Dynamic Optimal Hedge Ratio Design when Price and Production are stochastic with Jump

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Abstract

In this paper, we focus on the farmer's risk income, by using commodity futures, when price and output processes are correlated random represented by jump-diffusion models. We evaluate the expected utility of the farmer's wealth and we determine, at each instant of time, the optimal consumption rate and hedge position at given the time to harvest and state variables. We find a closed form optimal position of consumption and position rate in case of CARA utility investor. This result (see table 1.5) is a generalization of Ho (1984) result who consider the particular case where price and output are diffusion models.

Key words: Jump-diffusion process, futures, stochastic dynamic programming, Lévy measure, risk management

1. Introduction

Agricultural production is a risky business. Because agriculture is often carried out in the open air and always involves the management of living plants, it is particularly exposed to risk. Production risks are often the result of poor predictability in the medium or long term, weather conditions, and uncertainty about crop performance due to the impact of pests, disease or many other unpredictable factors. For several years, the countries of the world have intervened to help farmers cope with the risks associated with the production or the variation of the prices on the markets of agricultural commodities. Some national or international developments have led some governments to reorient their agricultural policies towards deregulation, and a more market-oriented approach. Much of the protection of some farmers against the uncertain future volatility of the markets has been removed. In addition, issues as risky as food security and environmental impacts are receiving increasing attention. Thus, future risk analysis and risk management in agriculture can be expected to receive increased attention (see Stulz (1984)-Stulz (1996) and some references therein).

In addition, prices of agricultural inputs and outputs are often overlooked when a farmer has to decide how much input to use, or how much produce to produce, so that income risk due to market volatility or production is diminished. In addition to the price risk, it takes into account the exchange rate risk in the context of the import of inputs or the export of agricultural production. Note that, governments can be another source of institutional risk (political risk, sovereign risk, etc.) for farmers. Changes in the rules that affect agricultural production would have, as a corollary, implications for profitability. For example, changes in the income tax provisions or the availability of various incentive payments. Horticultural producers may be severely affected by new restrictions on the use of pesticides, just as owners of intensive live animal farms may be affected by the introduction of restrictions on the use of drugs for the prevention and control of pesticides. the treatment of diseases. One can also cite the contractual risks, that is to say the risks inherent in the relations between the trading partners. For example, the unexpected breakdown of agreements among participants in supply chains is a significant source of risk in modern agribusiness.

Many people or organizations who need to concern themselves with risk in agriculture include: farmers, farm advisers, commercial firms selling to or buying from farmers, agricultural research workers, and policy makers and planners. Policymakers and planners of public or private agricultural policies also need to consider risks and farmers' responses to risks. Thus, models that include risk could provide better predictions of farmer behavior than those that do not. It should be noted that agricultural policy development and planning are themselves risky activities. It is also necessary for farmers to plan the risks they face. The well-being of the farmer, as well as the survival of the farm business, may depend on how agricultural risks are managed. Occurrence of extreme risks could jeopardize the survival of an agricultural activity. The ambition of many farmers to return the farm to a flourishing condition in the coming years may be frustrated if risk management is neglected. Farmers must also recognize that risk and risk aversion influence their production decisions or more generally management. Mention should also be made of the risks associated with innovation. Farmers must not neglect the risks inherent in moving from an existing production system to another system that is supposed to be "superior". It should be mentioned that the adoption of untested "improved" technology may involve a high degree of risk to the farmer, especially if the adoption of the technology requires a substantial capital investment. This risk may be higher if the farmer has no direct experience of the new method.

Similarly, agricultural traders who buy from farmers may well consider the farmers' willingness to reduce market risks. Thus, some of them will be willing to accept a lower price for their production, if the buyer is willing to offer a futures contract at a guaranteed price. The buyer and the seller may benefit from such an arrangement. The purchase or sale of derivatives may be useful in reducing price risk for both future and future products. The most important examples are the futures market hedging of commodities or the purchase or sale of call or put options (see Sakong et al. (1993)), depending on the farmer's need for hedging (see Anderson and Danthine (1980) and Anderson and Danthine (1983)). The hedging on the futures market is quite similar to the forward contract sale but with a number of differences. One important difference is that futures contracts are standardized and largely traded contracts, so that prices are determined more competitively than for a specific contract between a single farmer and a single trader. Thus, the farmer can get a better deal by hedging on the futures market than by contract sales. In fact, the seller of a futures contract undertakes to deliver a quantity fixed in the contract, of a defined category of the product traded at a given date. The buyer undertakes to receive the delivery of this quantity of product on this date at the price fixed in the contract. In practice, the delivery of the goods often does not take place. For example, a farmer may hedge to reduce future price risk by selling a futures contract for a product that is as close as possible to the product the farmer expects to have. For more details concerning Futures Hedging under price or production risks see Moschini and Hennessy (2001), Harvey Lapan (1991), Lapan and Moschini (1994) and Lioui and Poncet (1996) - Lioui and Poncet (2003). The cocoa producer plans to sell the cocoa in September. In May the producer decides to hedge on the futures market and so sells a futures contract for an amount of cocoa approximately equal to total production. For convenience, assuming that the October futures price at that moment is 500 euros/ton. By selling such a contract the farmer is agreeing to supply the specified quantity of cocoa of the specified quality in October at the contract price.

It is also important to take into account the risk aversion of the farmers (see Morgenstern and Von Neumann (1953) and Lien and Hardaker (2001) for more details). The function of absolute risk aversion to risk can be categorized according to its evolution in relation to the increase in wealth, such as the increase in absolute risk aversion (IARA), the absolute risk aversion (CARA) or decreasing absolute risk aversion (DARA). As a result, relative risk aversion can be categorized as increasing, constant or decreasing with wealth (IRRA, CRRA and DRRA, respectively). Constant risk aversion (CARA) means that preferences are unchanged if a constant sum is added or subtracted from all payments. Constant relative risk aversion (CRRA) means that preferences among at-risk perspectives are unchanged if all gains are multiplied by a positive constant. A more general and flexible utility function that characterize risk aversion is proposed in Aït-Sahalia et al. (2009). In this paper, we consider as in Ho (1984), that the farmer has an aversion to the constant risk of CARA type.

Risk hedging can be thought of as the process of simultaneously selecting the futures position and the underlying asset positions to build a portfolio of assets. The futures markets on commodities are these organized markets with a clearing which eliminates counterparty risk, liquidity risk and allows hedge against adverse exchange rate movements. To do this the agent must establish a strategy for risk management. What strategy should he choose ? Considering the approach of Morgenstern and Von Neumann (1953) who demonstrated in that the maximization of expected utility is a criterion for rational decision. The optimum consumption and portfolio rules in a continuous-time model has been introduced in Merton (1971). In the same spirit, Ho (1984) maximizes the consumption and wealth utility function of a farmer in the context of agricultural commodities . The latter paper use a model of consumption and investment in continuous time to determine the optimal behavior to have in the markets by a US wheat producer subject to the risks of prices and quantities through the monthly data for the period 1977 - 1980.

