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A Lipsetian theory of voluntary power handover

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ABSTRACT

We consider an autocracy where the ruling elite control both the resource wealth and education policies. Education prompts economic growth and enriches the budget of the elite. However, education also increases the “awareness of citizens” – capturing their reluctance to accept a dictatorship and their labor market aspirations – and forces the elite to expand redistribution or handover the power. A power handover leads to a more democratic regime, where the elite retains (at least partially) its economic power. This trade-off is the backbone of our Lipsetian theory of voluntary power handover. This theory provides new insights on the positive relationship between economic development, education, and democratization, and on the negative relationship between inequality and democratization. Finally, we revisit the resources-curse hypothesis within our setting.

JEL classification: D72, I25, O11, O43

Keywords: Institutional change Human capital Lipset’s theory Resource curse

1. Introduction

Over the last century, a large number of regime changes has transformed the international political scene. Arguably, the world is now significantly more democratic than before, as documented by the large increase in democratic country-wide elections for both the legislature and their chief executives (Bormann and Golder, 2013). In this paper, we investigate the economic determinants of these regime changes. More precisely, we propose a Lipsetian theory of power handover to explore the roles of resource wealth and human capital accumulation.

According to the resource curse hypothesis, resource abundance leads on average to low economic growth and slow development (see in particular, Sachs and Warner, 1995). One of the key suggested mechanisms is that “resource-rich countries may inadvertently—and perhaps deliberately—neglect the development of their human resources” (Gylfason, 2001, p. 850). Then, contrary to the traditional human capital-driven development (Lucas, 1988), resource abundance leads to underinvestment in education and, therefore, lower long-term growth.

Yet, the resource curse hypothesis falls short of explaining the observed heterogeneity in the relationship between resource abundance and development, suggesting that other factors are determinant in explaining this relationship. Anecdotally, two resource-rich countries like Botswana and the Democratic Republic of Congo have experienced divergent de-

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Fig. 1. The relationship between GDP growth rate and resource wealth (log). Each circle represents the average GDP growth rate and resource wealth of a country over a 10-year period. All countries on the top; non-democratic countries on the bottom.

This paper explores a novel mechanism that combines access to resources with human capital accumulation.\(^2\) The key intuition is that resource abundance allows the autocratic regime to fund a sizeable education system, which in turn may boost economic development and ultimately lead to democratization. Our mechanism is closely related to the “modernization theory” put forward by Lipset (1959, 1960) in two highly influential contributions. Lipset suggested that development causes democratization. His view is that democracy requires a significant civic engagement, a political culture of negotiation, and the recognition of the need for compromises. As these values are typical of developed societies, characterized by high

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2 For example in Boucekkine et al. (2016), redistribution of resource windfalls takes the form of consumption subsidies and there is no public education. Caselli and Tesei’s model is even more basic.
education, urbanization, individual mobility, and low inequality, he concludes that economic development is conducive to democratization.

The empirical validity of the Lipsetian hypothesis is controversial. The colonial origins of institutions is suggestive that institutions may be the ultimate cause of development, and not the reverse (see Acemoglu et al., 2001). Moreover, the stability of rich democracies, as opposed to poorer ones, may be responsible for the positive relationship between economic development and democratization (Przeworski et al., 2000). However, human capital and culture may be considered an even more fundamental source of growth than institutions (see Barro, 1999) or (Glaeser et al., 2004). As of now, it is fair to say that the empirical debate on the Lipsetian view is not yet settled. The VDEM data confirm the well-known positive relationship between the level of democracy – here measured by the Polyarchy index – and the level of education – here given by average years of education in adult population (Fig. 2). More interestingly, when restricting to non-democratic countries (i.e., with Polyarchy index less than 0.5 in all periods) two types of autocracies emerge (Fig. 2). The first type kept investment in education very low (with education level less than 4). The second type is characterized by large and increasing investment in education (rapidly closing the gap to the level of democratic countries).

An even harder challenge is to build theories incorporating the Lipsetian mechanisms. Two decades ago, Barro (1999) wrote that: “given the strength of the Lipset/Aristotle hypothesis as an empirical regularity, it is surprising that convincing theoretical models of the relation do not exist.” Here, we take up this challenge and reexamine Lipset’s modernization theory by disentangling economic development into its components—resource wealth, education, and inequality—and by analyzing their interaction and implications for democratization.

