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Group Cooperation against an Incumbent

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Group Cooperation against an Incumbent

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Abstract: In this paper, I study the ability of group members to cooperate against an incumbent in a repeated rent-seeking game and where group members and the incumbent have different valuations of the prize. I first consider that group members use Nash Reversion Strategies (NRS) to support cooperative behavior and show that full cooperation within the group is more easily sustained as a Stationary Subgame Perfect (Nash) Equilibrium (SSPE) as either group size, or the heterogeneity in the valuation of the prize, increases. In turn, I show that full cooperation within the challenger group can also be sustained as a Weakly Renegotiation-Proof Equilibrium (WRPE). Yet, an increase in group size makes it more difficult to sustain within-group cooperation but an increase in the relative valuation of the prize by group members still facilitates group cooperation.

Keywords: Collective Action; Group Cooperation; Repeated Game; Trigger Strategies; Renegotiation

JEL Classification: C73; D72; D74

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1 Introduction

Most of economic, social and political activities involve groups (countries, political parties, firms, administrative units, etc....) with shared common interests within groups and, potentially, conflicting interests across groups. The effectiveness of a group in carrying out its objective then crucially depends on how it deals with its collective action – or free-rider – problem. Olson's (1965) celebrated theory argues that this problem makes larger groups less effective than smaller groups and that, ultimately, collective action must break down when group size becomes very large.[1] The difficulties facing large groups seem to be also persistent in a dynamic setting. Despite the potential of trigger strategies to induce mutual cooperation in a repeated game (Axelrod, 1981), the common wisdom is that cooperation becomes more difficult to sustain as the size of the collectivity increases (e.g., Hardin, 1982; Olson, 1982; Sandler, 1992; Taylor, 1982).

A limitation of the traditional collective action theory, is that it analyses the collective action problem within a single group isolated form the rest of society. However, in many situations, collective action is often undertaken to counter similar actions by competing groups. Collective action might also be necessary for defending collective resources against an external entity or may also be undertaken for challenging the dominant position of an incumbent. In this paper, I analyze the logic of ongoing collective action in a repeated game setting in which an interest group – called the *challenger* group – invests some efforts to take over an incumbent for the award of a collective prize. The incumbent is referred to as such in that it does not have any collective action problem to solve. Also, the collective prize has the characteristics of a (pure) public good and all members of the challenger group have the same valuation of the prize, which can be different from that of the incumbent. One might think for example of several green NGOs making lobbying efforts in order to induce governments to pass a law on environmental protection in a specific production sector. The coveted public good could also be something more political as in the case of a parliamentary coalition challenging the power of the incumbent party.

In this paper, I will use a simple model where the *challenger* group's probability of winning the prize is given by the classic function introduced by Tullock (1980). In this context, I first investigate the effectiveness of Nash Reversion Strategies (NRS), *à la* Friedman (1971), to support full cooperation within the group. These strategies prescribe that any deviation from the cooperative level of effort is met with permanent reversion to the non-cooperative outcome. The difficulty of supporting cooperation within the group is then measured by the lowest discount factor supporting the optimal level of group effort as a subgame perfect outcome. I show that cooperation within the group is more easily sustained as either the relative valuation of the prize by group members or group size increases. This last result is in contrast with the common wisdom that group-size is detrimental to the effectiveness of trigger strategies to support cooperation. The intuition is that the Nash punishment threat becomes relatively more effective as group size increases. Next, I investigate whether full cooperation within the group can be sustained as a Weakly Renegotiation-Proof Equilibrium (WRPE), in the sense of Farrell and Maskin (1989). With perfect and frictionless renegotiation, full cooperation can indeed be sustained as a WPRE even in very large groups although it becomes more difficult to sustain as group size increases. However, an increase in the relative valuation of the prize by group members still makes it easier to sustain within-group cooperation. Thus, overall, I find only partial support for the Olson's theory.

[1] For a review of the literature on collective action theory *à la* Olson (1965), see Pecorino (2015).

My analysis is related to the literature on tacit cooperation in repeated games applied to public economics. McMillan (1979) was the first to apply NRS to a supergame model of private provision of a public good but without investigating the impact of an increase in the number of agents. In a prisoner's dilemma game with n players, Bendor and Mookherjee (1987) show that punishment strategies are not very valuable to sustain an efficient outcome in large groups when there is imperfect monitoring. Pecorino (1999) focuses on the effect of an increase in the number of contributors on their ability – as measured by the critical value of the discount parameter – to overcome free-riding in a model of private provision of a pure public good. He cannot derive monotonicity results but shows that cooperation can be maintained when the number of contributors goes to infinity for several specifications of the individual payoff function. Haag and Lagunoff (2007) analyze a collective action game played by a collection of heterogeneous individuals in terms of time preferences. They show that homogeneous and larger groups are more cooperative on average for a large class of collective action games.

All these works analyze the collective action problem within a group isolated from the rest of society. In contrast, in the present analysis, group action aims at contesting similar action by a unitary opponent. In another paper (Cheikbossian, 2012), I analyze the ability of group members to cooperate in rent-seeking for a private prize – fully divisible among group members – in a context of competition between two groups of unequal size. The two groups face a collective action problem and all individuals have the same valuation of the prize. I then show that the set of parameters for which cooperation can be sustained within the larger group as a subgame perfect outcome is as large as that for which cooperation can be sustained in the smaller group. Again, in the present analysis, I consider that one group only has a collective action problem to solve but the two entities have different valuations for the public prize. More importantly, I consider that group members can renegotiate in case within-group cooperation breaks down.

The present analysis is thus also related to the literature on the concept of 'weak-renegotiation proofness' applied to oligopoly models and to the issue of international agreements. Using Farrell and Maskin (1989)'s solution concept of WRPE for two-players games, Farrell (2000) shows that full collusion as a WRPE is impossible in repeated Bertrand competition when there are more than three firms, or more than nine firms in repeated Cournot competition (with linear demand and cost functions). Barrett (1999) develops a model of international cooperation where each country has a binary choice between cooperating or defecting and also shows that cooperation can only be supported by a 'small' number of countries. In a similar model, Asheim *et al.* (2006) show that two international agreements can do better than a single one for involving a larger number of countries in an agreement. Still in a model where players have a binary choice, Froyn and Hovi (2008) show that full participation can be sustained as WRPE provided that only a limited number of countries are permitted to punish a defection. Finally, Asheim and Holtsmark (2009) show that this result carries over a more general model where players or countries have a continuum of (emission) choices. Yet, they consider a very simple model since the Nash equilibrium of the stage game is an equilibrium in dominant strategies. In other words, the optimal choice of one player does not depend on the choices made by the other players. Furthermore, all players can participate to the cooperative agreement. In my analysis, there exist strategic interactions in that a change in a group member's action induces a change in the equilibrium actions not only of the other group members but also of the opponent.

2 The model

2.1 The stage game

I start by specifying the details of the stage game G. There are $n+1$ risk-neutral agents, indexed by $i = 0, 1, 2, ..., n$ who compete for a public prize. Agent 0 is a unitary entity – called the *incumbent* – and has a valuation V_I for the prize. Agents $i = 1, 2, ..., n$ are grouped into a common entity – called the *challenger* group – and join their efforts to take over from the incumbent in the award of the prize. All group members have the same valuation V_G of the prize, which is supposed to be higher than that of the incumbent, i.e. $V_G \geq V_I$. These valuations are publicly known. Let $x_i \in \Re_+$ be the rent-seeking expenditure/effort expended at unit cost by group member i, and $X = \sum_{i=1}^{n} x_i \in \Re_+$ be the sum of rent-seeking efforts of group members. The rent-seeking expenditure/effort of the external entity is denoted $Y \in \Re_+$. Following much of the contest literature, I assume that the probability of winning the prize $p : \Re_+^2 \to [0, 1]$ for group members is given by a contest success function, which has the Logit form, i.e.

$$p(X, Y) = \begin{cases} X/(X+Y) \text{ if } (X, Y) \neq (0, 0), \\ \\ 1/2 \text{ if } (X, Y) = (0, 0). \end{cases} \tag{1}$$

The preferences of all players are represented by an additively separable utility function, i.e.

$$v_i = p(X, Y) V_G - x_i, \tag{2}$$

for member $i = 1, 2, ..., n$ of the *challenger* group, and

$$v_0 = (1 - p(X, Y))V_I - Y, \tag{3}$$

for the *incumbent.*

I first analyze the one-shot equilibrium outcome in which group members do not cooperate in the contest with the opponent. Let $X_{-i} = \sum_{j \neq i} x_j$ be the sum of rent-seeking efforts of the group members excluding that of member i. Maximizing v_i with respect to x_i subject to the non-negativity constraint $x_i \geq 0$ yields group member i's best response to X_{-i} and to Y, i.e. $r_i(X_{-i}, Y)$, which is given by the following first order-condition[2]

$$\frac{Y}{(x_i + X_{-i} + Y)^2} V_G = 1, \tag{4}$$

with equality for $x_i > 0$.

Similarly, maximizing v_0 with respect to Y subject to the non-negativity constraint $Y \geq 0$, yields the best-response function of the incumbent to the collective effort X of the challenger group, i.e. $R_0(X)$, which is given by the following first order-condition

$$\frac{X}{(X+Y)^2} V_I \leq 1, \tag{5}$$

with equality for $Y > 0$.

[2] Note that the marginal return to an additional unit of individual rent-seeking effort (as well as to an additional unit of group rent-seeking effort) is decreasing in effort. Hence, each player's problem is strictly concave and the first-order conditions are both necessary and sufficient for characterizing the best-response functions of the players.