Anderson and Danthine (1983), Marcus and Modest (1984), Ho (1984) and Hey (1987) first develop dynamic Hedging models which assumes that producers can revise their hedge position during the growing season. Ho (1984) allowed for hedge positions to be continuously adjusted over time. Karp (1988) extended the Anderson and Danthine's models by adding stochastic production. Karp (1987) has developed a continuous model similar to Ho (1984). Unfortunately, paper Ho (1984) assumes that the evolution of commodity prices follows a Brownian-type continuous diffusion process. Indeed, the Brownian-type processes do not really take into account, the occurrence of jumps or strong turbulence in the evolution of the price of convenience. It should be noted that jumps diffusion process are among the stochastic processes, that are most often used in financial econometrics, to model the dynamics of asset prices. Discontinuities in these random processes are modeled with jumps. In addition, asset prices sometimes have breaks in their evolution, especially during periods of turbulence, such as the occurrence of bubbles or financial crises. In our paper extends Ho's model by assuming that the evolution of price follows a stochastic process with Levy jump, and that the logarithmic variation of the quantity produced by the farmer follows a normal distribution. Fore more details on jumps diffusion processes (see for instance Aït-Sahalia et al. (2009), Cont and Tankov (2004), Hanson (2007) and some references therein).

In this paper, we study a normative model of optimal hedging strategy of the farmer coupled with its consumption behavior in a continuous time jumps-diffusion framework. Some papers such as Rolfo (1980) has analyzed the farmer's hedging strategies in a one-period context. However, as mentioned in Ho (1984), in a one-period model some rather restrictive conditions are imposed on the individual's behavior. The one-period model assumes that the farmer ignores all past information. As a result, the farmer does not adjust over time its consumption and its hedging strategy against the risk of price and production through future contracts. These conditions can be relaxed in a continuous-time jump-diffusion framework. In an intertemporal framework, the farmer's hedging problem may be based on the assumptions that follows. We assume that the farm income (at harvest) depends on both the spot price and the output (in bushels), the farmer is subject to both price and output uncertainties during the production period. The farmer can hedge in commodity futures market, but he cannot eliminate both the price and production uncertainties by any hedging strategy. Other papers have also looked at intertemporal setups. In a discrete-time framework, Neuberger (1999) examines hedging long-term commodity supply commitment with multiple short term futures contracts. Duffie and Stanton (1992) price continuously resettled contingent claims, which bear some similarities with continuous-time instantaneous forward contracts, as the current market value of such claims is always zero. Smith and Stulz (1985) classify rationales for hedging into two categories: costs and risk aversion. They devote entire section IV in their paper to hedging motivated by risk aversion of managers. Mathematically, manager maximizes an objective function (expected utility) which is already concave. In that case individual preferences result directly in hedging behavior.

Our contribution is to revisit the work of Thomas Ho in the same context, taking into account jumps in price trends. Under our new assumptions regarding the stochastic evolution of price and quantity, added to that of Ho (1984), we obtain a coverage ratio (see the last line of Table 1.5), which is quite different from that found in Ho's paper.

This article is divided into four sections. This article is divided into four sections. The first section entitled "Modeling" study the optimal portfolio and determine the design pf its optimal decisions, and the last section presents the final conclusion and remarks and possible some possible orientations of Farm Risk management. The first section of the appendix entitled "jump-diffusion process" introduces the preliminaries concepts of stochastic calculus for jump-diffusion model, and the section of the appendix entitled "Stochastic Dynamic Programming" provides tools for dynamic programming process jump-diffusion.

2. Modeling

2.1. Assumptions

A-1
- The farmer's optimal behavior is determined in a continuous-time, finite horizon framework. The harvest time T is assumed to be known.
- At the beginning of the period (the planting season), the farmer makes his production decision on the quantity of crops, Q bushels, to be produced. At the end of the period (the harvest time), he sells all the crops at the prevailing spot price P (per bushel), and hence, his farm income is given by PQ.

A-2
- During the production period (time between the planting and harvest seasons) the farm income is subject to two sources of uncertainty:
  1. the price at which the crop will be sold (Price risk), P
  2. the quantity of the crop which will be sold (output risk)Q.

- At each time t, the farmer forms expectations of the spot price P_t and output Q_t at harvest, and then at the next instant, with more information, the farmer revises the expectations:
- P_t and Q_t are Itô-Lévy processes.

$$dQ_t = \sigma_Q Q_t dZ_t \tag{1}$$

$$\frac{dP_t}{P_{t_-}} = \sigma_P d\omega_t + (e^J - 1)dN_t - \lambda E(e^J - 1)dt \tag{2}$$

$$dZ_t d\omega_t = \rho dt \tag{3}$$

σ_Q and σ_P are the constant instantaneous standard deviation of the anticipated output and anticipated spot price, respectively, ρ is the instantaneous correlation coefficient, dZ_t and $d\omega_t$ are standardized Wiener processes and dN_t is a Poisson process with arbitrary distributed jump amplitude J. The symbol E in front of $(e^J - 1)$ stands for the expectation under physical measure and the constant variable λ is the jump intensity of the Poisson process.The two stochastic processes N_t, ω_t and J are mutually independent. For more details see Zhang et al. (2012) et Ho (1984).

A-3 There are frictions in the real sector.

i) During the production period, the farmer cannot affect the output level by committing further investment (buy more acreage) or disinvestment (land abandonment or selling off portions of the business).

ii) Significant agency costs prohibit the farmer from shifting the uncertainty of farm income to the market through issuance of shares.

A-4
- The futures market is assumed to be perfect:
  - Participants can trade costlessly and continuously
  - The futures contracts are perfectly divisible.
  - The contracts have the settlement date at the time of harvest.
  - mark-to-market settlements occur continuously through time, so that the net value of the contract always remains zero.
- Each contract calls for a delivery of one bushel.Those holding long positions promise to take delivery of the underlying crop and make payment at the futures price of the contract; those holding short positions promise to make delivery of the underlying com- modity and receive payment at maturity at the futures price of the contract.

- Let F_t denote the equilibrium settlement price on the contract at time t. F_t is stochastic, and it is characterized by $F_t = P_t$ for more details see Mahul and Vermersch (2000). Thus $dF_t = dP_t$.