We study the paradigmatic case of autocratic elite with full political and economic power. In line with the recent literature on democratization games (see Acemoglu and Robinson, 2006) and consistently with empirical evidence (Aidt and Franck, 2015), the elite anticipate and react to the existence and extent of revolutionary threats. They can act to avoid revolutions in two ways. The elite can introduce appropriate redistribution and wage-setting policies to placate the incentives to revolt of the citizens. Alternatively, they can start the democratization process and voluntarily dismiss their power. In the latter case, the benefits and costs for the elite are defined by a sharing rule, resulting from a negotiation between the citizens (now achieving the political power) and the elite for the control of the natural resources. Given our assumptions, we can only study cases of voluntary power handover. As outlined by Boucekine et al. (2019), since 1960 pro-democratic transitions have been recurrent but revolutions tend to be much less frequent than other types of transitions: only 30 revolutions out of 227 pro-democratic transitions (13% of total).

Citizens are hand-to-mouth workers: they are employed in the national industry and consume in each period their labor income and transfers. If their consumption and thus life satisfaction does not reach a specific threshold, workers would revolt. A key ingredient is to assume this threshold is endogenous and increasing with respect to the human capital of the workers. This can be rationalized by two separate components. The first component is the subsistence level of consumption; it can also include cultural and social aspects capturing the willingness to revolt, such as the degree of individualism or collectivism of workers (Gorodnichenko and Roland, 2015, see). The second component relies on the idea that, as citizens become more educated, they also become more aware of the political situation (see Zaller, 1992). Consequently, and besides its influence on the sharing rule, the education policy set by the elite has two direct opposite effects. On the one side, human capital is a production factor in the economy and triggers economic growth. On the other side, it has the above—described awareness cost, making workers more demanding.

With moderate returns to education, our theory predicts two possible scenarios. The first corresponds to elite who decide to keep investment in education low, rely massively on resource export, and redistribute to citizens just enough to stay in power. As a result, inequality remains large and the autocracy persists. The second scenario emerges when the elite undertake a path of education and development leading to an institutional change in a finite time horizon. Despite their full political and economic power, the elite may indeed opt for a massive education policy. In this second scenario, the economy experiences a progressive reduction of the dependence on resource export and the level of inequality continuously shrinks. However, the elite, facing increasing political and redistributive claims of the citizens, will eventually handover power.

These predictions are in line with Lipset’s theory of institutional change. The democratization path is triggered by the correct balance between economic returns to citizens’ education and their increasing claims for consumption. In particular, a high level of human capital is both a prerequisite for – and a consequence of – institutional change. It is a prerequisite as the elite dismiss their power only if the economy has the potential to become rich and guarantee to the elite sufficient economic returns in the post-autocratic regime. It is a consequence because the elite anticipate the costs and benefits of the democratization path and optimally decide whether or not to reach the human capital level that triggers a regime change.

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3 On this econometric debate, see: Acemoglu et al. (2005); Epstein et al. (2006); Castello-Clement (2008); Papaioannou and Siourounis (2008), Che et al., 2013.

4 Or, as suggested by Campante and Chor (2012) to explain the Arab Spring events, they may have higher income expectations and require better working opportunities.

5 This is a characteristic trait of autocratic regimes, partially explained by the low life expectancy of citizens.

6 Our results are consistent with the central role of education in the post-soviet transitions: Papaioannou and Siourounis (2008) find that democratization is “more likely to emerge in affluent and especially educated societies,” while education is also a key factor determining the intensity and the pace of democratic reforms. These also provide further support to the idea that dictators may, in some circumstances, adopt growth enhancing policies, as emphasized recently by Shen (2007); Cervellati and Sunde (2014), and De Luca et al. (2015).
Finally, our Lipsetian theory has some concise implications regarding the role of natural resources which rationalize the contrasting empirical evidence on the institutional resource curse (see again Alexeev and Conrad, 2009), and (Haber and Menaldo, 2011): resource wealth cannot alone trigger the democratization process. Even when a country is resource rich, the democratization path may be suboptimal and dominated by permanent dictatorship. This case emerges when, for example, the education sector is not very effective and/or when the elite expect significant losses following the regime change.