Due to the public-good nature of collective rent-seeking, first-order conditions only determine group effort. I look, however, for a symmetric equilibrium such that all group members make the same level of effort. The symmetric equilibrium is thus a couple $\left(x^N, Y^N\right)$ such that $R_0(X^N) = Y^N$ and $r_i(X^N_{-i}, Y^N) = x^N$ for all $i = 1, 2, ..., n$ (and where $X^N_{-i} + x^N = X^N = nx^N$).

Solving the system given by (4) and (5) and letting λ be the ratio of the valuations of the prize, i.e. $\lambda = V_G/V_I \geq 1$, we have that

$$x^N = \frac{\lambda}{n\left[1+\lambda\right]^2} V_G, \tag{6}$$

while $Y^N = X^N/\lambda$.

The equilibrium winning probability for the challenger group is thus given by

$$p(X^N, Y^N) = \frac{\lambda}{1+\lambda}, \tag{7}$$

which is independent of the size of the *challenger* group. Indeed, with the lottery contest success function and linear cost functions, the increased free-rider problem induced by a larger group size is exactly compensated by the larger number of contributors (see Katz et al., 1990, and Ursprung, 1990).

Under non-cooperation, each group member has thus the following utility

$$v^N = \frac{\lambda\left[n(1+\lambda)-1\right]}{n\left[1+\lambda\right]^2} V_G, \tag{8}$$

while that of the incumbent is given by

$$v_0^N = \frac{V_I}{[1+\lambda]^2}. \tag{9}$$

2.2 The optimal solution

Consider now a situation where there can be full cooperation within the challenger group. In other words, group members can jointly choose individual efforts so as to maximize the aggregate welfare of their group, i.e., they choose X so as to maximize $\sum_i v_i = np\left(X, Y\right) - X$, given Y. The *collective* best response of the challenger group to Y, i.e. $R(Y)$, is given by the following first-order condition

$$\frac{nY}{\left(X+Y\right)^2} V_G \leq 1, \tag{10}$$

with equality for $X > 0$.

Similarly, the best-response function of the incumbent to the collective effort X of the challenger group, i.e. $R_0(X)$, is given by the following first-order condition

$$\frac{X}{\left(X+Y\right)^2} V_I \leq 1, \tag{11}$$

with equality for $Y > 0$.

An interior equilibrium is a couple of efforts (X^C, Y^C) such that $R(Y^C) = X^C$ and $R_0(X^C) = Y^C$. Assuming that the members of the challenger group share equally the collective effort, and denoting by $x^C = X^C/n$ this common individual effort under within-group cooperation, I obtain

$$x^C = \frac{\lambda n}{\left[1+\lambda n\right]^2} V_G, \tag{12}$$

and $Y^C = x^C/\lambda$.

As a result the equilibrium winning probability for the challenger group is given by

$$p(X^C, Y^C) = \frac{\lambda n}{1+\lambda n}. \tag{13}$$

Under cooperation within the challenger group, each group member has thus the following utility

$$v^C = \left[\frac{\lambda n}{1+\lambda n}\right]^2 V_G, \tag{14}$$

while that of the incumbent is given by

$$v_0^C = \frac{V_I}{[1+\lambda n]^2}. \tag{15}$$

3 The repeated game

3.1 Preliminaries

The $n+1$ agents play an infinitely repeated game with discounting. Time is discrete and dates are denoted by $t = 0, 1, 2...$Let $G^\infty(\delta)$ be the repeated game obtained by repeating G infinitely often, and where $\delta \in (0,1)$ is the discount parameter per period for each player. I assume that the effort level produced by all group members and the incumbent are perfectly observed by all agents. Let $Y(t) \in \Re_+$ and $x(t) \equiv (x_1(t), x_2(t), ..., x_n(t)) \in \Re_+^n$ be, respectively, the effort of the *incumbent* and the vector of individual efforts within the *challenger group* in period t. A public history in period $t \geq 1$ is thus $h(t) = ((x(0), Y(0)), (x(1), Y(1)), ..., (x(t-1), Y(t-1)))$ or $h(t) = \{(x(s), Y(s))\}_{s=0}^{t-1}$. The initial history is the null set, $H_0 \equiv \{\varnothing\}$, and H_t the set of t-period histories. A pure strategy for group member i in $G^\infty(\delta)$, for $i = 1, 2, ..., n$, is an infinite sequence of functions $\sigma_i = \{\sigma_i^t\}_{t=1}^\infty$, where $\sigma_i^t : H_t \to \Re_+$ is a mapping from the set of all possible public histories up to $t-1$ (included) into the set of effort levels. Similarly, a pure strategy in $G^\infty(\delta)$ for the incumbent is defined on the same sets, i.e. $\sigma_0 = \{\sigma_0^t\}_{t=1}^\infty$, where $\sigma_0^t : H_t \to \Re_+$. The strategy profile is denoted by $\sigma \equiv (\sigma_0, \sigma_1, \sigma_2, ..., \sigma_n)$.

Any strategy profile σ generates an outcome path $S = \{x(t), Y(t)\}_{t=0}^\infty$, defined inductively by $\{x(0), Y(0)\} = \sigma(\varnothing)$ and $(x(t), Y(t)) = (\sigma_0^t, \sigma_1^t, ..., \sigma_n^t)$ for all $t \geq 1$. A path of rent-seeking efforts S thus implies an infinite stream of stage-game payoffs $\{v_i(x(t), Y(t))\}_{t=0}^\infty$ for group member $i = 1, 2, ..., n$ and $\{v_0(x(t), Y(t))\}_{t=0}^\infty$ for the incumbent. Let $v_i(t) \equiv v_i(x(t), Y(t))$ for $i = 0, 1, 2, ...n$ – with $i = 0$ for the *incumbent* and $i = 1, 2, ..., n$ for the members of the *challenger group*. The average discounted payoff of player i for the outcome path $S = \{x(t), Y(t)\}_{t=0}^\infty$ is given by

$$V_i^\delta(S) = (1-\delta)\sum\nolimits_{t=0}^\infty \delta^t v_i(t) \tag{16}$$

Thus, the average discounted payoff of player i, for $i = 0, 1, 2, ..., n$ in $G^\infty(\delta)$ obtained with the strategy profile σ is $V_i^\delta(\sigma) = V_i^\delta(S(\sigma))$. σ is a Nash equilibrium if σ_0 is a best response to $(\sigma_1, \sigma_2, ..., \sigma_n)$ and if σ_i, for $i = 1, ..., n$, is a best response to $\sigma_{-i} = (\sigma_0, \sigma_1, ..., \sigma_{i-1}, \sigma_{i+1}, ..., \sigma_n)$. And it is a subgame perfect equilibrium in $G^\infty(\delta)$ if after every history, the continuation of σ is a Nash equilibrium in the corresponding subgame. I will restrict attention to Stationary Subgame Perfect (Nash) Equilibria (SSPE), i.e., equilibria in which after any history, a stationary profile of strategies is played thereafter, and which also satisfy the additional requirement of symmetry within the *challenger* group, in the sense that all group members make the same level of effort at every history.

3.2 Nash Reversion Strategies

I first consider that the members of the *challenger* group use *Nash Reversion Strategies* (NRSs) *à la* Friedman (1971) in order to support cooperative behavior. NRSs prescribe that group members cooperate in the first period and in all subsequent periods if all members cooperated in the previous period. If any member deviated in the previous period, then all group members revert to the single-shot Nash equilibrium forever, independently of the behavior of the incumbent. Yet, group members' actions must be mutual best responses and must also be individual best-responses to the action of the *incumbent* – in case of non-cooperation within the group – or must constitute a collective best-response function to the action of the *incumbent* – in case of cooperation within the group. Furthermore, it is supposed that the *incumbent* plays in every period his static best-response to the group members' strategies – whether they cooperate or not – regardless of the history (i.e. $\sigma_Y^t = R_0(x(t))$ for all t). To put this informally, there is always non-cooperation between the *challenger* group and the *incumbent*.

In general, in infinitely repeated games, the set of SSPE is very large. I thus focus on the "best" SSPE from the viewpoint of group members, that is the one sustaining the joint-payoff maximizing level of rent-seeking effort x^C.[3] Formally, I thus focus on the strategy profile characterized by: $\sigma_i^t = x^C$ for $i = 1, 2, ..., n$ and $\sigma_0^t = R_0(X^C) = Y^C$ if $t = 0$ or if $h(t) = \left\{(x^C(s), Y^C(s))\right\}_{s=0}^{t-1}$ for $t \geq 1$; $\sigma_i^t = x^N$ for $i = 1, 2, ..., n$ and $\sigma_0^t = R_0(X^N) = Y^N$ otherwise. This strategy profile is denoted by $\tilde{\sigma}$.

Let x^D be the effort level of a contributing member who considers defecting from the cooperative phase while other play $\tilde{\sigma}$. The payoff of the deviator would thus be given by

$$v^D = \frac{(n-1)x^C + x^D}{(n-1)x^C + x^D + Y^C} V_G - x^D, \tag{17}$$

where x^C is given by (14) and where $x^C = \lambda Y^C$. The deviator chooses x^D so as to maximize v^D. The first-order condition to this problem implies that

$$\frac{\partial v^D}{\partial x^D} = \frac{Y^C}{\left[(n-1)x^C + x^D + Y^C\right]^2} V_G - 1 \leq 0. \tag{18}$$

In any period in which all group members contribute the joint-maximizing level of effort x^C, each member's best possible deviation from $\tilde{\sigma}$ is given by:

$$x^D = \begin{cases} = \dfrac{\sqrt{n}(1+\lambda n) - n\left[1 + \lambda(n-1)\right]}{(1+\lambda n)^2} V_G \text{ if } n = 2, \\ \\ 0 \text{ if } n \geq 3. \end{cases} \tag{19}$$

Thus each member's best possible deviation from $\tilde{\sigma}$ is to cut her contribution to 0 for any $n \geq 3$, independently of λ. In contrast, $x^D > 0$ for $n = 2$, independently of λ.[4]

Substituting (21) into (19) and using (14), I obtain the (best) deviation payoff for any group member,

[3] I must insist that this SSPE is Pareto-efficient only from the point of group members. This is reason why I use the term "best" SSPE.