A-5 The farmer has a cash account:W_t Wt that may be positive (negative) on which a constant rate of interest r is received (paid).Any net cash flow resulting from either position in futures contracts or from consumption is deposited or withdrawn from this cash account. Therefore, the borrowing rate equals the lending rate.

A-6 During the period, at each instant t, the farmer has to determine optimal consumption rate
c_t^*: optimal consumption rate and
x_t^* the futures position : such that the expected additive utility of consumption is maximized

$\max_{c,x} E[\int_0^T U(c,t)dt + B(Y_T,T)]$ with $Y_T = P_T Q_T + W_T$

where $\forall t \geq 0$ Y_t the total wealth at time t, $P_t Q_t$ is the crop at time t, W_t: the cash account value at time t, $B(.,T)$ is the terminal utility of wealth and is assumed to be a concave function and $U(c,t)$ an instantaneous utility function for consumption such that
$: \begin{cases} \frac{\partial U}{\partial c} > 0 \\ \frac{\partial^2 U}{\partial c^2} < 0. \end{cases}$

2.2. Objective function and the Optimal Decisions

- Let x_t denote the number of contracts held by the farmer at time t in a short position
- Let dF_t: variation due to an increase in the settlement price of the futures
- $x_t dF_t$: denote the amount, in cash, to pay at the clearing corporation by the farmer.
- the change in the farmers cash account is the sum of three cash flows:
  - The interest earned from the cash account: $rW_t dt$
  - The consumption : $-c_t dt$
  - the mark-to-market settlement of his futures position
- This is summarized by the following budget constraint equation

$$dW_t = (rW_t - c_t)dt - x_t dF_t \tag{4}$$

- To derive optimal decisions, x_t^* and c_t^*, we utilize the Bellman stochastic dynamic programming technique.
  The objective function

$$J^T(W,F,Q,t) = \max_{c_t,x_t,t<T} E_t[\int_t^T U(c,s)ds + B(Y_T,T)] \tag{5}$$

  where E_t is an expectation operator, conditional on $W(t) = W, F(t) = F, Q(t) = Q$ and $c_s > 0$.

- Substituting the minimization operator by maximization one into (75) J^T is the solution of Hamilton-Jacobi-Bellmann equation

$$J_t^T + \max_{c,x}[\mathscr{H}(y,t)] = 0 \tag{6}$$

  this can also be written as follows

$$\max_{c,x}[dJ^T + U(c,t)dt] = 0 \tag{7}$$

  satisfying the boundary condition

$$J^T(W,F,Q,T) = B(F_T Q_T + W_T, T). \tag{8}$$

- The boundary condition impose that the optimized derived utility equal to, (at time T), the terminal utility of wealth, since the consumption equal to zero when the wealth equal to zero to. This is the initial value to our dynamic programming. Thus, this problem is a time backwardation problem. See Beckmann and Czudaj (2013) for further details

3. Solution

3.1. Evaluation of dynamics dW_t, dY_t and dJ^T

Let us evaluate dW_t.

since $\begin{cases} dF_t = dP_t \\ F_t = P_t, \end{cases}$ then

$$\frac{dF_t}{F_{t_-}} = \sigma_F d\omega_t + (e^J - 1)dN_t - \lambda E(e^J - 1)dt \tag{9}$$

$$dQ_t = \sigma_Q Q_t dZ_t \tag{10}$$

$$dZd\omega = \rho dt \tag{11}$$

Substituting dF_t into (4), we obtain

$$dW_t = (rW_t - c_t)dt - x_t dF_t \tag{12}$$

$$= (rW_t - c_t)dt - x_t F_{t\_}\sigma_F d\omega_t + x_t F_{t\_}[\lambda E(e^J - 1)dt - (e^J - 1)dN_t] \tag{13}$$

Thus

$$dW_t = [rW_t - c_t + x_t\lambda F_{t\_}E(e^J - 1)]dt - x_t F_{t\_}\sigma_F d\omega_t - x_t F_{t\_}(e^J - 1)dN_t \tag{14}$$

Let us evaluate a dynamic dY_t of the wealth farmer Y_t .
Since

$$Y_t = P_t Q_t + W_t = F_t Q_t + W_t, \tag{15}$$

then

$$dY_t = dF_t Q_t + dW_t \tag{16}$$

Applying (57) to dF_tQ_t, such that (16) takes the form

$$dY_t = F_t dQ_t + Q_t dF_t + d[F_t, Q_t] + dW_t \tag{17}$$

Applying (62) to $d[F_t, Q_t]$, such that (17) takes the form

$$dY_t = F_t dQ_t + Q_t dF_t + d < F_t, Q_t >^c + \Delta F_s \Delta Q_s + dW_t \tag{18}$$

Since $d < F_t, Q_t >^c = \rho\sigma_F\sigma_Q F_{t-}Q_t dt$ and $\Delta Q_s = 0$, then (17) takes the form:

$$\begin{aligned} dY_t = [(rW_t - c_t) - \lambda(Q_t - x_t)E(e^J - 1)F_{t-} + \rho\sigma_F\sigma_Q F_{t-}Q_t]dt \\ + \sigma_F(Q_t - x_t)F_{t-}d\omega_t + \sigma_Q F_{t-}Q_t dZ_t \\ + (Q_t - x_t)F_{t-}(e^J - 1)dN_t \end{aligned} \tag{19}$$

Let (ω^1, ω^2) be two independent standard Wiener Processes satisfying : $\omega_t = \omega_t^1$ and $Z_t = \rho\omega_t^1 + \sqrt{1-\rho^2}\omega_t^2$. Thus (19) takes the form:

$$\begin{aligned} dY_t = \underbrace{[(rW_t - c_t) - \lambda(Q_t - x_t)E(e^J - 1)F_t + (\rho\sigma_F\sigma_Q F_t Q_t)]}_{\alpha_1} dt \\ + \underbrace{[(Q_t - x_t)F_t\sigma_F + \rho\sigma_Q F_t Q_t]}_{\alpha_2} d\omega_t^1 + \underbrace{\sigma_Q F_t Q_t\sqrt{1-\rho^2}}_{\alpha_3} d\omega_t^2 \\ + (Q_t - x_t)F_t(e^J - 1)dN_t \end{aligned} \tag{20}$$

Let us evaluate dJ^T.
Applying (71) to $J^T(Y_t, t) = \max_{c_t, x_t} E_t[\int_t^T U(c,s)ds + B(Y_T, T)]$, we obtain:

$$\begin{aligned} dJ^T = J_t^T dt + J_Y^T \alpha_1 dt + \frac{1}{2}J_{Y^2}^T(\alpha_2^2 dt + \alpha_3^2 dt) \\ + \lambda dt\int[J^T(Y_t + (Q_t - x_t)F_t(e^J - 1)z, t) - J^T(Y_t, t)]\nu(dz) \end{aligned} \tag{21}$$

as $E_t(d\omega_t^i) = 0$ for $i = 1;2$.