The remainder of the paper is organized as follows. Section 2 briefly reviews the related literature and puts our research into context. Section 3 presents the model of the economy. Sections 4 and 5 study all the possible solutions and then compare them to determine the optimal choices. Section 6 discusses extensions. Section 7 concludes. All the proofs are gathered in the Appendix.
2. Related literature

Despite the abundant and mixed empirical results, there are only few attempts to capture Lipset’s theory in a theoretical framework. Bourguignon and Verdier (2000) introduce an endogenous political economy decision mechanism that depends on the education of citizens: the ruling oligarchs set the education policy anticipating their effects on the economic growth, on inequality, on the political participation of citizens, and on the structure of political power. They show that a high initial per capita income is associated with a larger likelihood of a country to be in a democracy or to face a quick transition; initial inequality has, instead, the opposite effects. In Bourguignon and Verdier’s model, the elite optimally chooses the number of poor to be educated: this increases the future number of voters and dilutes the elite’s political power, while providing educational externalities. In contrast, we model education policies as investments in human capital accumulation and, rather than introducing a voting system and a redistributive tax, we assume that the elite maintain full power, but faces increasing redistribution requests. Closely related to this mechanism, Eicher et al. (2009) present a framework where education raises political awareness (the electorate becomes better at monitoring the government). The authors focus on the interaction between education, corruption, and development and show that a poverty trap may emerge for intermediate levels of education. More recently, Glaeser et al. (2007) convey the idea that education raises the benefits of civic engagement pretty much as social capital, therefore leading to a larger social and political involvement. They further argue that education does not only favor the emergence of democracy, but also helps stabilizing it. On a similar line, Jung and Sunde (2014) have investigated the Lipset claim that democracy is more likely in countries with more equal distributions of resources. Two main novelties characterize our contribution. First, by disentangling the role of classic correlates of economic development—i.e. income, education, inequality, natural resources—we provide a new set of predictions that can be used to reassess the empirical evidence. Second, by introducing natural resources, we bring together the literature on institutional change, the literature on education-driven endogenous growth, and the literature on the natural resource course.

3. The model

Time is continuous, i.e. $t \in \mathbb{R}_+$; the time index is omitted where no confusion may arise.

Society consists of a ruling elite and workers. The elite have control of a constant windfall of (natural) resources $R > 0$. Resources have two alternative uses. A part of it is exported on the international primary good market and sold at the exogenous price $p^e > 0$—let export quantity be $X(\geq 0)$—and, for the remaining part, it is supplied internally to the manufacturing sector—let domestic supply be $Q \leq R - X$.

The manufacturing sector is perfectly competitive. Firms employ resources $Q$ and the human capital $H$ of workers to produce a homogeneous commodity $Y$. The production function is Cobb-Douglas, i.e. $Y \leq f(Q, H) = AQ^aH^{1-a}$ with $\alpha \in (0, 1)$.

The resource rent of the elite then consists of the international and domestic sales of resources. The use of this rent is threefold: a part $C$ is allocated for the elite’s consumption; another part $\Theta$ is transferred to (or, if negative, expropriates from) the workers; the remaining part $E$ is invested in education. Formally, the elite’s budget constraint is:

$$ p^eX + pQ \geq C + \Theta + E. $$

where $p = p_C$ is the competitive price of resources.

Investment in the education sector increases human capital of the workers according to the following accumulation function:

$$ \dot{H} = h(E, H) = hE - \delta H $$

where $h > 0$ measures the effectiveness of the education investment and $\delta \geq 0$ is the depreciation rate of human capital.

The transfer and the education investment allow the elite to supervise the level of life satisfaction and resentment of the workers. Workers—normalized to unity—inelastically supply their human capital $H$ to the manufacturing sector and earn the equilibrium wage $w = (1 - \alpha)\frac{Y}{H}$. Their income is completed by the transfer $\Theta$ and is entirely consumed in each period. As transfers can be negative, part of their wage income might be taxed away by the elite.

