_{t,t}] - c\right) - \frac{\lambda}{2}\left\{Q_T^2 V[F_{t,t}] - 2Q_t Q^*_{t+1,T} Cov(F_{t,t}, sp_t)\right\} \tag{4.18}$$

   where the optimal quantity in futures contracts and insurance policy are respectively given by

$$Q_T^{*,v} = -\frac{E[F_{t,t}] \ + \ c}{\gamma V[F_{t,t}]} + \frac{Q^*_{t+1,T} Cov(F_{t,t}, sp_t)}{\gamma V[F_{t,t}]} \tag{4.19}$$

   and

$$v_T^* = 1 - \frac{E[F_{t,t}] Q_t^{*,v}}{F_{t-1,T}\bar{Q}_{t-1}}. \tag{4.20}$$

Let Q^*_t, $t \in \{1, \ldots, T\}$, be the optimal quantity to rollover from nearby futures contract to new one with,

$$Q_t^* = \begin{cases} Q_t^{*,\phi}, & \text{if crop yield is lower than expected;} \\ Q_T^{*,v}, & \text{otherwise.} \end{cases} \tag{4.21}$$

Over the hedging horizon, $[0, T]$, the optimal quantity depends upon risk aversion, transaction costs, the spread and either the basis or the futures price at end of period. Specially, when the crop yields are lower than expected, indemnity is paid to the producer based on the proportion, v_T^* as compensation. This guarantees the producer in the situations when drastic weather conditions hold leading to low revenue. The component with the spread, sp_t reflects profit and loss relation in futures market at the same time, scaled by the expected optimal hedge at future dates. Particularly, at any time t prior to final time T, the optimal quantity, Q_t, differs from the optimal quantity at T, Q_T with adjustment term of the hedge at future dates. Hence, at the end of hedging horizon, the optimal quantity does not include the spread since the producer will close the hedge and will not consider another futures contract in this strategy.

5 Empirical applications

Recall that two categories of price data where used in Chapter 1, the nearby contract prices with the front contract as proxy for spot price and expiry month prices and we use the last nearby as futures contract to compute the hedge ratios. In order to compares strategies of hedge ratios with the sequential hedging strategy, we restrict the period of analysis to two years, that is from August 1^{st}, 2013 to July 31^{th}, 2015. We exclude soybeans meal commodity as the results look similar to the soybean case.

Table 1 – Estimation of Hedge ratios

	minV		MV		SK		MEG		LPM		VaR		CVaR	
Commodity	est.	eff.	est.	eff.	est.	eff.	est.	eff.	est.	eff.	est.	eff.	est.	eff.
Corn	1.081	0.797	1.915	0.322	1.916	0.322	1.162	0.737	1.062	0.797	1.091	0.797	1.194	0.798
	0.000	1.471	0.001	3.903	0.001	3.903	0.000	1.907	0.000	1.307	0.000	1.557	0.000	1.544
	0.007	1e-04	0.012	1e-04	0.012	1e-04	0.008	1e-04	0.008	1e-04	0.008	1e-04	0.021	1e-04
Oat	1.127	0.494	1.604	0.406	1.605	0.406	1.253	0.417	1.336	0.477	1.139	0.494	1.148	0.434
	1e-04	0.730	2e-04	1.146	2e-04	1.146	1e-04	0.865	1e-04	0.945	1e-04	0.742	1e-04	0.782
	0.018	1e-04	0.020	1e-04	0.020	2e-04	0.018	1e-04	0.019	1e-04	0.018	1e-04	0.018	1e-04
R. rice	0.8002	0.472	2.395	-1.40	2.399	-1.42	0.481	0.326	0.354	0.326	0.818	0.472	0.838	0.397
	-1e-04	2.077	7e-04	5.657	7e-04	5.655	-3e-04	0.187	-4e-04	0.086	-1e-04	2.198	-1e-04	2.219
	0.009	0.001	0.019	3e-04	0.019	3e-04	0.011	0.000	0.010	0.000	0.009	1e-04	0.009	1e-04
Soybeans	-0.11	0.006	1.067	-0.66	1.067	-0.66	-0.13	0.009	-0.10	0.006	-0.09	0.005	-0.07	0.003
	-5e-04	-0.20	0.000	1.886	0.000	1.886	0.000	1.286	-5e-04	-0.18	-5e-04	-0.16	-5e-04	-0.16
	0.016	0.000	0.020	1e-04	0.020	1e-04	0.016	0.000	0.016	0.000	0.015	0.000	0.017	1e-04
Wheat	1.101	0.825	1.841	0.452	1.840	0.453	0.815	0.698	0.844	0.780	1.108	0.825	1.121	0.7253
	1e-04	2.899	4e-04	3.922	4e-04	3.924	0.000	0.932	0.000	0.827	1e-04	2.959	1e-04	2.5589
	0.007	1e-04	0.013	2e-04	0.013	2e-04	0.009	1e-04	0.008	1e-04	0.007	1e-04	0.008	1e-04
Cocoa	1.227	0.838	-2.32	-6.19	-2.33	-6.19	1.283	0.816	1.235	0.838	1.209	0.838	1.259	0.738
	-1e-04	-13.1	0.002	-3.38	0.002	-3.38	-1e-04	-12.8	-1e-04	-13.4	0.000	-12.4	0.000	-11.8
	0.004	-3e-04	0.027	0.000	0.027	0.000	0.004	0.000	0.004	0.000	0.004	0.000	0.004	0.000
Coffee	1.125	0.979	0.840	0.916	0.839	0.916	1.124	0.857	1.082	0.977	1.123	0.979	1.133	0.868
	0.000	-0.40	1e-04	1.576	0.000	1.575	0.000	0.506	0.000	0.806	0.000	-0.37	0.000	-0.37
	0.004	-1e-04	0.007	0.000	0.007	0.000	0.003	0.000	0.004	0.000	0.003	0.000	0.004	0.000
Cotton	1.146	0.536	1.420	0.505	1.420	0.505	0.714	0.431	0.814	0.491	1.149	0.536	1.367	0.503
	0.000	2.030	0.000	2.026	0.000	2.026	0.000	1.341	0.000	1.541	0.000	2.033	0.000	2.033
	0.009	1e-04	0.009	1e-04	0.009	1e-04	0.009	1e-04	0.010	1e-04	0.009	1e-04	0.009	1e-04