End this paragraph by the dynamic programming equation.
Substituting (21) into $\max_{c,x}[dJ^T + U(c,t)dt] = 0$, we obtain this following result:

$$\max_{c_t,x_t}\Bigg(J_t^T + J_Y^T[(rW_t - c_t) - \lambda(Q_t - x_t)E(e^J - 1)F_t + (\rho\sigma_F\sigma QF_tQ_t)]$$
$$+\frac{1}{2}J_{Y_t^2}^T F_t^2[(Q_t - x_t)^2\sigma_F^2 + 2\rho(Q_t - x_t)\sigma_F\sigma_Q Q_t + \sigma_Q^2 Q_t^2]$$
$$+\lambda\int[J^T(Y_t + (Q_t - x_t)F_t(e^J - 1)z,t) - J^T(Y_t,t)]\nu(dz) + U(c_t)\Bigg) = 0. \quad (22)$$

As equation (22) exist, the optimal decisions (c^*, x^*) satisfy the first order conditions.

3.2. Evaluation of c_t^* and x_t^*

Let us evaluate c_t^*.
The first order condition is obtained by deriving the terms into maximization operator with respect to c. We obtain:

$$-J_Y^T + U_c(c,t) = 0 \quad (23)$$

thus, t The optimal consumption rate is determined such that the marginal utility of consumption equates the marginal derived utility in wealth.
we obtain:

$$U_c(c^*,t) = J_Y^T(Y,t) \quad (24)$$

. This result was obtained by Ho (1984) in case of no jump into dynamic processes.

Hence c^* is determined independently of the hedging decisions:

$$c^* = [\frac{\partial U}{\partial c}]^{-1}[\frac{\partial J^T(Y,t)}{\partial Y}]. \quad (25)$$

Let us evaluate x_t^*.

Let us write the second condition of first order and derive the optimal position x_t^*.
To do this, we utilize an utility functions defined by: $U(c,t) = e^{-\beta t}V(c)$ and $G(Y,t) = e^{-\beta t}L(Y)$ where V is the utility function of consumption, L is the utility function of wealth. The choice of U and G (as product of two functions with separable variables) is motivated by the fact that the consumption and wealth processes are Markov processes.
More, we suppose that these two utility functions are exponential utility function type: $V(c) = -\frac{1}{q}\exp(-qc)$ and $L(y) = -\frac{K}{q}\exp(-rqy)$ where $q > 0$ and K a positive constant.
Let us evaluate c_t^* in this case.

since $\frac{\partial L(y)}{\partial y} = rK\exp(-rqy) = -rqL(y)$ and $\frac{\partial^2 L(y)}{\partial y^2} = r^2q^2L(y)$, thus (25) takes the form:

$$c^* = rY - \frac{1}{q}\log(rK). \tag{26}$$

Substituting these two functions into (22), we obtain:

$$\begin{aligned}0 = \max_{c_t,x_t}\Bigg(& V(c_t) - \beta L(Y_t) - rqL(Y_t)[(rW_t - c_t) + \rho\sigma_F\sigma_Q F_t Q_t - \lambda(Q_t - x_t)E(e^J - 1)F_t] \\ & + \frac{1}{2}r^2q^2L(Y_t)[(Q_t - x_t)^2\sigma_F^2F_t^2 + 2\rho(Q_t - x_t)\sigma_F\sigma_Q F_t^2 Q_t + \sigma_Q^2F_t^2Q_t^2] \\ & + \lambda\int[e^{-rq[(Q_t - x_t)F_t(e^J-1)z]}L(Y_t) - L(Y_t)]\nu(dz)\Bigg).\end{aligned} \tag{27}$$

We divide this term by $rqL(Y_t)$. since $qL(Y_t) < 0$, thus the max is replaced by the min (27) takes the form:

$$\begin{aligned}0 = \min_{c_t,x_t}\Bigg(& \frac{V(c_t)}{rqL(Y_t)} - \frac{\beta}{rq} - [(rW_t - c_t) + \rho\sigma_F\sigma_Q F_t Q_t - \lambda(Q_t - x_t)E(e^J - 1)F_t] \\ & + \frac{1}{2}rq[(Q_t - x_t)^2\sigma_F^2F_t^2 + 2\rho(Q_t - x_t)\sigma_F\sigma_Q F_t^2 Q_t + \sigma_Q^2F_t^2Q_t^2] \\ & + \frac{\lambda}{rq}\int[e^{-rq[(Q_t - x_t)F_t(e^J-1)z]} - 1]\nu(dz)\Bigg).\end{aligned} \tag{28}$$

Let us evaluate K :
We deduce its value by replacing (c_t, x_t) by (c_t^*, x_t^*) into (28). So we obtain:

$$\begin{aligned}K = \frac{1}{r}\exp\Bigg(& 1 - \frac{\beta}{r} - q[(rW_t - rqY) + \rho\sigma_F\sigma_Q F_t Q_t - \lambda(Q_t - x_t)E(e^J - 1)F_t] \\ & + \frac{1}{2}rq^2[(Q_t - x_t)^2\sigma_F^2F_t^2 + 2\rho(Q_t - x_t)\sigma_F\sigma_Q F_t^2 Q_t + \sigma_Q^2F_t^2Q_t^2] \\ & + \frac{\lambda}{r}\int[e^{-rq[(Q_t - x_t)F_t(e^J-1)z]} - 1]\nu(dz)\Bigg).\end{aligned} \tag{29}$$

To evaluate x_t^*, we distinguish two case: $\lambda = 0$ and $\lambda \neq 0$.

First case: $\lambda = 0$, i.e, the dynamic of price is pure diffusive process .
The condition of first order, i.e, the derivative of second term of equation (28) with respect to x is zero, applying it to (28) we obtain:

$$rq[(Q - x)\sigma_F^2F^2 + \rho\sigma_Q\sigma_F F^2 Q] = 0 \tag{30}$$

ainsi

$$x_{t,P}^* = (1 + \rho\frac{\sigma_Q}{\sigma_F})Q. \tag{31}$$

Remark: 3.1. The optimal hedge ratio $\frac{x_{t,P}^*}{Q} = (1 + \rho\frac{\sigma_Q}{\sigma_F})$ depend to the correlation ρ between the two uncertainties, the price P and the quantity Q. If the correlation is negative, the farmer'revenue is less uncertain, thus the farmer would not hedge his entire position in futures market.