[4] Indeed, $x^D \geq 0$ if and only if $[(1+\lambda n)/(1+\lambda(n-1))] \geq \sqrt{n}$. On the one hand, the left-hand term of this inequality is increasing in λ and thus reaches a maximum when λ is going to infinity, in which case the inequality reduces to $n/(n-1) \geq \sqrt{n}$ or $\sqrt{n} \geq (n-1)$, which does not hold for any $n \geq 3$. On the other hadn, the left-hand term of the above-mentioned inequality reaches a minimum in $\lambda = 1$, in which case it reduces to $(n+1)/n \geq \sqrt{n}$, that is $3 > 2\sqrt{2}$ (when $n = 2$).

that is

$$v^D = \begin{cases} = \dfrac{1+(1+\lambda n)\left[n(1+\lambda)-2\sqrt{n}\right]}{(1+\lambda n)^2} V_G \text{ if } n=2, \\ \\ \dfrac{\lambda(n-1)}{1+\lambda(n-1)} V_G \text{ if } n \geq 3. \end{cases} \tag{20}$$

The one-period net gain to deviating from the cooperative agreement within the group is $\left(v^D - v^C\right)$, while the per-period net benefit to maintaining cooperation within the group is $v^C - v^N$. Thus, the discounted value of avoiding a (permanent) breakdown of cooperation within the group is given by $[\delta/(1-\delta)]\left(v^C - v^N\right)$. Therefore, no group members have an incentive to deviate from the cooperative agreement if and only if

$$v^D - v^C \leq \frac{\delta}{1-\delta}\left[v^C - v^N\right]. \tag{21}$$

Focusing on situations where self-enforcement is a binding constraint on the abilities of group members to cooperate, the critical value of the discount parameter above which cooperation can be sustained as a SSPE through NRSs is then $\delta = \left(v^D - v^C\right)/\left(v^D - v^N\right)$. Substituting (10), (16) and (22) into this expression, I obtain

$$\delta^N(2) = \frac{2(3-2\sqrt{2})(1+2\lambda)(1+\lambda)^2}{8(2-\sqrt{2})\lambda^3 + 4(8-5\sqrt{2})\lambda^2 + (23-16\sqrt{2})\lambda + 2(3-2\sqrt{2})}, \tag{22}$$

if $n = 2$, and

$$\delta^N(n)\,|_{n\geq 3} = \frac{n(1+\lambda)^2\left[\lambda n(n-2)+(n-1)\right]}{(1+\lambda n)^2\left[\lambda(n^2-n-1)+(n-1)^2\right]} \tag{23}$$

if $n \geq 3$.

I have the following result.[5]

Result 1: (i) $\delta^N(n)$ *is decreasing in* n *for any* $n \geq 3$ *independently of* λ; (ii) $\delta^N(n)$ *is decreasing in* λ *for any* $n \geq 2$ *independently of* n.

The sustainability of within-group cooperation depends on the impact of increasing n or λ on the one-period net gain of deviating from the cooperative agreement – given by $v^D - v^C$ – relative to the net benefit of maintaining within-group cooperation – given by $v^C - v^N$. I focus on the situation where $n \geq 3$. I have

$$v^C - v^N = \frac{\lambda(n-1)\left[\lambda n^2(1+\lambda)-(1+\lambda n)\right]}{n(1+\lambda)^2(1+\lambda n)^2} V_G, \tag{24}$$

and

$$v^D - v^C = \frac{\lambda\left[\lambda n(n-2)+(n-1)\right]}{\left[1+\lambda(n-1)\right](1+\lambda n)^2} V_G. \tag{25}$$

As shown in the Proof of Result 1 (in the Appendix), the *net benefit of cooperation* – given by $v^C - v^N$ – is increasing in n. This reflects the fact that an increase in group size exacerbates the free-rider problem, which in turn makes the punishment threat relatively more effective. As regards the impact of group size on the incentive to deviate, that is on the *net benefit of defection*, – given by $v^D - v^C$ – it is decreasing in n for any $n \geq 4$. The incentive to deviate from the cooperative outcome is determined by the benefit of withdrawing his/her individual contribution and by the resulting negative impact on the probability of success of the group. But this last is concave in aggregate effort. Hence, the decrease in the probability of

[5] All the Proofs of the results are given in the Appendix.

success due to an individual defection becomes less pronounced as n increases, which would reinforce the incentive to defect. However, the concavity of the probability of success also implies that, when n increases, it increases the optimal level of aggregate effort but by a lower extent than the size of the group. In turn, an individual contribution and thus the benefit of withdrawing his/her contribution also decreases with n, which would relax the incentive to deviate. It turns out that the decrease in the benefit of withdrawing his/her individual contribution dominates that of the associated cost in terms of probability of success. As a result, the *net benefit* of defection decreases with group size and this effect goes in the same direction than the increase in the punishment threat for facilitating the sustainability of group cooperation. This is in contrast with the general presumption that decentralized strategies of reciprocity may fail to enforce cooperation in large groups. In the present analysis, where group members face an external entity, I have that an increase in group size unambiguously facilitates group cooperation. Furthermore, cooperation remains an equilibrium in very large groups if their members place any weight at all on the future since, then, the critical discount factor $\delta^N(n)\,|_{n\geq 3}$ converges to 0 as n goes to infinity.

Also, an increase in the valuation of the prize by group members compared to that of the incumbent – i.e. an increase in λ – makes within-group cooperation less difficult to sustain.[6] As shown in the appendix, an increase in λ increases the net benefit of cooperation and thus the effectiveness of the punishment threat as does an increase in group size. However, it also increases the net benefit of deviating from the cooperative outcome. That is, as λ increases, the benefit of withdrawing his/her individual contribution becomes more important than the cost in terms of probability of success. As a result the net benefit of defection increases but this effect is always dominated by the increased punishment threat as λ rises, which in turn makes within-group cooperation easier to sustain.

3.3 Renegotiation

Although infinite Nash reversion is subgame perfect, it is not very realistic because it is unforgiving and, furthermore, it hurts the punishers just as the deviator. Indeed, it is hard to believe that agents remain forever in a Pareto dominated equilibrium without renegotiating back to a cooperative outcome. I thus now consider another strategy profile which is robust to renegotiation. Specifically, I analyze whether full cooperation can be sustained as a *Weakly Renegotiation-Proof Equilibrium* (WRPE), in the sense of Farrell and Maskin (1989). To be WPRE, a strategy profile must satisfy two requirements: (i) The first is that of subgame perfection which requires that no player can gain by a one-period deviation after any history; (ii) The second requirement is that, after any history, there must not exist two continuation equilibria such that all players are better off in one continuation equilibrium than in the other. Otherwise, the players would renegotiate from one to the other.

A strategy profile satisfying these two requirements is *Penance-p,* which has been proposed by Asheim et al. (2006) and Froyn and Hovi (2008) in the context of a prisoner's dilemma game. Our contribution here is to extend the use of this concept to a game of group cooperation with continuous payoffs and strategic interactions between players and, more importantly, with one player not participating to the agreement. Specifically, the *Penance-p* subgame perfect strategy profile, denoted by $\tilde{\sigma}(p)$, specifies (i) that a group member contributes the joint-maximizing level of effort except if another group member has been the sole

[6] Both $v^C - v^N$ and $v^D - v^C$ depend on λ *and* on $V_G = \lambda V_I$. We consider that V_I is fixed and thus the increase in λ is only driven by the increase in V_G.

deviator from *Penance-p* in the previous period and (ii) that if a deviation occurs then p group members – the *punishing* members – stop contributing to collective action, whereas the $n-p$ other group members continue to act cooperatively. This means that they produce the joint-maximizing level of group effort of a group consisting of $n-p$ members, while the p punishing members do not contribute at all to collective action but still benefit from the collective action of their group. Punishments last only one period and so this strategy profile is robust to renegotiation if *not all* group members strictly gain by restarted full cooperation instead of carrying out the punishment.

Again, I focus on the "best" WRPE from the viewpoint of group members, that is the one sustaining the joint-payoff maximizing level of rent-seeking effort x^C. Recall that $x(t) \equiv (x_1(t), x_2(t), ..., x_n(t)) \in \Re^n_+$. Thus, the *Penance-p* strategy profile can be formally described as follows

$$\sigma_i^t = \begin{cases} x^C \text{ if } t=0 \text{ or if } (x(t-1), Y(t-1)) = (x^C, Y^C), \\ x^P \text{ if } x_i(t-1) \neq x^C \text{ or if } x_j(t-1) \neq x^C \text{ for } j \neq i \text{ and } i \notin P_j, \\ 0 \text{ if } x_j(t-1) \neq x^C \text{ for } j \neq i \text{ and } i \in P_j, \end{cases} \tag{26}$$

for $i = 1, 2, ..., n$.

x^P is joint-payoff maximizing level of rent-seeking effort of a group of size $(n-p)$, and where P_j is the set of punishing members when member j deviates from $\tilde{\sigma}(p)$. As mentioned above, the number of punishing members is the same for all $i = 1, 2, ..., n$ and is denoted by p. Finally, as in the previous section, the incumbent is assumed to play in every period his static best-response to the group members' strategies regardless of the history, that is $\sigma_0^t = R_0(x(t))$ for all t.

3.3.1 The subgame-perfection requirement

Consider first the requirement of subgame perfection. There are two kinds of histories to check: When no deviation has taken place in the previous round $t-1$ and when a deviation did occur in $t-1$.