For each approach, the first column is labeled "est." as estimates of hedge ratio, average return and standard deviation for the corresponding hedge portfolio.
The second column displays the estimates of effectiveness measures respectively Ederington [14], $\mathbf{HE}_{\text{ED}}$, Howard and D'Antonio [23], $\mathbf{HE}_{\text{SH}}$, and L-performance, $\mathbf{HE}_{\text{LP}}$.
We consider the riskless interest rate to be $r = 0$.

Table 2 – Optimal sequential hedging strategy

Commodity	Q^*_0	$Q^*_{T_0}$	$\hat{\mu}$	$\hat{\sigma}$	$\mathbf{HE}_{\text{ED}}$	$\mathbf{HE}_{\text{SH}}$	$\mathbf{HE}_{\text{LP}}$
Corn	1.613	2.735	0.001	0.021	0.313	2.735	1.206
Oat	2.501	1.308	0.007	0.010	0.402	1.808	2.571
R. rice	1.776	1.049	-0.002	0.018	0.676	2.049	5.832
Soybeans	-2.586	-3.104	0.000	0.019	0.595	1.176	3.315
Wheat	3.956	5.176	0.004	0.016	0.292	3.810	1.975
Cocoa	-0.923	2.810	-0.001	0.013	0.682	1.210	0.173
Coffee	1.699	5.144	0.002	0.015	0.362	2.144	7.110
Cotton	4.027	2.385	0.004	0.014	0.427	1.385	2.134

We set transaction costs at zero ($c = 0$); $\hat{\mu}$ and $\hat{\sigma}$ are the estimates of respectively mean and volatility of hedge portfolio returns and $\mathbf{HE}_{\text{LP}}$ is L-performance effectiveness measure.

Table 1 displays estimates of hedge ratios as presented in Section **??**. It is clearly apparent that mean-variance and skweness-kurtosis based methods seem to be more consistent that other in that Sharpe ratio measure is higher for the hedge ratios in the first two method than the others. Besides, these two hedge ratios are closed. The other methods also look similar with more differences. This classification is more pronounced in case of rough rice and cocoa when opposite position is suggested by the two classes of hedge ratios. In the optimization program, the hedge ratios from mean-variance and skewness-kurtosis methods maximize the objective function while the other methods minimize their objective function. This shows how hedging strategy depends on the optimization program with regard to the producer's preference. Besides, as expected, even hedge ratios may vary substantially from one approach to another, there is no clear cut to stand for method of optimization based on effectiveness measures. Indeed, the L-performance effectiveness measure is closed to zero for all the strategies.

The sequential hedging strategy requires assumption of distribution of spot and futures prices. Instead, we use historical returns over the three period for both spot and futures prices.We set the Q_0 to the average output until july 2012. Herein, the rollover dates is chosen arbitrary[12] at the end of July for all commodities.

Table 2 exhibits results of optimal sequential hedging strategy which are more consistent than those in Table 1. Measures of L-performance effectiveness are all significantly greater than zero for all the commodities. Besides, the $\mathbf{HE}_{\text{ED}}$ measure is sensibly lower for the sequential strategy, what suggests the superiority of L-performance over the other effectiveness measures.