Second case: $\lambda \neq 0$.

Like Aït-Sahalia et al. (2009), we choose a Levy measure to obtain a closed form.

Consider the Levy measure defined by: $\nu(dz) = \beta e^{-\eta z} \mathbb{I}_{\{z \geq 0\}} dz$ where β and η are strictly positive constant. This measure satisfies $\int_{\mathbb{R}} \min(1,|z|)\nu(dz) < \infty$

The calculation of integral term into (28) gives:

$$\frac{\lambda}{rq}\int [e^{-rq(Q_t-x_t)F_t(e^J-1)z}-1]\nu(dz) = \frac{\lambda}{rq}\int_0^{+\infty}[e^{-rq(Q_t-x_t)F_t(e^J-1)z}-1]ce^{-\eta z}dz \tag{32}$$

$$= \frac{\lambda\beta}{rq}\int_0^{+\infty}[e^{-[rq(Q_t-x_t)F_t(e^J-1)+\eta]z}-e^{-\eta z}]dz \tag{33}$$

$$= \frac{\lambda\beta}{rq}\Big[\frac{1}{rq(Q_t-x_t)F_t(e^J-1)+\eta}-\frac{1}{\eta}\Big]. \tag{34}$$

The second condition of first order applying to (28) give us:

$$-\lambda E(e^J-1)F_t - rq(Q_t-x_t)\sigma_F^2F_t^2 - rq\rho\sigma_F\sigma_Q F_t^2 Q_t + \lambda\beta\Big[\frac{F_t(e^J-1)}{(rq(Q_t-x_t)F_t(e^J-1)+\eta)^2}\Big] = 0. \tag{35}$$

(35) it is an implicit form of cubic equation in $(Q_t - x_t)$ contains x_t. Otherwise x_t is solution of (35).

Setting $S = (Q_t - x_t)$, (35) takes the form $aS^3 + bS^2 + cS + d = 0$ and setting $X = S + \frac{b/a}{3}$, the equation takes the form $X^3 + pX + q = 0$. The discriminant is $\Delta = \frac{p^3}{27} + \frac{q^2}{4}$. We solve in case where $\Delta < 0$ (this implies that $p < 0$). The equation has three real solutions. Since we minimize in the objective function , we consider the smallest solution defined by:

$$(Q_t - x_t^*) = -\frac{b/a}{3} + \sqrt{\frac{-4p}{3}}cos(\frac{1}{3}arccos(-q\sqrt{\frac{-27}{4p^3}}-\frac{2\pi}{3})) \tag{36}$$

By replacing p and q by its values we obtain:

$$\begin{aligned}
x_t^* =& (1+\frac{1}{3}\rho\frac{\sigma_Q}{\sigma_F})Q + \frac{2\eta}{3rqF} + \frac{\lambda E(e^J-1)}{rq\sigma_F^2F}(e^J-1)\\
&+\Bigg(+\frac{4}{9}\frac{\rho^2\sigma_Q^2Q^2}{\sigma_F^2} + \frac{8}{3}\big(\frac{2}{3}(e^J-1)-1\big)\frac{\eta\lambda E(e^J-1)}{r^2q^2\sigma_F^2F^2} - \frac{8\eta\rho\sigma_Q Q}{3rq\sigma_F F} + \big(\frac{16}{3}-\frac{4}{(e^J-1)}\big)\frac{\eta^2}{3r^2q^2F^2} + \frac{16\rho\sigma_Q Q\eta}{9\sigma_F rqF}\\
&+\frac{8\lambda\rho\sigma_Q QE(e^J-1)}{9rq\sigma_F^3F}(e^J-1) + \frac{4\lambda^2E^2(e^J-1)}{9r^2q^2\sigma_F^4F^2}(e^J-1)^3\Bigg)^{\frac{1}{2}} cos\Bigg(\frac{1}{3}arccos\Bigg(\Bigg(-\frac{2}{27}\rho^3\frac{\sigma_F^3Q^3}{\sigma_F^3} - \frac{16\eta^3}{27r^3q^3F^3}\\
&-\frac{2\lambda^3E^3(e^J-1)}{27r^3q^3\sigma_F^3F^3}(e^J-1)^3 - \frac{\lambda E(e^J-1)}{r^3q^3(e^J-1)\sigma_F^2F^3} + \frac{\lambda c}{r^2q\sigma_F^2F^3} - \frac{\eta^2\rho\sigma_Q Q}{r^2q^2\sigma_F^2F^2(e^J-1)}\\
&-\frac{2\lambda E(e^J-1)(e^J-1)}{9rq\sigma_F^2F}\big(\frac{\rho^2\sigma_Q^2Q^2}{\sigma_F^2}+\frac{4\eta^2}{r^2q^2F^2}\big) - \frac{2\rho\sigma_Q Q}{9\sigma_F}\big(\frac{\lambda^2E^2(e^J-1)(e^J-1)^2}{r^2q^2\sigma_F^4F^2}+\frac{4\eta^2}{r^2q^2F^2}\big)\\
&-\frac{4\eta}{9rqF}\big(\frac{\lambda^2E^2(e^J-1)(e^J-1)^2}{r^2q^2\sigma_F^4F^2}+\frac{\rho^2\sigma_Q^2Q^2}{\sigma_F^2}\big) - \frac{2\lambda^2E^2(e^J-1)(e^J-1)^2}{9r^2q^2\sigma_F^4F^2}\big(\frac{\rho\sigma^Q Q}{\sigma_F}+\frac{2\eta}{rqF}\big)
\end{aligned}$$

$$
- \frac{8\lambda\eta E(e^J-1)(e^J-1)\rho\sigma_Q Q}{r^2q^2\sigma_F^3F^2} + \frac{4\eta\rho\sigma_Q Q}{3rq\sigma_F F} + \frac{4\eta E(e^J-1)\lambda}{3r^2q^2\sigma_F^2F^2} + \frac{2\eta^2}{3r^2q^2(e^J-1)F^2}\Big)
$$

$$
\times \frac{3\sqrt{3}}{2}\Big(+ \frac{1}{3}\frac{\rho^2\sigma_Q^2Q^2}{\sigma_F^2} + 2\big(\frac{2}{3}(e^J-1)-1\big)\frac{\eta\lambda E(e^J-1)}{r^2q^2\sigma_F^2F^2} - \frac{2\eta\rho\sigma_Q Q}{rq\sigma_F F} + \big(\frac{4}{3} - \frac{1}{(e^J-1)}\big)\frac{\eta^2}{r^2q^2F^2} + \frac{4\rho\sigma_Q Q\eta}{3\sigma_F rqF}
$$

$$
+ \frac{2\lambda\rho\sigma_Q QE(e^J-1)}{3rq\sigma_F^3F}(e^J-1) + \frac{\lambda^2E^2(e^J-1)}{3r^2q^2\sigma_F^4F^2}(e^J-1)^3\Big)^{\frac{-3}{2}} - \frac{2\pi}{3}\Big)\Big). \quad (37)
$$