The workers’ life satisfaction and resentment depend on their consumption. If their income is not large enough, workers might not be able to afford enough consumption and may decide to contest the power of the elite. More precisely, there is a threshold level of consumption that triggers a revolt. This threshold consists of two components. Following the long tradition of Francois Quesnay and Adam Smith, the first component interprets a subsistence level of consumption $s > 0$ (for a recent application, see Galor and Weil, 2000). The second component captures the idea that, as workers get more

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7. This is consistent with the fact that most governments of MENA countries (or Central and Eastern European transitional countries) operate large parts of the natural resource industries. While governments exercise control over these industries, this doesn’t prevent them from privatizing these sectors, as for example in Egypt (during the last years of presidency of Hosni Mubarak) or in Russia and the post-Soviet republics.

8. We might have incorporated (physical) capital accumulation. But this is to a large extent irrelevant for the Lipsetian theory we want to develop. In addition, it would make the analysis much trickier and prevent us from giving a full characterization of the optimal solution.

9. Subsistence need not be limited to nutrition, clothing, and housing needs. Inspired by Gorodnichenko and Roland (2015), differences in $s$ across countries may capture cultural and social aspects such as the degree of individualism/collectivism of society. Alternatively, differences in $s$ may also capture
and more educated, they raise their consumption requirements and resentment. This is consistent with the idea, borrowed from the political sciences literature, that education increases the workers awareness about the oppression of a dictatorial regime (see Zaller, 1992). This “political awareness” component thus depends on the workers’ level of human capital. For simplicity, it is a linear function of human capital with slope \( \phi > 0 \), referred to as the political awareness parameter. Our approach is very much in line with the literature, see in particular Eicher et al. (2009) who consider, in a different setting, that education makes people more politically aware. Then, workers decision to revolt is summarized as follows:

\[
\begin{cases} 
\text{revolt} & \text{if } wH + \Theta < s + \phi H \\
\text{no revolt} & \text{otherwise}
\end{cases}
\]  

(3)

A key aspect of our model is the capacity of the elite to fully internalize workers’ incentives to revolt. Consequently, the elite can always avoid a costly revolt, but may choose to leave the office voluntarily, which we interpret as democratization. Let \( T \in \mathbb{R}_+ \cup \{\infty\} \) be the time at which the autocratic regime comes to an end (permanent autocracy holds when \( T = \infty \)). In this scenario, what the elite earn at the date of the regime change is denoted by \( S(H(T)) \), with \( H(T) \) the stock of human capital at the date of voluntary power handover. This term captures the present value of the benefits accruing to the elite in the democratic regime. It is important to note that hereafter, we assume that \( S'(\cdot) > 0 \). Several arguments support this assumption. First, the elite should expect that they will keep some control over the economic activities, and then that their economic interests will extend over the democratic regime. Given that wealth is an increasing function of human capital in our setting, it seems natural to assume that \( S(\cdot) \) is increasing in the amount of human capital available at the date of the transition. A second explanation, which is closer to Lipset’s view, is the following: as explained in the Introduction, human capital is tightly connected with negotiation and absence of violence. Thus, the elite expect democratization in an economy with highly educated workers to be characterized by less political violence and lower economic expropriation. We will deal with specifications of \( S(\cdot) \) that are consistent with both views (see Remark 3 below).

To sum-up, we model the elite’s capacity to control the occurrence of a regime change by (i) introducing the no revolt constraint (3) in the their optimization program, and (ii) taking \( T \), the date of a regime change if any, as a control variable. Finally, the intertemporal well-being of the elite is given by:

\[
U = \int_0^T e^{-\rho t} u(C) \, dt + e^{-\rho T} S(H(T)),
\]

(4)

with \( T \leq \infty \), and where the instantaneous utility function is \( u(C) \equiv \frac{(C^{1-\gamma})}{1-\gamma} \) with \( \gamma \in (0, 1) \) and \( \rho > 0 \) is the discount rate.

Before moving to the analysis, we further discuss three aspects of our model.