No deviation in period $t-1$.

In this history, *Penance-p* prescribes that all group members continue to cooperate. If however, one member deviates in period t and reverts to *Penance-p* in $t+1$ then his/her deviation payoff is v^D given by (22) in period t, while in period $t+1$ he/she obtains the payoff of group cooperation with $n-p$ contributing members. Substituting n by $n-p$ into (16), I obtain

$$v^P = \frac{[\lambda(n-p)]^2}{[1+\lambda(n-p)]^2} V_G. \tag{27}$$

In period $t+2$ onward, full cooperation is restored so that member i obtains v^C, given by (16) in that period and in any future period. Therefore, any group member has no incentives to unilaterally deviate from the cooperative outcome unless he/she can obtain a larger discounted sum of payoffs in period t and $t+1$ by deviating, that is if and only if $v^D + \delta v^P \leq (1+\delta)v^C$ or $\delta \geq \delta^P$ with $\delta^P \equiv \left(v^D - v^C\right) / \left(v^C - v^P\right)$.

Recall that the best deviation payoff v^D from the cooperative phase depends on whether $n=2$ or $n \geq 3$. If $n=2$, then $p=1$. Indeed p must always be strictly positive otherwise defection would never be punished and so each member would always defect. And it must also be lower than 2, otherwise the strategy profile

would correspond to Nash reversion strategies and the renegotiation-proofness requirement would not be satisfied. Then, using (16), the first expression in (22) and (29), I obtain for $n = 2$ and $p = 1$,

$$\delta^P(2,1) = \frac{(3-2\sqrt{2})(1+2\lambda)(1+\lambda)^2}{\lambda^2(3+4\lambda)}. \tag{28}$$

Now consider that $n \geq 3$. In this case, the member who defects from the cooperative phase cuts her contribution to 0 (see 21), so that the best deviation payoff is given by the second expression in (22). Also using (16) and (29), I then obtain the following threshold value of the discount parameter

$$\delta^P(n,p)\,|_{n\geq 3} = \frac{[1+\lambda(n-p)]^2\,[\lambda n(n-2)+(n-1)]}{\lambda p\,[1+\lambda(n-1)]\,[2\lambda n(n-p)+2n-p]}. \tag{29}$$

We now have to verify the subgame-perfect requirement, when a single deviation has taken place in period $t-1$.

A deviation in period $t-1$.

In this history, *Penance-p* prescribes that in period t, there are p *punishing* members who do not contribute to the collective action of the group. I first determine under which condition the $n-p$ group members (including the member who deviated in $t-1$) that are prescribed to cooperate to the benefit of the whole group by contributing x^P have no incentive to deviate from *Penance-p* in period t.

If $n = 2$ (and thus $p = 1$) or $n - p = 1$, the best deviation payoff from the punishment phase, denoted v^{DP} is equal to v^P because v^P is the optimal payoff of the unique member who makes rent-seeking efforts in t (because he/she deviated in $t-1$). Thus, the incentive compatibility constraint on the punishment path is always satisfied if $n - p = 1$.

Now suppose that $(n-p) > 1$ and let x^{DP} be the effort level of a contributing member who defects from the punishment in period t. The best deviation payoff of a contributing member in the punishment phase who decides to deviate depends on whether $n-p = 2$ or $n-p \geq 3$ and is given by (??) with n replaced by $n-p$, that is[2]

$$v^{DP} = \begin{cases} = \dfrac{4\lambda^2+(3-2\sqrt{2})(1+2\lambda)}{(1+2\lambda)^2}V_G \text{ if } n-p=2, \\ \\ \dfrac{\lambda\,(n-p-1)}{1+\lambda(n-p-1)}V_G \text{ if } n-p\geq 3. \end{cases}. \tag{30}$$

Therefore, each of the $n-p$ contributors in period t has no incentive to unilaterally deviate from *Penance-p* unless he/she can obtain a larger discounted sum of payoffs in period t and $t+1$ by deviating, that is if and only if $v^{DP} + \delta v^P \leq v^P + \delta v^C$ or $\delta \geq \delta^{DP}$ with $\delta^{DP} \equiv \left(v^{DP} - v^P\right) / \left(v^C - v^P\right)$.

Thus, using (16), (29) and (35), the minimum discount factor above which no contributing member in the punishment phase has an incentive to deviate is given by

$$\delta^{DP}(n)\,|_{n-p=2} = \frac{(3-2\sqrt{2})(1+2\lambda)(1+\lambda n)^2}{(n-2)\,[n+2+4\lambda n]\,\lambda^2}, \tag{31}$$

if $n-p = 2$ and

$$\delta^{DP}(n,p)\,|_{n-p\geq 3} = \frac{(1+\lambda n)^2\,[\lambda(n-p)(n-p-2)+(n-p-1)]}{\lambda p\,[\lambda(n-p-1)+1]\,[2\lambda n(n-p)+2n-p]}, \tag{32}$$

if $n-p \geq 3$.

I now establish the following result that will prove useful in the rest of the analysis.

Result 2: *The minimum discount factors $\delta^P(n,p)\,|_{n\geq 3}$ and $\delta^{DP}(n,p)\,|_{n-p\geq 3}$ above which the strategy profile $\tilde{\sigma}(p)$ satisfies the subgame-perfect requirements are both decreasing in the number of punishing members p.*

I also need to verify that the punishing members who do not contribute at all to collective action in the punishment period have no incentives to deviate by producing a positive level of effort. If the p punishing members adhere to the punishment in period t and do not participate to collective action, then the payoff of each of them is given by the probability of success (multiplied by V_G) of a cooperative group consisting of $n-p$ members, that is from (15),

$$v^{PA} = \frac{\lambda\,(n-p)}{1+\lambda(n-p)} V_G. \tag{33}$$

If $n-p \geq 3$, then each group member's best possible deviation is to cut her contribution to 0, which corresponds exactly to what is prescribed by *penance-p* for the p punishing members. So, in this case, the punishing members do not have any incentives to deviate in the punishment phase, all the more it will trigger the punishment again in $t+1$.

If $n-p \leq 2$, then the best deviation for a contributing member is to make a positive level of effort. However, a non-contributing member still cannot do better than not contributing at all to collective action independently of n and p. Indeed, let x^A be the effort level of a punishing member who considers defecting in the punishment period t. His/her payoff would be given by

$$v^A = \frac{(n-p)x^P + x^A}{(n-p)x^P + x^A + Y^P} V_G - x^A, \tag{34}$$

where $x^P = \lambda Y^P$. $\partial v^A/\partial x^A$ is decreasing in x^A and thus a punishing member does not have any incentive to deviate from *penance-p* by producing a positive level of effort if and only if $\left(\partial v^A/\partial x^A\right)|_{x^A=0} \leq 0$, that is if $V_G/\left[1+\lambda(n-p)\right]^2 \leq Y^P$. Since $Y^P = x^P/\lambda = \left[(n-p)/\left[1+\lambda(n-p)\right]^2\right] V_G$, this inequality is always satisfied for $n-p \geq 1$.

3.3.2 The renegotiation-proofness requirement

I now turn to the renegotiation-proofness requirement. It implies an upper bound on the number of punishing countries. Indeed, full cooperation is weakly renegotiation-proof if not *all* group members are strictly worse off with the punishment than with renegotiation (i.e. returning immediately to cooperation without punishment). Suppose member i deviates in $t-1$. Clearly, in period t the $n-p$ members (including the defecting member of period $t-1$) who contribute to group effort are worse off with the punishment than with renegotiation. Thus, weak-renegotiation proofness requires that the p punishing members are at least as well off with punishment as with renegotiation.

If the p punishing members adhere to the punishment in period t and do not participate to collective action, then the payoff of each of them is given by v^{PA}. If however, full cooperation is restored immediately, then all members obtain v^C given by (16). In either cases, all group members obtain v^C in $t+1$ onward. Thus, weak renegotiation-proofness requires that $v^{PA} \geq v^C$ or $p \leq \bar{p}$, with

$$\bar{p} \equiv \frac{n(1+\lambda n)}{1+2\lambda n}. \tag{35}$$

Intuitively, the number of punishing members must not be too large, otherwise the effective level of collective action in the punishment path is too low to prevent renegotiating back to the fully cooperative outcome. One can also easily verify that $\bar{p} \in (n/2, (n+1)/2)$ for any $\lambda > 1$. Thus, the maximal number of punishing members satisfying the renegotiation-proofness requirement will be given by:

$$\bar{p} = n/2 \text{ if } n \text{ is even or } \bar{p} = (n-1)/2 \text{ if } n \text{ is odd.} \tag{36}$$

From Result 2, I have that $\delta^P(n,p)\,|_{n\geq 3}$ is decreasing in the number of punishing members. Thus, substituting (42) into (31), the lowest values of the discount factor satisfying the incentive compatibility constraint on the cooperative phase are given by

$$\delta^P(n) = \left| \begin{array}{l} \dfrac{[2+\lambda(n+1)]^2\,[\lambda n(n-2)+(n-1)]}{\lambda(n-1)\,[1+\lambda(n-1)]\,[1+3n+2\lambda n(n+1)]} \text{ if } n = 3,5,7,... \\ \\ \dfrac{[2+\lambda n]^2\,[\lambda n(n-2)+(n-1)]}{\lambda n^2(3+2\lambda n)\,[1+\lambda(n-1)]} \text{ if } n = 4,6,8,.... \end{array} \right. \tag{37}$$

$\delta^{DP}(n,p)\,|_{n-p\geq 3}$ is also decreasing in the number of punishing members provided that $n-p \geq 3$. It can be easily verified that this inequality reduces to $n \geq 5$ if n with $\bar{p} = (n-1)/2$ if n is odd; while it reduces to $n \geq 6$ if n is even with $\bar{p} = n/2$, if n is even. Thus, under these conditions and substituting (42) into (37), the lowest values of the discount factor satisfying the incentive compatibility constraint on the punishment phase are given by

$$\delta^{DP}(n) = \left| \begin{array}{l} \dfrac{2\,[1+\lambda n]^2\,[\lambda(n+1)(n-3)+2(n-1)]}{\lambda(n-1)\,[2+\lambda(n-1)]\,[1+3n+2\lambda n(n+1)]} \text{if } n = 5,7,9... \\ \\ \dfrac{2\,[1+\lambda n]^2\,[\lambda n(n-4)+2(n-2)]}{\lambda n^2(3+2\lambda n)\,[2+\lambda(n-2)]} \text{ if } n = 6,8,10. \end{array} \right. \tag{38}$$

Thus, provided that $n \geq 5$, *Penance-p* strategies with full cooperation within the challenger group is a WPRE if and only if $\delta \geq \text{Max}\left\{\delta^P(n), \delta^{DP}(n)\right\}$.