3.3. Comparative tabular

	Thomas HO et al.	Nyassoke et al.
Empirical facts	**Pure diffusion model**	**Jump-diffusion model**
Great and sudden variations in prices	Difficulty: need to a great volatility for the model	Generic property of the model
Markets are incomplete; some risks can not be hedged	Markets are complete	Markets are incomplete
Some strategies are better	All strategies lead to zero residual risk depending of choice of measure	Hedging is obtained by solving a optimization portfolio problem
Utility function of wealth:	CARA type	CARA type
Optimal consumption:	$c^* = [\frac{\partial U}{\partial c}]^{-1}[\frac{\partial J^T(Y,t)}{\partial Y}]$	$c^* = [\frac{\partial U}{\partial c}]^{-1}[\frac{\partial J^T(Y,t)}{\partial Y}]$
Optimal position:	$x_P^* = (1+\rho\frac{\sigma_Q}{\sigma_F})Q$	$x_t^* = (1 + \frac{1}{3}\rho\frac{\sigma_Q}{\sigma_F})Q + \frac{2\eta}{3rqF} + \frac{\lambda E(e^J-1)}{rq\sigma_F^2 F}(e^J - 1) + \Big(+ \frac{4}{9}\frac{\rho^2\sigma_Q^2 Q^2}{\sigma_F^2} + \frac{8}{3}\big(\frac{2}{3}(e^J - 1) - 1\big)\frac{\eta\lambda E(e^J-1)}{r^2q^2\sigma_F^2F^2} - \frac{8\eta\rho\sigma_Q Q}{3rq\sigma_F F} + \big(\frac{16}{3} - \frac{4}{(e^J-1)}\big)\frac{\eta^2}{3r^2q^2F^2} + \frac{16\rho\sigma_Q Q\eta}{9\sigma_F rqF} + \frac{8\lambda\rho\sigma_Q QE(e^J-1)}{9rq\sigma_F^3 F}(e^J - 1) + \frac{4\lambda^2E^2(e^J-1)}{9r^2q^2\sigma_F^4F^2}(e^J - 1)^3\Big)^{\frac{1}{2}} cos(\frac{1}{3}arccos(-q\sqrt{\frac{-27}{4p}} - \frac{2\pi}{3}))$

4. Conclusion

This work proposes an investment-consumption model, in continuous-time, which determine the optimal instantaneous consumption and optimal hedging position using futures by the farmer in managing income risk (price risk and production uncertainty). In our model we assume that the price process is a jump-diffusion process and it generalizes Ho (1984) who assumed that this price process is a diffusion process. We justify that the optimal instantaneous consumption is unchanged in both models. Assuming the farmer preferences for consumption and wealth are represented by exponential utility functions, we determine a closed formula of the optimal hedge ratio. Our principal result is the last line of the precede table 1.5 (see also the remark 1.1)

An open question is the implementation of the theoretical results with real data from Cameroonian cocoa market, to provide a hedging policy to stakeholder.

A. Jump-diffusion process

A.1. Counting measure of jumps

Definition A.1. *Let be* $t \geq 0$, $A \in \mathscr{B}(\mathbb{R}^d \setminus \{0\})$ *and the random quantity*

$$N(t,A) = \#\{0 \leq s \leq t; \Delta X_s \in A\} = \sum_{0 \leq s \leq t} \mathbb{I}_A(\Delta X_s). \tag{38}$$

For all ω*, the function of sets* $A \mapsto N(t,A)(\omega)$ *is a counting measure on* $\mathscr{B}(\mathbb{R}^d \setminus \{0\})$*. So*

$$A \mapsto \mathbb{E}[N(t,A)] \tag{39}$$

is a Borel measure on $\mathscr{B}(\mathbb{R}^d \setminus \{0\})$*. One denote* $\mu(.) = \mathbb{E}[N(1,.)]$ *is called intensity measure of process* X *.*

For all $t \geq 0$ and A lower bounded , we define a compensated random jump measure $\tilde{N}$ by :

$$\tilde{N}(t,A) = N(t,A) - t\mu(A). \tag{40}$$

Remark: A.1. $N(t,A)$ is a shorthand measure notation for the measure set $N([0,t] \times A)$.

A.2. Space-time Poisson process

Also called general compound Poisson processes, marked Poisson point processes. The space-time Poisson process is a generalization of Poisson process denoted by

$$\mathscr{N}(dt,dq) = \mathscr{N}(]t,t+dt],]q,q+dq]) \tag{41}$$

Poisson mark space Q :

$$\int_Q \mathscr{N}(dt,dq) = dN(t;Q) \tag{42}$$

et

$$\int_0^t \int_Q \mathscr{N}(ds,dq) = \int_0^t dN(s,Q) = N(t;Q) \tag{43}$$

.

Example: if $Q = \{1\}$, the number of jumps in $]t, t+dt]$ is deduced from:

$$\int_Q \mathscr{N}(dt,dq) = \mathscr{N}(dt,\{1\}) = N(dt) = dN(t;1) = dN(t) = dN_t \tag{44}$$

A.3. Jump-diffusion and conditional infinitesimal moments

The Poisson and Wiener processes provide stochastic differential equations (SDE) in continuous time for simple jump-diffusion state process $X(t)$,

$$dX(t) = f(X(t),t)dt + g(X(t),t)d\omega(t) + h(X(t),t)dN(t), \tag{45}$$

Where $X(0) = x_0$, with a set of continuous function $\{f, g, h\}$ taken as coefficients and eventually non linear.

N, a random Poisson measure with intensity $dt \bigotimes d\mu$ on $\mathrm{I\!R}_+ \times (\mathrm{I\!R}^d \setminus \{0\})$.
If $h : \mathrm{I\!R}^d \to \mathrm{I\!R}^n$ is a function Borel-measurable and if $A \in \mathscr{B}(\mathrm{I\!R}^d \setminus \{0\})$ satisfies $\mu(A) < +\infty$, one defined, for all $t \geq 0$ and $\omega \in \Omega$, Poisson integral of h by

$$\int_A h(z)N(t,dz) = \sum_{z\in A} h(z)N(t,\{z\}). \tag{46}$$

Ito processes are jump-diffusion processes. Levy processes are essentially jump-diffusion processes, but extended to processes with infinite rate of jump. Jump-diffusion processes with constants coefficients are Levy processes.