Remark 5.1. Besides, note that adding an insurance contract to futures contract in rollover hedging seems to decrease the number of futures contracts when low crop is expected. This effect of insurance contract is illustrated in the hedging strategies for the first two years and the first year respectively for oat and cotton. Therefore, combining futures and insurance contracts will further reduce market and production risk. Specially, insurance contract is addressed in low crop yield situation.

[12] in reality case, rollover date are published

Conclusion

The purpose of hedging has received many contributions in literature of futures market. For storable commodities, the hedging issue is of specific in that the asset is often non-traded. Futures contracts are the usual instruments to cover the farmer from the losses, but their use requires appropriate hedging strategies because of market moves. Hence, there is no guarantee of achieving the goal of reducing the price risk with only futures hedge. Indeed, when harvest fails, the the losses increase at final time and farmer may go bankruptcy. Other hedging strategies have been studying; the rollover strategy that consists in lock in price for longer time period by sequentially closing position in nearby contract and taking other in new futures contract along long time period.

We have investigated different approaches of hedging with futures contract in agricultural markets. We have first described the existing approaches that do not consider output risk due to production contingencies like bad weather condition, pests infestation for stored goods. In these strategies output is considered as deterministic and strategies strongly depend on the approaches. Besides, there is no clear cut for a best approach over the other based the existing effectiveness measures

Since most of producers in agriculture strongly rely on revenue from their activity, production risk is relevant. Hence, management of risk should include production risk contribute to avoid substantial losses on final income. In addition, the rollover strategy is subject to larger production risk within the intercrop periods and then and additional basis risk.

We derive sequential optimal hedging with rollover process that takes into account production risks. The strategy requires both futures market and insurance contract that are combined to further guarantee the producer a level of gross revenue. The resulting hedge depends on spread between nearby futures and new futures contracts as well as either the basis or the expected futures price at end of each period such that when the crop yields are lower than expected indemnity is paid to producer as compensation by insurance company.In order to distinguish the hedging approaches, we estimate hedging ratio from described static hedging strategies for commodity data at hands.

The results show how difficult is to select the best strategies based on the existing effectiveness measure. However, the application of L-performance measure is significant on the sequential strategy with insurance contract. Hedging in static framework only requires the price distribution at initial and final dates of each period to compute various moment for various hedge ratios. Futures prices are settled daily and one may gain additional information about price behavior by using price pattern over the hedging horizon.

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A Estimation of hedge ratios

In practice, the estimation of hedge ratio depends on the methods that is adopted to compute the hedge ratio. Herein, we describe some estimations methods for existing approach for hedge ratio with no quantity risk.

The minimum-variance hedge ratio is simply estimated by linear regression of spot returns on futures returns

$$r_{s,t} = a + \beta r_{f,t} + \varepsilon_t, \tag{A.1}$$

where a the intercept, β an estimate of h_{MV}, ε the error term and t is the observation time. While the linear regression is easy to implement by ordinary least square technique, it relies on no exhaustive assumptions which makes the estimated hedge ratio critical on statistical basis. Error term in equation (A.1) is often heteroskedastic and ordinary least square approach is based on unconditional mean and variance instead.

In the expected utility approach, appropriate utility function and distribution are usually guested to achieve closed form solution. Otherwise, numerical approximation usually allows to derive the hedge ratio.

The estimation of the mean-extended-Gini hedge ratio, is usually based on empirical distribution function of R_h

$$\widehat{\Gamma}_h(\delta) = \frac{\delta}{N}\left\{\sum_{i=1}^{N} r_{h,i}[1-\widehat{G}(r_{h,i})]^{\delta-1} - \frac{1}{N}\left(\sum_{i=1}^{N} r_{h,i}\right)\left(\sum_{i=1}^{N}[1-\widehat{G}(r_{h,i})]^{\delta-1}\right)\right\} \tag{A.2}$$

where N is the sample size and $r_{h,1},\ldots,r_{h,N}$, the observations of hedge portfolio returns. Then mean-extended-Gini coefficient, $\widehat{\Gamma}_h(\delta)$ is minimized as risk measure function. Alternatively, Shalit [41] had used another formula whose estimation is as follows

$$\widehat{h}_{\text{MEG}} = \frac{\sum_{i=1}^{N}(r_{s,i}-\bar{r_s})(d_i-\bar{d})}{\sum_{i=1}^{N}(r_{f,i}-\bar{r}_f)(d_i-\bar{d})} \tag{A.3}$$

with $d_i = [1-\widehat{G}(r_{h,i})]^{\delta-1}$ and $\bar{d} = \sum_{i=1}^{N} d_i/N$.