Let us now consider that $n \leq 4$. Again, with a group of two members, subgame-perfection only requires that each member has no incentive to deviate from the cooperative path, i.e. $\delta \geq \delta^P(2,1)$, where $\delta^P(2,1)$ is given by (30). The renegotiation-proofness requirement is also satisfied since $p = 1$. Now, for a group of three members, the renegotiation-proofness requirement also implies that $p = 1$. And subgame-perfection requires that $\delta \geq Max\left\{\delta^P(3), \delta^{DP}(3)\,|_{n-p=2}\right\}$, where $\delta^P(3)$ and $\delta^{DP}(3)\,|_{n-p=2}$ are given by (42) and (36), respectively. For $n = 4$, the renegotiation proofness requirement is such that $p \leq 2$. Since $\delta^P(n,p)\,|_{n\geq 3}$ is decreasing in p, $\delta^P(4)$ most relaxes the incentive compatibility constraint on the cooperative phase. In this case, *Penance-p* strategies with full cooperation within the challenger group is a WPRE if and only if $\delta \geq \text{Max}\left\{\delta^P(4), \delta^{DP}(4)\,|_{n-p=2}\,, \delta^{DP}(4,1)\,|_{n-p\geq 3}\right\}$, where $\delta^P(4)\,|_{n_e\geq 4}$, $\delta^{DP}(4)\,|_{n-p=2}$, and $\delta^{DP}(4,1)\,|_{n-p\geq 3}$ are given by (41), (36) and (37), respectively.

I have the following Result.

Result 3: *The strategy profile $\tilde{\sigma}(\bar{p})$ with full cooperation within the challenger group is a* WPRE *if and only if:*

(i) $\delta \geq \delta^P(2,1)$ *when* $n = 2$;

(ii) $\delta \geq \delta^P(n)$ *when* $n = \{3, 4\}$;
(iii) $\delta \geq \delta^P(5)$ *when* $n = 5$ *and* $\lambda \leq \tilde{\lambda}$ *or* $\delta \geq \delta^{DP}(5)$ *when* $n = 5$ *and* $\lambda \geq \tilde{\lambda}$, *with* $\tilde{\lambda} \simeq 1.13$;
(iv) $\delta \geq \delta^{DP}(n)$ *when* $n \geq 6$.

As mentioned above, Froyn and Hovi (2008) in a model where players have a binary choice between cooperating and defecting, and Asheim and Holstmark (2009) in a model with continuous choices – but without strategic interactions between players – find that full cooperation can be sustained a WPRE through *penance* strategies. I show that this result extends to a more general model with strategic interactions between players and in a context where full cooperation within a group is undertaken to counteract the action of a unitary opponent. Furthermore, contrary to the above mentioned analysis, the number of players determines which incentive compatibility constraint is more stringent than the other. Observe that the inequality $\delta^{DP}(n) \geq \delta^P(n)$ reduces to $v^{DP} - v^P \geq v^D - v^C$ since $\delta^{DP}(n) \equiv \left(v^{DP} - v^P\right) / \left(v^C - v^P\right)$ and $\delta^P(n) \equiv \left(v^D - v^C\right) / \left(v^C - v^P\right)$. $v^{DP} - v^P$ represents the net benefit of deviation for a contributing member on the punishment path, while $v^D - v^C$ represents the net benefit of deviation for each member on the cooperative path. When group size is larger than 5, the best possible deviation from either the cooperative phase or the punishment phase is to cut his/her contribution to 0. In this case, as shown and explained in the previous subsection, the net benefit of defection decreases with group size – that is with the number of contributing members. It follows that there are more incentives to deviate from the punishment path than from the fully cooperative outcome (and hence $\delta^{DP}(n) \geq \delta^P(n)$). This result is reversed when the challenger group has few members (less than 5). In this case, the best possible deviation from the punishment path is to make a positive level of effort since contributing members are not enough to make it profitable full defection. In turns, it makes defection from the punishment path less attractive than defection from the equilibrium path.

I now characterize the impact of an increase in group size n, or in the relative valuation of the prize by group members given by λ, on the difficulty of sustaining within-group cooperation, as measured by the lowest discount factor supporting the optimal level of group effort as a WRPE. For $n \leq 5$, I have the following result.

Result 4: *Suppose that* $n \leq 5$, *then*:

(i) $\delta^P(2,1) < \delta^P(4) < \delta^P(3)$;
(ii) $\delta^P(5) > \delta^P(4)$ *and* $\delta^{DP}(5) > \delta^P(4)$;
(iii) $\delta^P(2,1)$, *and* $\delta^P(n)$ *for* $n = \{3, 4, 5\}$ *and* $\delta^{DP}(5)$ *are decreasing in* λ.

Thus, for a group of a small size, increasing the number of group members generally makes full cooperation more difficult to sustain as a WRPE except when group size increases from 3 to 4 members. Also, whether a group of 5 members has more or less difficulty to sustain within-group cooperation as a WRPE compared to a group of 3 members is indeterminate and depends on the exact value of λ. Intuitively, a group of 3 members is that for which the punishment is relatively the least severe since, in that case, the number of punishing members in proportion of group size – that is $p/n = 1/3$ – is the smallest compared to a group of 2, 4 or 5 members. Yet, an increase in the valuation of the prize by group members compared to that of the incumbent – i.e. an increase in λ – unambiguously makes within-group cooperation less difficult to sustain as a WRPE. The question now is whether these results remain valid for larger groups with an

arbitrary number of members. For $n \geq 5$, I have the following result.

Result 5: *Suppose that* $n \geq 5$, *then*: (i) $\delta^{DP}(n+1) < \delta^{DP}(n)$ *if* n *is odd*; (ii) $\delta^{DP}(n)$ *is increasing in* n *independently of* λ; (iii) $\delta^{DP}(n)$ *is decreasing in* λ *independently of* n.

Point (i) of this result echoes the previous one for a group of less than five members and where the incentive compatibility constraint along the cooperative path is more stringent than that of the punishment path. For a group of more than 5 members, the binding constraint is that corresponding to the punishment path. But the intuition remains the same. If the challenger group has an odd number of members, then integrating one more member facilitates within-group cooperation because this allows increasing the number of punishing members also by 1, thus increasing the proportion of group members punishing a deviation. Nevertheless, an increase in group size generally raises the threshold value of the minimum discount factor above which within-group cooperation on the efficient level of group effort can be sustained as a WRPE. As just mentioned above, the net benefit of defection from the punishment path, that is $v^{DP} - v^{P}$, is decreasing in the number of contributing members in the same way as does the net benefit of defection from cooperation with group size. However, and in contrast to the situation where group members use Nash Reversion Strategies, the punishment threat also becomes less effective as group size increases due to the renegotiation proofness requirement. In fact, Result 5 shows that $v^{C} - v^{P}$ is decreasing even more with group size than $v^{DP} - v^{P}$, thus increasing $\delta^{DP}(n)$ and making full cooperation more difficult to sustain as a WRPE as group size increases. In other words, the constraint of renegotiation-proofness brings us back to the general presumption that maintaining cooperation becomes more difficult as the size of the collectivity increases (see, e.g., Hardin 1982; Olson 1982; Sandler 1992; Taylor 1982). Yet, in contrast to Olson (1965)'s conjecture full cooperation can still be maintained in arbitrarily large groups provided the discount factor of group members is sufficiently high.

As for the impact of the heterogeneity in the valuation of the prize on the difficulty of sustaining full cooperation as a WRPE, I obtain the same result than that obtained when group members use simple Nash Reversion Strategies without any possibility of renegotiation. The cooperative level of individual effort – whether there are n or only $(n-p)$ contributing members – increases with λ and, hence, withdrawing his/her individual contribution becomes more and more tempting even though a defection also decreases the probability of winning the public prize. However, the net benefit of maintaining full cooperation relative to the punishment phase also increases with λ, thus making the punishment threat more effective. And, as shown by Result 5, the renegotiation-proofness requirement with a limiting number of punishing members does not alter the increased effectiveness of the punishment threat for making within-group cooperation easier to sustain, as the relative valuation of the prize by group members increases.

4 Concluding remarks

In a repeated contest game, I analyze the ability of group members to cooperate in order to challenge the position of an incumbent for the award of a public prize. The results show that group cooperation can be more easily sustained through simple Nash Reversion Strategies (NRS) as the relative valuation of the prize by group members, or group size, increases. The intuition is that bad outcomes under a static setting make for effective punishments in a repeated game setting. Next, I consider that group

members can renegotiate, without costs, returning back to the cooperative outcome in case of a defection. I show that group cooperation can still be maintained independently of the size of the group as a Weakly Renegotiation-Proof Equilibrium (WRPE). Furthermore, the renegotiation-proofness requirement does not change the result that a greater heterogeneity in the valuation of the prize makes it easier within-group cooperation. However, the comparative static result with respect to group size is reversed. In particular, the critical discount factor above which full cooperation can be maintained converges to 0 under NRS while, under the constraint of renegotiation-proofness, it converges to 1 as the size of the collectivity goes to infinity. Therefore, it would be necessary to investigate other strategies of cooperation against against an opponent that are in between the impossibility of renegotiation and perfect or frictionless renegotiation. And the (im)perfectness of renegotiation should depend, among other features, on group size. Thus, the next step should be to introduce a cost of renegotiation as a function of group size in this simple framework, before considering a generalization of the model.