Assuming Poisson and Wiener processes are independents. The conditional moments are determined as follows:

$$E[dX(t)|X(t) = x] = (f(x,t) + \lambda(t)h(x,t))dt, \tag{47}$$

and

$$Var[dX(t)|X(t) = x] = (g^2(x,t) + \lambda(t)h^2(x,t))dt, \tag{48}$$

The jump in state process is given by

$$[X](Tk) \equiv X(T_k^+) - X(T_k^-) = h(X(T_k^-), T_k^-) \tag{49}$$

where

$$[X](t) = h(X(t),t)dN(t), \tag{50}$$

The infinitesimal moment and jump properties are very useful for modeling approximations of real applications, by providing a basis for estimating the coefficient functions f , g, and h, as well as some of the process parameters, at least in the first approximation, through comparison to the empirical values of the basic probability corresponding of the stochastic integral equation.

A.4. Stochastic Jump-Diffusion Chain Rule

The state process is decomposed into continuous changes,

$$d_{(cont)}X(t) = f(X(t),t)dt + g(X(t),t)d\omega(t) \tag{51}$$

and discontinuous or jump changes,

$$d_{(saut)}X(t) = [X](t) = h(X(t),t)dN(t) \tag{52}$$

such that

$$dX(t) = d_{(cont)}X(t) + d_{(saut)}X(t). \tag{53}$$

Thus, the change of a composite function of the state process $X(t)$, $dF(X(t),t)$, can be decomposed into the sum of continuous and discontinuous changes.
The function $F(x,t)$ is assumed to be at least twice continuously differentiable in x and once in t. This leads to

$$d_{(cont)}F(X(t),t) \simeq F_t(X(t),t)dt + F_x(X(t),t)d_{(cont)}X(t) + \frac{1}{2}F_{xx}(X(t),t)(d_{(cont)}X(t))^2, \tag{54}$$

For continuous composite and to

$$d_{(saut)}F(X(t),t) = (F(X(t)+h(X(t),t),t) - F(X(t),t))dN(t) \tag{55}$$

for jump composite.
Combining the continuous and discontinues changes, we derive:

$$\begin{aligned} dF(X(t),t) =& F(X(t)+dX(t),t+dt) - F(X(t),t) \\ =& F_t(X(t),t)dt + F_x(X(t),t).(f(X(t),t)dt + g(X(t),t)d\omega(t)) \\ &+ \frac{1}{2}F_{xx}(X(t),t).g^2(X(t),t)dt \\ &+ (F(X(t)+h(X(t),t),t) - F(X(t),t))dN(t). \end{aligned} \tag{56}$$

A.5. Quadratic covariation

Proposition A.1. *If X, Y are Levy processes, then*

$$X_tY_t = X_0Y_0 + \int_0^t X_{s-}dY_s + \int_0^t Y_{s-}dX_s + [X,Y]_s \tag{57}$$

where $[X,Y]$ is the quadratic covariation.

If N is a random Poisson measure on $[0,T]\times\mathbb{R}$ and $(\omega_t)_{t\in[0,T]}$, a Wiener process, both are independent. If

$$X_t^i = X_0^i + \int_0^t \phi_s^i d\omega_s + \int_0^t \int_{\mathbb{R}^d} \psi^i(s,y)N(ds,dy) \qquad i=1,2 \tag{58}$$

then the quadratic covariation $[X^1, X^2]$ is equal to

$$[X^1, X^2]_t = \int_0^t \phi_s^1 \phi_s^2 ds + \int_0^t \int_{\mathbb{R}^d} \psi^1(s,y)\psi^2(s,y)N(ds,dy). \tag{59}$$

or

$$[X^1, X^2]_t = \int_0^t \phi_s^1 \phi_s^2 ds + \sum_{0 \leq s \leq t} \Delta X_s^1 \Delta X_s^2. \tag{60}$$

Under differential form :

$$d[X^1, X^2]_t = \phi_t^1 \phi_t^2 dt + \int_{\mathbb{R}^d} \psi^1(t,y)\psi^2(t,y)N(dt,dy). \tag{61}$$

or

$$d[X^1, X^2]_t = \phi_t^1 \phi_t^2 dt + \Delta X_t^1 \Delta X_t^2. \tag{62}$$

B. Stochastic Dynamic Programming

In this section the essential is provided by Hanson (2007)

B.1. Model and principle of optimality

The general jump-diffusion SDE (45) is reformulated with an additional process, the control process $U(t)$ on $\mathscr{U}$, thus the dynamic'state is defined by :

$$\begin{aligned} dX(t) =& f(X(t),U(t),t)dt + g(X(t),U(t),t)d\omega(t) \\ &+ \int_Q h(X(t),U(t),t,q)\mathscr{N}(dt,dq;X(t),U(t),t) \end{aligned} \tag{63}$$

where $\omega(t)$ is a wiener process or diffusion process, $N(t;Q,X(t),U(t),t)$ is a Poisson process or jump process with jump amplitude mark random variable Q, and $\mathscr{N}(dt,dq;X(t),U(t),t)$ is the Poisson random measure.

The objective function has a control formulation which is the combination of a final cost at time t_f and cumulative instantaneous costs, given the initial data $(t, x(t))$. That is

$$V[X,U,t_f](x,t) = \int_t^{t_f} C(X(s),U(s),s)ds + S(X(t_f),t_f). \tag{64}$$

It is a functional of the processes $X(t)$ and $U(t)$, where $C(x,u,t)$ is the scalar instantaneous or **running cost function** on the time horizon $]t, t_f]$ given the state at t and $S(x,t)$ is the final cost function; both are assumed continuous. This is the Bolza form of the objective.