The lower partial moment hedge is approximated either on basis of the empirical distribution or the kernel estimation, Lien and Tse [34]. The empirical distribution approach leads to

$$\widetilde{\ell}_n(c,r) = \frac{1}{N}\sum_{r_{h,i}<c}(c-r_{h,i})^n, \tag{A.4}$$

and the kernel estimation consists in substituting the probability density function of the portfolio returns by a kernel density function[13],

$$\widehat{\ell}(n,\bar{r},G) = \frac{1}{N\varpi}\sum_{i=1}^{N}\int_{-\infty}^{\bar{r}}(\bar{r}-r)^n\, k\left(\frac{r-r_i}{\varpi}\right)\mathrm{d}r, \tag{A.5}$$

with k is the kernel function and ϖ is the bandwidth. By plugging $z=(r-r_i)/\varpi$ into the integral, we have

$$\widehat{\ell}_n(\bar{r},G) = \frac{1}{N}\sum_{i=1}^{N} l_n(c,r_{h,i}), \tag{A.6}$$

with

$$l_n(c,r_{h,i}) = \int_{-\infty}^{(c-r_{h,i})/\varpi}(c-z\varpi-r_{h,i})^n\, k(z)\mathrm{d}z. \tag{A.7}$$

Setting $n=2$ and assuming that the portfolio returns and the futures returns are independent, then hedge ratio is the same as the of semi-variance will be the same as the minimum variance hedge ratio, Lien and Tse [35].

Traditional way to estimate the hedge ratios from VaR and CVaR is numerical optimization, unless convenient distributions is use to get closed form solution [**?**],[**?**].

[13]The density function of R_h can be estimated by the kernel method

$$\widehat{g}(R_h) = \frac{1}{N\varpi}\sum_{i=1}^{N} k\left(\frac{R_h-r_{h,i}}{\varpi}\right).$$

B L-performance measure

The L-performance is a ratio of a trimmed L-moment. of order 1 and a trimmed L-moment of order 2 (Darolles et al. [12]), and are easily estimated from sample counterpart. The trimmed L-moments λ_p, $p \in N$ are expectations of linear functions of the conceptual order statistics $\tilde{X}_{1:N} < \ldots < \tilde{X}_{N:N}$. A conceptual random sample is a set $\tilde{X}_1, \ldots, \tilde{X}_N$ of independent with the same distribution as random X. The p-trimmed L-moments of order 1 are defined from a conceptual random sample of size $2p+1$, $p \in N$ and is equal to the expectation of the median of a conceptual random sample, for $p \geq 0$, (Darolles et al. [12]);

$$\begin{aligned} \lambda_{1,p} &= E\left[\tilde{X}_{(p+1):(2p+1)}\right], \\ &= \int_0^1 Q(u) P_{1,p}(u) \mathrm{d}u, \end{aligned} \tag{B.1}$$

where Q is the quantile function defined as the general inverse of the cumulative distribution function and $P_{1,p}$ is a nonnegative polynomial[14] with unit mass.

$$P_{1,p}(u) = \frac{(2p+1)!}{p!} u^p (1-u)^p.$$

The trimmed L-moments for $p \geq 1$ can be defined even when the expectation of X does not exist, Darolles et al. [12].

The L-moments of order 2 measure the expected slope of conceptual order statistics, as a function of rank i. Consider ranks $q+1$ and $2p-q+1$, equally distant by $|p-q|$ from the middle rank $n+1$ of the conceptual sample. The (q,p)-trimmed L-moment of order 2 defined for $p \geq 0,\ 0 \leq q \leq p-1$ by

$$\begin{aligned} \lambda_{1,q,p} &= E\left[\tilde{X}_{(2p-q+1):(2p+1)} - \tilde{X}_{(2p+1):(2p+1)}\right], \\ &= \int_0^1 Q(u) P_{1,q,p}(u) \mathrm{d}u \end{aligned} \tag{B.2}$$

where the polynomials $P_{1,q,p}$ generate the space of odd functions

$$P_{2,q,p}(u) = \frac{(2p+1)!}{q!(2p-q)!} \left[u^{2p-q}(1-u)^q - u^q(1-u)^{2p-q}\right]$$

The L-performance is a ratio of a trimmed L-moment of order 1 and a trimmed L-moment of order 2:

$$L_{q,p} = \frac{\lambda_{1,p}}{\lambda_{1,q,p}}, \qquad p \geq 0,\ 0 \leq q \leq p-1 \tag{B.3}$$

For different values of (quantile) trimming parameters p and $q/(p+1)$, we get a battery of L-performances.

[14] More detail on the polynomials in Darolles et al. [12]