5 Appendix

5.1 Proof of Result 1

I first evaluate the impact of group size n and of the relative valuation of the prize by group members λ on the threshold value of the discount factor $\delta^N(n)$. Regarding the impact of group size, I have

$$\frac{\partial \delta^N(n)}{\partial n} = -\frac{(1+\lambda)^2 \left\{ \begin{array}{c} n^3(n-1)(n-3)\lambda^3 + n\left[n^2\left(n^2-3n+1\right)+3(n-1)\right]\lambda^2 \\ +\left[n^2\left(n^2-n-3\right)+5n-1\right]\lambda+(n-1)^2 \end{array} \right\}}{(1+\lambda n)^3\left[\lambda(n^2-n-1)+(n-1)^2\right]^2}, \tag{A1}$$

which is strictly negative for any $n \geq 3$.

The impact of the ratio of valuations λ on $\delta^N(n)$ is given by

$$\frac{\partial \delta^N(n)}{\partial \lambda} = -\frac{n(n-1) \left\{ \begin{array}{c} n\left[n(n^2-2n-1)+3\right]\lambda^3 + \left[n^3(2n-5)+n(6n-5)+1\right]\lambda^2 \\ +\left[n^3\left(n-2\right)+5n(n-1)-2\right]\lambda+n^2(n-2)+3(n-1) \end{array} \right\}}{(1+\lambda n)^3\left[\lambda(n^2-n-1)+(n-1)^2\right]^2}, \tag{A2}$$

which is also strictly negative for any $n \geq 3$.

I now evaluate the impact of group size n and of the relative valuation of the prize by group members λ on the net benefit to maintaining within-group cooperation $v^C - v^N$ and on the net benefit of deviating from the cooperative outcome $v^D - v^C$. Calculating the derivative of $v^C - v^N$ with respect to n, I obtain

$$\frac{\partial\left(v^C - v^N\right)}{\partial n} = \frac{\lambda\left[n^3\lambda^3 + n^2(4n-3)\lambda^2 + n(2n^2-3)\lambda - 1\right]}{n^2(1+\lambda)^2(1+\lambda n)^3} V_G, \tag{A3}$$

which is clearly positive for any $n \geq 3$.

The derivative of $v^D - v^C$ with respect to n is given by

$$\frac{\partial\left(v^D - v^C\right)}{\partial n} = -\frac{\lambda\left[n(n^2-4n+2)\lambda^3 + n(n-4)\lambda^2 - (1+\lambda n)\right]}{\left[1+\lambda(n-1)\right]^2(1+\lambda n)^3} V_G, \tag{A4}$$

One can see that this derivative is negative for any $n \geq 4$ and $\lambda \geq 1$.

Now, I consider that V_I remains unchanged and I calculate the derivative of $v^C - v^N$ with respect to $\lambda = V_G/V_I$. I have

$$\frac{\partial\left(v^C - v^N\right)}{\partial \lambda} = \frac{(n-1)\left[n^2(n+3)\lambda^3 + n\left(n^2+4n+1\right)\lambda^2 + 3n(n-1)\lambda - 2\right]}{n(1+\lambda)^3(1+\lambda n)^3} V_G, \tag{A5}$$

which is clearly positive.

The derivative of $v^D - v^C$ with respect to λ is given by

$$\frac{\partial\left(v^D - v^C\right)}{\partial\lambda} = \frac{\left[n(2n^2-6n+3)\lambda^2 + (4n^2-8n+1)\lambda + 2(n-1)\right]}{\left[1+\lambda(n-1)\right]^2(1+\lambda n)^3} V_G \tag{A6}$$

which is also clearly positive for any $n \geq 3$.

Calculating the derivative of $\delta^N(2)$, with respect to λ, I have

$$\frac{\partial\delta^N(2)}{\partial\lambda} = -\frac{(3-2\sqrt{2})(1+\lambda)\left[\lambda\left[5+4\lambda(5+4\lambda)\right]-1\right]}{\left[8(2-\sqrt{2})\lambda^3 + 4(8-5\sqrt{2})\lambda^2 + (23-16\sqrt{2})\lambda + 2(3-2\sqrt{2})\right]^2}, \tag{A7}$$

I now compare $\delta^N(2)$ and $\delta^N(3)$. Recall that in contrast to $n \geq 3$, the group member who defects from the cooperative outcome chooses a positive level of effort when $n = 2$. Using (25), $\delta(3)$ is given by

$$\delta^N(3) = \frac{3\left(1+\lambda\right)^2\left(2+3\lambda\right)}{\left(4+5\lambda\right)\left(1+3\lambda\right)^2}, \tag{A8}$$

Calculating the difference between $\delta^N(2)$ and $\delta^N(3)$, I have

$$\delta^N(2) - \delta^N(3) = -\frac{\left(1+\lambda\right)^2\left[11.29\lambda^4 + 0.82\lambda^3 - 16.90\lambda^2 - 7.37\lambda + 0.69\right]}{\left(4+5\lambda\right)\left(1+3\lambda\right)^2\left[4.69\lambda^3 + 3.72\lambda^2 + 0.37\lambda + 0.34\right]}, \tag{A9}$$

which can be positive or negative depending on λ.

5.2 Proof of Result 2

Observe that $\delta^P \equiv \left(v^D - v^C\right) / \left(v^C - v^P\right)$ depends on p only thought v^P, which is itself decreasing in p for any $n \geq 2$ and $p \geq 1$. Indeed, the derivative of v^P with respect to p is given by

$$\frac{\partial v^P}{\partial p} = -\frac{2(n-p)\lambda^2}{\left[1+\lambda(n-p)\right]^3} V_G, \tag{A10}$$

which is negative. It follows that $\delta^P(n,p)\,|_{n\geq 3}$ is decreasing in the number of punishing members p.

I now turn to the impact of the number of punishing members p on the threshold value of the discount factor $\delta^{DP}(n,p)\,|_{n-p\geq 3}$. I have

$$\frac{\partial\delta^{DP}(n,p)\,|_{n-p\geq 3}}{\partial p} = -\frac{\left(1+\lambda n\right)^2\left[1+\lambda(n-p)\right]\left[\Phi_0(n,p) + \Phi_1(n,p)\lambda + \Phi_2(n,p)\lambda^2\right]}{\lambda p^2\left[\lambda(n-p-1)+1\right]^2\left[2\lambda n(n-p) + 2n - p\right]^2}, \tag{A11}$$

where

$$\begin{aligned}
\Phi_0(n,p) &= 2(n-1)(n-p) + p^2, \\
\Phi_1(n,p) &= (n-p)\left[4n(n-p-2) + p(4+p) + 2\right], \\
\Phi_2(n,p) &= 2n(n-p)\left[n(n-2p-3) + p(4+p) + 2\right].
\end{aligned}$$

Clearly, $\Phi_0(n,p)$ and $\Phi_1(n,p)$ are strictly positive for any $n-p \geq 2$. Calculating the derivative of the term [.] in $\Phi_2(n,p)$ with respect to p, I have $\partial\Phi_2(n,p)/\partial p = 4 - 2(n-p)$, which is negative for any $n-p \geq 2$. Thus, the term [.] in $\Phi_2(n,p)$ reaches a maximum in $p = 1$, in which case, I have $\Phi_2(n,1) = n^2 - 5n + 7$, which is positive for any $n \geq 3$. As a result, $\Phi_2(n,p)$ is positive for any $n \geq 3$. In turn, $\partial\delta^{DP}(n,p)\,|_{n-p\geq 3}\,/\partial p$ is negative and thus $\delta^{DP}(n,p)\,|_{n-p\geq 3}$ is a decreasing function of the number p of punishing members for any $n-p \geq 3$ (implying $n \geq 4$).

5.3 Proof of Result 3

Let first consider a group of three members, i.e. $n = 3$ so that $p = 1$. Using (42), $\delta^P(3)$ is given by

$$\delta^P(3) = \frac{2 + \lambda(7 + 6\lambda)}{\lambda(5 + 12\lambda)}, \tag{A12}$$

while using (36) $\delta^{DP}(3)\,|_{n-p=2}$ is given by

$$\delta^{DP}(3)\,|_{n-p=2} = \frac{(3 - 2\sqrt{2})(1 + 2\lambda)(1 + 3\lambda)^2}{\lambda^2(5 + 12\lambda)}. \tag{A13}$$

Calculating the difference, we have

$$\delta^P(3)\,|_{n\geq 3} - \delta^{DP}(3)\,|_{n-p=2} = \frac{(1 + 2\lambda)\left[2\lambda(2 + 3\lambda)(3\sqrt{2} - 4) - (3 - 2\sqrt{2})\right]}{\lambda^2(5 + 12\lambda)}, \tag{A14}$$

which is clearly positive.