Let us end this section with the Principle of optimality

The expected cost for (64) is:

$$E_{(\omega,N)]t,t_f]}\Big[\int_t^{t_f} C(X(s),U(s),s)ds + S(X(t_f),t_f)\Big] \tag{65}$$

Hence, the optimal expected cost for (64) is :

$$\min_u E_{(\omega,N)]t,t_f]}\Big[\int_t^{t_f} C(X(s),U(s),s)ds + S(X(t_f),t_f)\Big] \tag{66}$$

the optimal expected cost (66) can be decomposed as follows:

Proposition B.1. *Under the hypothesis of decomposition rule and properties of jump-difffusion processes,*

$$\begin{aligned} v^*(x,t) = \min_{U]t,t+\delta t]} \Big[& E_{(\omega,N)]t,t+\delta t]}\Big[\int_t^{t+\delta t} C(X(s),U(s),s)ds \\ & + v^*(X(t+\delta t),t+\delta t)\Big|X(t)=x,U(t)=u\Big]\Big] \end{aligned} \tag{67}$$

Thus, we have formally derived the fundamental recursive formula of *stochastic dynamic programming* :

Proof:

$$\begin{aligned} v^*(x,t) &= \min_{U]t,t+\delta t]}\Big[E_{(\omega,N)]t,t+\delta t]}\Big[\int_t^{t+\delta t} C(X(s),U(s),s)ds \\ &\quad + \min_{U]t+\delta t,t_f]}\Big[E_{(\omega,N)]t+\delta t,t_f]}\Big[\int_{t+\delta t}^{t_f} C(X(s),U(s),s)ds \\ &\quad + S(X(t_f),t_f)|X(t+\delta t),U(t+\delta t)\Big]\Big]\Big|X(t)=x,U(t)=u\Big]\Big] \\ &= \min_{U]t,t+\delta t]}\Big[E_{(\omega,N)]t,t+\delta t]}\Big[\int_t^{t+\delta t} C(X(s),U(s),s)ds \\ &\quad + v^*(X(t+\delta t),t+\delta t)\Big|X(t)=x,U(t)=u\Big]\Big]. \end{aligned} \tag{68}$$

□

B.2. Hamilton-Jacobi-Bellman Equation (HJBE) for Stochastic Dynamic Programming

Using the Principle of Optimality (67) and by taking the limit of small δt, replacing δt by dt, we can systematically derive the partial differential equation of stochastic dynamic programming, also called the stochastic Hamilton-Jacobi-Bellman (HJB) equation, for the general, multi-dimensional Markov dynamics case. From the increment form of the state differential

$$dX(t) = X(t+dt) - X(t), \tag{69}$$

we consider the expansion of the state argument

$$X(t+dt) = X(t) + dX(t) \tag{70}$$

about $X(t)$ for small $dX(t)$ and about the explicit time argument $t+dt$ about t in the limit of small time increments dt, using an extension of Taylor approximations extended to include discontinuous (i.e, Poisson)

$$
\begin{aligned}
v^*(x,t) =^{dt} \min_u \Big[E_{(d\omega,dN)(t)} \Big[& C(x,u,t)dt + v^*(x,t) + v_t^*(x,t)dt \\
& + \partial_x v^*(x,t).\big(f(x,u,t)dt + g(x,u,t)d\omega(t)\big) \\
& + \frac{1}{2}\partial_{xx} v^*(x,t).d\omega(t)g(x,u,t)\big(g(x,u,t)d\omega(t)\big) \\
& \int_Q \big(v^*(x+\hat{h}(x,u,t,q),t) - v^*(x,t)\mathscr{N}(dt,dq,;x,u,t)\big)\Big]\Big]
\end{aligned}
\tag{71}
$$

Recall that $dN(t;x,u,t) = \int_Q \mathscr{N}(dt,dq;x,u,t)$ where the first t argument of dN is the time implicit to the Poisson process, while the second t argument is an explicit time corresponding to the implicit state and control parametric dependence and $\hat{h}(x,u,t,q)$ is jump amplitude with a corresponding multiplicative factoring of the Poisson random measure.
The next step is to take a conditional expectation over the isolated differential Wiener and Poisson processes by considering these expectations:

$$
\begin{cases}
E[d\omega(t)] = 0 \\
E[d\omega(t)d\omega(t)] = dt \\
E[\mathscr{N}(dt,dq;x,u,t)] = \lambda(t;x,u,t)dt\Phi_Q(dq;x,u,t) = \lambda(t;x,u,t)\Phi_Q(q;x,u,t)dqdt.
\end{cases}
\tag{72}
$$

With (72), (71) is transformed as follows

$$
\begin{aligned}
v^*(x,t) =& v^*(x,t) + v_t^*(x,t)dt + \min_u \big[C(x,u,t)dt \\
& + \partial_x v^*(x,t).\Big(f(x,u,t)dt + g(x,u,t)E_{d\omega}[d\omega(t)]\Big) \\
& + \frac{1}{2}\partial_{xx} v^*(x,t).g(x,u,t)g(x,u,t)E_{d\omega}[d\omega(t)d\omega(t)] \\
& \int_Q \big(v^*(x+\hat{h}(x,u,t,q),t) - v^*(x,t)E_{\mathscr{N}}[\mathscr{N}(dt,dq,;x,u,t)]\big)\Big]\Big] \\
=& v^*(x,t) + v_t^*(x,t)dt + \min_u \Big[C(x,u,t)dt + \partial_x v^*(x,t)(f(x,u,t)dt + 0) \\
& + \frac{1}{2}\partial_{xx} v^*(x,t).g(x,u,t)g(x,u,t)dt \\
& + \lambda \int_Q \big(v^*(x+\hat{h}(x,u,t,q),t) - v^*(x,t)\big).\Phi_Q(dq;x,u,t)dt\Big]
\end{aligned}
\tag{73}
$$

We note that the $v^*(x,t)$ value on both sides of the equation cancel and then the remaining common multiplicative factors of dt also cancel.
The Hamiltonian (technically, a pseudo-Hamiltonian) functional is given by:

$$
\begin{aligned}
\mathscr{H}(x,u,t) \equiv& C(x,u,t) + \partial_x v^*(x,t).f(x,u,t) \\
& + \frac{1}{2}g^2(x,u,t)\partial_{xx} v^*(x,t) \\
& + \lambda(t;x,u,t)\int_Q \big[v^*\big(x+\hat{h}(x,u,t,q),t\big) - v^*(x,t)\big].\phi_Q(q;x,u,t)dq,
\end{aligned}
\tag{74}
$$

Also, with sufficiently small dt, $U]t, t+dt]$ has been replaced by the conditioned control vector u at t.

Theorem B.1. ***Hamilton-Jacobi-Bellman Equation (HJBE) for Stochastic Dynamic Programming (SDP)*** *If $v^*(x,t)$ is twice differentiable in x and once differentiable in t, while the operator decomposition rules are valid, then*

$$0 = v_t^*(x,t) + \min_u[\mathscr{H}(x,u,t)] = v_t^*(x,t) + \mathscr{H}^*(x,t) \tag{75}$$

The optimal control, if it exists, is given by

$$u^* = u^*(x,t) = argmin_u[\mathscr{H}(x,u,t)] \tag{76}$$

subject to any control constraints.

(75) is not an ordinary PDE.

The equation (75) is the Hamilton-Jacobi-Bellman equation, is also called simply the Bellman equation, or the stochastic dynamic programming equation or the PDE of stochastic dynamic programming, or in particular, the PIDE of stochastic dynamic programming where PIDE denotes a partial integral differential equation.

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