**Remark 1.** The elite are particularly powerful. They are able to control the consumption/income of workers, and thus their willingness to revolt, in three different ways: (i) directly, by setting the transfer \( \Theta \); (ii) indirectly, by deciding how many resources to supply to the national industry \( Q \); and (iii) dynamically, by investing more or less in education \( E \) and thus setting their level of human capital. Furthermore, they control the political transition process and choose the timing \( T \) (possibly infinite) for the institutional change. Note that our results are strengthened by this simplifying assumption: despite the elite’s exceptional power, we show that the elite might decide to lead the country to democratization.

**Remark 2.** The resource windfall needs to be sufficiently large. To make the problem interesting, Assumption 1 below ensures that the elite can sustain the dictatorial regime when human capital is zero (i.e. when the consumption demanded by the workers is smallest).

**Assumption 1.** \( p^R > s \): The value of resources is larger than the subsistence consumption of the workers and gives the elite some freedom in how to allocate such wealth.

**Remark 3.** In Sections 4 and 5, we take a linear specification of the scrap value: \( S(H(T)) = \pi H(T) \) with \( \pi > 0 \). This assumption is enough to convey the key idea that at the moment of transition, the elite benefit from a higher stock of human capital in the country. The parameter \( \pi \) can be interpreted as a sharing rule that gives the piece of the cake that goes to the elite at the time of a regime change. This choice is made for the sake of exposure since we get simple closed-form solutions in this case. An alternative to the linear, and exogenous, scrap value case is examined in Section 6. As announced in the Introduction, we provide with a Nash bargaining foundation for an endogenous sharing rule, leading to a generally nonlinear scrap value. We ultimately show that the main properties arising from the linear scrap value case remain valid.

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10 Importantly, there are also arguments for the effect of education on consumption requirements and resentment being negative. There is, in fact, anecdotal and empirical evidence that education has been used by several authoritarian regimes as an instrument to control people (mostly through indoctrination). If “indoctrination” is sufficiently large, it may happen that also the combined effect of the listed mechanisms be negative. Importantly, this does not affect our main results, with the sole exception of the (otherwise negative) relationship between education of workers and inequality.

11 In their study of the interaction between education, corruption and development, the authors indeed assume that the probability to get reelected, for a corrupt government, is decreasing in the share of educated people.

12 Accounting for the uncertainty surrounding the occurrence of a revolt, or its success, does not alter the qualitative nature of the results. We present a simple extension of the analysis to a risky world in the Appendix B.1.
4. Permanent dictatorship vs institutional change

The elite seek to maximize utility (4), subject to the budget constraint (1), equilibrium prices $p$ and $w$, the dynamics of human capital (2), and the revolution decisions of workers (3). To do so, the elite sets optimally the use of resources $Q$, own consumption $C$, transfers $\Theta$, and education $E$. Yet, substituting $\Theta$ from the non-revolt condition of workers (3), the optimization problem of the elite can be written as an optimal stopping problem, where $T$ is the time until which the constraint is met. Formally:

$$\max_{\{Q,t,T\}} \int_0^T e^{-\rho t} u \left( p^* (R - Q) + AQ^\alpha H^{1-\alpha} - E - s - \phi H \right) dt + e^{-\rho T} \pi H(T)$$

s.t. $H = hE - \delta H$

$H(T) = H_T$ is free

$E, R - Q \geq 0$

$H(0) = H_0$ is given

Maximizing the criterion with respect to national resource supply $Q$, requires that national prices equalize international ones, i.e. $p = p^*$, and sets the optimal ratio between resources and human capital. Optimal investment in education equalizes the marginal benefit from education with the marginal cost of investing in education (in terms of foregone consumption). The optimal time $T < \infty$ for violating the no-revolt condition equalizes the elite’s value of staying in the current dictatorial regime and the value of the salvage function. The optimality conditions are displayed in Appendix A.1.

Since the optimal stopping problem is non-convex, we proceed in two steps. First, we study the dynamics of: i) the system when education investments are strictly positive. Second, we study the cases of: ii) zero investments in education; and iii) alternating periods of positive and zero investments. As both i) and ii) may be solutions of our optimization problem (see Section 5), we also highlight how these potential solutions depend on the parameters of the model.