Now consider a group of four members. For $n = 4$, the renegotiation proofness requirement is such that $p \leq 2$. Since $\delta^P(n, p)\,|_{n\geq 3}$ is decreasing in p, the optimal number of punishing members that most relaxes the incentive compatibility constraint on the cooperative phase is given by $\delta^P(4)$. Using (42), we have

$$\delta^P(4) = \frac{(1 + 2\lambda)^2}{4\lambda(1 + 3\lambda)}. \tag{A15}$$

I now determine whether $p = 1$ or $p = 2$ most relaxes the incentive compatibility constraint on the punishment path. Using (36) and (37), we have

$$\delta^{DP}(4)\,|_{n-p=2} = \frac{(3 - 2\sqrt{2})(1 + 2\lambda)(1 + 4\lambda)^2}{4\lambda^2(3 + 8\lambda)}. \tag{A16}$$

and

$$\delta^{DP}(4, 1)\,|_{n-p\geq 3} = \frac{(2 + 3\lambda)(1 + 4\lambda)^2}{\lambda(1 + 2\lambda)(7 + 24\lambda)}. \tag{A17}$$

Calculating the difference, we have

$$\delta^{DP}(4, 1)\,|_{n-p\geq 3} - \delta^{DP}(4)\,|_{n-p=2} = \frac{(1 + 4\lambda)^2\left[79.53\lambda^3 + 78.72\lambda^2 + 15.08\lambda - 1.20\right]}{4(1 + 2\lambda)(3 + 8\lambda)(7 + 24\lambda)\lambda^2}, \tag{A18}$$

which is clearly positive, thus implying that $\delta^{DP}(4)\,|_{n-p=2} < \delta^{DP}(4, 1)\,|_{n-p\geq 3}$. Thus, $p = 2$ when $n = 4$, most relaxes the incentive compatibility constraint both on the cooperative and punishment phase. I thus need to compare $\delta^P(4)$ and $\delta^{DP}(4)\,|_{n-p=2}$. I have

$$\delta^P(4) - \delta^{DP}(4)\,|_{n-p=2} = \frac{(1 + 2\lambda)\left[7,76\lambda^3 + 7.14\lambda^2 + 1.11\lambda - 0.17\right]}{4(1 + 3\lambda)(3 + 8\lambda)\lambda^2}, \tag{A19}$$

which is also positive. Thus the minimum discount factor above which full cooperation can be obtained as WPRE in a group of four members is given by $\delta^P(4)$.

I now consider that $n \geq 5$. Let $\Delta(n) \equiv \delta^{DP}(n) - \delta^P(n)$. For n odd, we have

$$\Delta(n) = \frac{n(n + 1)[n(n - 5) + 2]\lambda^4 + \left[n^2(3n - 13) - 7n + 1\right]\lambda^3 - 2(3n + 1)\lambda(1 + 2\lambda) - 4}{\lambda\left[1 + \lambda(n - 1)\right]\left[2 + \lambda(n - 1)\right]\left[1 + 3n + 2\lambda n(n + 1)\right]} \tag{A20}$$

Also, the derivative of $\Delta(n)$ with respect to n is given by

$$\frac{\partial\Delta(n)}{\partial n} = \frac{(1 + \lambda n)\left[2 + \lambda(n + 1)\right]\left[\Phi_5(n)\lambda^5 + \Phi_4(n)\lambda^4 + +\Phi_3(n)\lambda^3 + \Phi_2(n)\lambda^2 + \Phi_1(n)\lambda + 12\right]}{\lambda\left[1 + \lambda(n - 1)\right]^2\left[2 + \lambda(n - 1)\right]^2\left[1 + 3n + 2\lambda n(n + 1)\right]^2} \tag{A21}$$

where

$$\begin{aligned}
\Phi_1(n) &= 2(17n-5), \\
\Phi_2(n) &= 2\left[n(18n-1)+1\right], \\
\Phi_3(n) &= \left[n^2(17n+21)+11n-1\right], \\
\Phi_4(n) &= 3n(n+1)\left[n(n+6)-3\right], \\
\Phi_5(n) &= 2n(n^2-1)(3n+1).
\end{aligned}$$

One can observe that $\Phi_i(n) \geq 0$ for $i = \{1,2,3,4,5\}$ so that the sign of this derivative is always positive. Hence, $\Delta(n)$ reaches a minimum in $n = 5$. In this case, one obtain

$$\Delta(5) = \frac{15\lambda^4 + 4\lambda^3 - 16\lambda^2 - 8\lambda - 1}{2\lambda\left[1+2\lambda\right]\left[1+4\lambda\right]\left[4+15\lambda\right]}, \tag{A22}$$

which is negative for $\lambda \leq \tilde{\lambda}$ and then positive for $\lambda \geq \tilde{\lambda}$, where $\tilde{\lambda}$ can be found numerically, i.e. $\tilde{\lambda} \simeq 1.13$. For $n = 7$, one obtain

$$\Delta(7) = \frac{\left[\lambda(16\lambda-3)-1\right]\left[2\lambda(7\lambda+4)+1\right]}{\lambda\left[1+3\lambda\right]\left[1+6\lambda\right]\left[11+56\lambda\right]}, \tag{A23}$$

which is strictly positive for any $\lambda \geq 1$. As a result, I have that $\delta^{DP}(n) > \delta^{P}(n)$ for any n odd and larger than 7 independently of λ.

Now let consider even values of n. We have

$$\Delta(n) = \frac{n^2[n(n-6)+4]\lambda^4 + n\left[3n^2-17n+6\right]\lambda^3 - 12n\lambda^2 - 6n\lambda - 4}{\lambda n\left[1+\lambda(n-1)\right]\left[2+\lambda(n-2)\right]\left[3+2\lambda n\right]} \tag{A24}$$

Also, the derivative of $\Delta(\bar{p})$ with respect to n is given by

$$\frac{\partial\Delta(n)}{\partial n} = \frac{(1+\lambda n)(2+\lambda n)\left[\Phi_5(n)\lambda^5 + \Phi_4(n)\lambda^4 + \Phi_3(n)\lambda^3 + \Phi_2(n)\lambda^2 + \Phi_1(n)\lambda + 12\right]}{\lambda n^2\left[1+\lambda(n-1)\right]^2\left[2+\lambda(n-2)\right]^2\left[3+2\lambda n\right]^2} \tag{A25}$$

where

$$\begin{aligned}
\Phi_1(n) &= 2(17n-12), \\
\Phi_2(n) &= 4\left[9n^2-8n+3\right], \\
\Phi_3(n) &= n(17n^2-2), \\
\Phi_4(n) &= n^2\left[3n^2+16n-18\right], \\
\Phi_5(n) &= 2n^3(3n-4).
\end{aligned}$$

One can observe that $\Phi_i(n) \geq 0$ for $i = \{1,2,3,4,5\}$ so that the sign of this derivative is always positive. Hence, $\Delta(n)$ reaches a minimum in $n = 6$. In this case, I have

$$\Delta(6) = \frac{\left[6\lambda^2+6\lambda+1\right]\left[6\lambda^2-3\lambda-1\right]}{9\lambda\left[1+2\lambda\right]\left[1+4\lambda\right]\left[1+5\lambda\right]}, \tag{A26}$$

which is strictly positive for any $\lambda \geq 1$. As a result, I have that $\delta^{DP}(n) > \delta^{P}(n)$ for any n odd and larger than 6 independently of λ.

5.4 Proof of Result 4

(i) Using (42), we have

$$\delta^P(3) - \delta^P(4) = -\frac{(1+2\lambda)\left[3+2\lambda(7+6\lambda)\right]}{4\lambda(1+4\lambda)(5+12\lambda)} > 0 \tag{A27}$$

and thus $\delta^P(3) > \delta^P(4)$.

Using (30) and (42), we also have

$$\delta^P(4) - \delta^P(2,1) = \frac{(1+2\lambda)\left[5.94\lambda^3 + 5.20\lambda^2 - 0.43\lambda - 0.69\right]}{4\lambda^2(1+3\lambda)(3+4\lambda)} > 0 \tag{A28}$$

and thus $\delta^P(3) > \delta^P(4) > \delta^P(2,1)$.

We also have

$$\delta^P(5) - \delta^P(3) = \frac{(4\lambda-9)(2+3\lambda)-3}{4\lambda(1+4\lambda)(5+12\lambda)}, \tag{A29}$$

which is of ambiguous sign. It follows that $\delta^{DP}(5) - \delta^P(3)$ is also of ambiguous sign.

However, we have that

$$\delta^P(5) - \delta^P(4) = \frac{11\lambda^2 + 7\lambda + 1}{4(1+3\lambda)(5+12\lambda)} > 0 \tag{A30}$$

and thus $\delta^P(5) > \delta^P(4)$.

Furthermore, we have that

$$\delta^{DP}(5) - \delta^P(4) = \frac{105\lambda^4 + 103\lambda^3 + 11\lambda^2 - 10\lambda - 21}{4\lambda(1+2\lambda)(1+3\lambda)(4+15\lambda)} > 0 \tag{A31}$$

and thus $\delta^{DP}(5) > \delta^P(4)$.

I now analyze the impact of increasing λ on these thresholds values of the discount parameter. I obtain,

$$\frac{\partial \delta^P(2,1)}{\partial \lambda} = -\frac{2(3-2\sqrt{2})(1+\lambda)\left[3+\lambda(9+7\lambda)\right]}{\lambda^3(3+4\lambda)^2} < 0 \tag{A32}$$

and

$$\frac{\partial \delta^P(3)}{\partial \lambda} = -\frac{2\left[27\lambda^2 + 24\lambda + 5\right]}{\lambda^2(5+12\lambda)^2} < 0 \tag{A33}$$

and

$$\frac{\partial \delta^P(4)}{\partial \lambda} = -\frac{8\lambda^2 + 6\lambda + 1}{4\lambda^2(1+3\lambda)^2} < 0 \tag{A34}$$

and

$$\frac{\partial \delta^P(5)}{\partial \lambda} = -\frac{15\lambda^2 + 8\lambda + 1}{4\lambda^2(1+4\lambda)^2} < 0 \tag{A35}$$

5.5 Proof of Result 5

I now consider that $n \geq 5$ and odd values of n. I have

$$\delta^{DP}(n+1) - \delta^{DP}(n) = -\frac{2\left[2+\lambda(n+1)\right]^2\left[2(n-1)+\lambda(n-3)(n+1)\right]\left[1+2\lambda(1+\lambda(n+1))\right]}{\lambda(n-1)(n+1)^2\left[2+\lambda(n-1)\right]\left[3+2\lambda(n+1)\right]\left[1+n(3+2\lambda(n+1))\right]} \tag{A36}$$

and thus $\delta^{DP}(n+1) < \delta^{DP}(n)$ when n is odd.

I now analyze the impact of increasing n on $\delta^{DP}(n)$. When n is odd and greater than 5, we have

$$\frac{\partial \delta^{DP}(n)}{\partial n} = \frac{2(1+\lambda n)\left[2+\lambda(n+1)\right]\left[\Phi_3(n)\lambda^3 - \Phi_2(n)\lambda^2 - \Phi_1(n)\lambda - \Phi_0(n)\right]}{\lambda(n-1)^2\left[2+\lambda(n-1)\right]^2\left[1+3n+2\lambda n(n+1)\right]^2} \tag{A37}$$

where

$$\begin{aligned}
\Phi_0(n) &= 6(n-1)^2, \\
\Phi_1(n) &= \left[n^2(5n-19) - (n+1)\right], \\
\Phi_2(n) &= n(n+1)\left[n^2 - 10n + 1\right], \\
\Phi_3(n) &= 2n(n^2-1)(n+3)
\end{aligned}$$

The term in the [.] in the numerator of (A37) is positive if $\Phi_3(n)\lambda^3 \geq \Phi_2(n)^2 + \Phi_1(n)\lambda + \Phi_0(n)$. We have that $\Phi_i(n) \geq 0$ for $i = 0, 1, 3$ while $\Phi_2(n)$ is negative for $n \leq 9$ and positive for $n \geq 11$. Since $\Phi_i(n) \geq 0$ for $i = 0, 1, 3$ and $\lambda \geq 1$, a sufficient condition for the above inequality to be satisfied is then that $\Phi_3(n)\lambda^2 \geq \Phi_2(n)\lambda^2 + \Phi_1(n)\lambda^2 + \Phi_0(n)\lambda^2$ or that $\Phi_3(n) - \Phi_2(n) - \Phi_1(n) - \Phi_0(n) \geq 0$, which is indeed the case since it is equal to $(n+1)\left[n(n^2+9n+11)-5\right] > 0$.

When n is even and greater than 6, we have

$$\frac{\partial \delta^{DP}(n)}{\partial n} = \frac{2(1+\lambda n)(2+\lambda n)\left[\Phi_3(n)\lambda^3 - \Phi_2(n)\lambda^2 - \Phi_1(n)\lambda - \Phi_0(n)\right]}{\lambda n^3 \left[2+\lambda(n-2)\right]^2 \left[3+2\lambda n\right]^2} \tag{A38}$$

where

$$\begin{aligned}
\Phi_0(n) &= 6(n-4), \\
\Phi_1(n) &= 5n^2 - 48n + 24, \\
\Phi_2(n) &= n(n^2 - 24n + 24) \\
\Phi_3(n) &= 4n^3.
\end{aligned}$$

The term in [.] in the numerator of (A38) is positive if $\Phi_3(n)\lambda^3 \geq \Phi_2(n)\lambda^2 + \Phi_1(n)\lambda + \Phi_0(n)$. We have that $\Phi_i(n) \geq 0$ for $i = 0, 3$. $\Phi_1(n)$ is negative for $n \leq 8$ and positive for $n \geq 10$ while $\Phi_2(n)$ is negative for $n \leq 22$ and positive for $n \geq 24$. If $n \leq 8$, a sufficient condition for the above inequality to be satisfied is $\Phi_3(n)\lambda^3 \geq \Phi_0(n)$, which is indeed the case since $\Phi_3(n) \geq \Phi_0(n)$ and $\lambda \geq 1$. If $n \geq 10$, a sufficient condition for the inequality to be satisfied is $\Phi_3(n)\lambda^2 \geq \Phi_2(n)\lambda^2 + \Phi_1(n)\lambda^2 + \Phi_0(n)\lambda^2$ or that $\Phi_3(n) - \Phi_2(n) - \Phi_1(n) - \Phi_0(n) \geq 0$, which is indeed the case since it is equal to $n(3n^2 + 19n + 18) > 0$.

I now analyze the impact of increasing λ on $\delta^{DP}(n)$. When n is odd, I have

$$\frac{\partial \delta^{DP}(n)}{\partial \lambda} = -\frac{2(1+\lambda n)\left[2+\lambda(n+1)\right]\left[\Phi_2(n)\lambda^2 + \Phi_1(n)\lambda + \Phi_0(n)\right]}{\lambda^2 (n-1)^2 \left[2+\lambda(n-1)\right]^2 \left[1+3n+2\lambda n(n+1)\right]^2} \tag{A39}$$

where

$$\begin{aligned}
\Phi_0(n) &= 2(3n+1)(n-1), \\
\Phi_1(n) &= (n-1)^2(5n+3), \\
\Phi_2(n) &= n(n+1)(n^2 - 2n + 9),
\end{aligned}$$

and thus $\partial \delta^{DP}(n)/\partial \lambda$ is negative.

When n is even, I have

$$\frac{\partial \delta^{DP}(n)}{\partial \lambda} = -\frac{2(1+\lambda n)(2+\lambda n)\left[\Phi_2(n)\lambda^2 + \Phi_1(n)\lambda + \Phi_0(n)\right]}{\lambda^2 n^2 \left[2+\lambda(n-2)\right]^2 \left[3+2\lambda n\right]^2} \tag{A40}$$

where

$$\begin{aligned}
\Phi_0(n) &= 6(n-2), \\
\Phi_1(n) &= 5n^2 - 22n + 24, \\
\Phi_2(n) &= n(n^2 - 6n + 24),
\end{aligned}$$

and thus $\partial \delta^{DP}(n)/\partial \lambda$ is also negative when n is even.

References

[1] Asheim, G., Froyn, C.B., Hovi, J., and F.C. Menz, (2006), "Regional versus Global Cooperation on Climate Control," *Journal of Environmental Economics and Management* 51(1), 93-109.

[2] Asheim, G.B., and B. Holtsmark, (2009), "Renegotiation-Proof Climate Agreements with Full Participation: Conditions for Pareto-Efficiency," *Environmental Resource Economics* 43, 519-533.

[3] Axelrod, R.,(1981), "The Emergence of Cooperation among Egoists," *American Political Science Review* 75, 306-18.

[4] Barrett, S., (1999), "A Theory of Full International Cooperation," *Journal of Theoretical Politics* 11(4), 519-541.

[5] Bendor, J. and D. Mookherjee, (1987), "Institutional Structure and the Logic of Ongoing Collective Action," *American Political Science Review* 81(1), 129-154.

[6] Cheikbossian, G., (2012), "The Collective Action Problem: Within-group Cooperation and BetIen-group Competition in a Repeated Rent-Seeking Game," *Games and Economic Behavior* 74, 68-82.

[7] Farrell, J., and E. Maskin, (1989), "Renegotiation in Repeated Games," *Games and Economic Behavior* 1, 327-360.

[8] Farrell, J., (2000), "Renegotiation in Repeated Oligopoly Interaction," in *Incentives, Organization and Public Economics: Papers in Honor of Sir James Mirrlees,* ed. P.J. Hammond and G.D. Myles, Oxford University Press.

[9] Friedman, J.W., (1971), "A Non-Cooperative Outcome for Supergames" *Review of Economic Studies* 38, 1-12.

[10] Froyn, C.B., and J. Hovi, (2008), "A climate Agreement with Full Participation," *Economic Letters* 99, 317-19.

[11] Haag, M. and R. Lagunoff, (2007), "On the Size and Structure of Group Cooperation," *Journal of Economic Theory* 135, 68-89.

[12] Hardin, R., (1982), *Collective Action,* Baltimore: John Hopkins University Press.

[13] Katz, E., Nitzan S. and J. Rosenberg, (1990), "Rent-seeking for Pure Public Goods," *Public Choice* 65, 49-60.

[14] McMillan, (1979), "Individual Incentives in the Supply of Public Inputs," *Journal of Public Economics* 12, 97-98.

[15] Olson, M., (1965), *The Logic of Collective Action,* Cambridge, MA: Harvard University Press.

[16] Olson, M., (1982), *The Rise and Decline of Nations,* New Haven, Yale University Press.

[17] Pecorino, P., (1999), "The Effect of Group Size on Public Good Provision in A Repeated Game Setting," *Journal of Public Economics* 72, 121-134.

[18] Pecorino, P., (2015), "Olson's Logic of Collective Action at Fifty," *Public Choice* 162, 243-62.

[19] Salant, S., Switzer, S., and R. Reynolds, (1983), Losses from Horizontal Merger: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium, *Quarterly Journal of Economics* 98(2), pp. 185-200.

[20] Sandler, T., (1992), *Collective Action: Theory and Applications,* Ann Arbor: University of Michigan Press.

[21] Taylor, M. (1982), *Community, Anarchy and Liberty,* Cambridge: Cambridge University Press.

[22] Tullock, G., (1980), "Efficient Rent-seeking," in Buchanan, J.M., Tollison, R.D., and G. Tullock (Eds.), *Toward a Theory of the Rent-seeking Society,* College Station: Texas A&M University Press, 97-112.

[23] Ursprung, H., 1990, Public Goods, Rent Dissipation and Candidate Competition, *Economics and Politics 2,* 115-132.

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