4.1. Education-driven institutional change

Define as follows the instantaneous return on human capital $\Omega$ and the instantaneous return on education investment $\chi$:

$$\Omega = \frac{1-\alpha}{\alpha} p^* \left( \frac{\omega R}{p^*} \right)^{1/\alpha} - \phi.$$ $\chi = h\Omega - \delta.$

The instantaneous return on human capital is the difference between the (equilibrium) gross return of human capital and the feedback effect of human capital on workers’ claims for democracy, given by the political awareness parameter $\phi$.

Any solution with strictly positive education $E > 0$ satisfies the following necessary conditions for optimality (where the superscript 1 indicates the regime with positive education):

$$\begin{align*}
C^1(t) &= C_0^1 e^{\chi t} - \frac{h(p^*(R - s))}{\chi} - \frac{\gamma hC^1_0 (1 - \gamma) e^{\chi t}}{\lambda^1(t)} e^{(\rho - \chi)t} - \frac{h(p^*(R - s))}{\chi} \\
H^1(t) &= H_0 + \frac{h(p^*(R - s))}{\chi} - \frac{\gamma hC^1_0 (1 - \gamma) e^{\chi t}}{\lambda^1(t)} e^{(\rho - \chi)t} - \frac{h(p^*(R - s))}{\chi} \\
\lambda^1(t) &= \frac{C_0^1 e^{\chi t}}{h} e^{(\rho - \chi)t}
\end{align*}$$ (5)

We establish the following result.

**Proposition 1.** Assume that the return on education are lower than the rate of the elite’s time preference:

$$\rho > \chi.$$ (6)

(i) There is no solution combining permanent dictatorship and positive education.

(ii) There may be a solution combining institutional change and positive education.

(a) This solution is characterized by the accumulation of human capital, institutional change in finite time $T = T( H_0, R, \pi )$, and an end-point stock of human capital:

$$H_T = \frac{h}{\rho - \chi} \left( \frac{\gamma (h\pi)^{1/2}}{1 - \gamma} + p^*R - s \right) > 0.$$ (7)

(b) The necessary condition for the existence of such solution for all $H_0 \in [0, H_T]$ is:

$$p^*R - s > (h\pi)^{-\frac{1}{2}}.$$ (8)

---

13 The optimal resource supply of the elite to the national industry would determine a price wedge between international and internal resource prices in case of costly redistributive transfers. In this case, the elite would find it more profitable to redistribute income to workers by oversupplying resources and, indirectly, determining a wage increase. While this extension is potentially relevant for an empirical assessment, the results discussed are not affected.
(c) Sufficient conditions for the existence of a unique solution are (8) and \( \chi > \chi \), with \( \chi \in (0, \rho) \) the unique solution of:

\[
e^{\frac{\chi}{\chi}} = \frac{\rho (1 - \gamma)(\rho - \chi (1 - \gamma)) + \gamma^2 \chi}{(1 - \gamma)(\rho - \chi)^2}.
\]  

(9)

The ranking between \( \rho \) and \( \chi \) turns to be crucial in understanding the nature of the potential solutions, under positive education. Imposing \( \rho > \chi \) is at the same time necessary to show the existence of a solution with democratization in finite time, and sufficient to discard an outcome featuring permanent dictatorship. Under the opposite condition \( \chi \geq \rho \), autocracy is too growth-friendly. The elite can invest in education and, due to the high returns, this stimulates growth of output (and citizens’ consumption) while being compatible with the respect of the no-revolution constraint, which is in fact never binding. However, the solution with permanent dictatorship and positive education is not relevant because it would either violate the resource constraint, or imply resource imports to become infinite (with \( X \) tending to \(-\infty\)). Thus, a necessary condition for the elite to be willing to leave power in finite time is that the return on education investment is quite moderate and they exhibit a high enough level of impatience.

It is worth noting that these are two characteristic traits of resource-rich non-democratic regimes. Indeed, it is well documented that (small) resource-rich economies usually display high discount rates because of their dependence on resource revenue (low diversification) and (volatile) resource price, and their political instability (Nigeria, Venezuela), among other factors (see Hooper, 2018, for a discussion). At the same time, and focusing on the MENA region, if resource-rich countries have invested heavily in education over the past decades, it is clear that the returns of this policy in terms of growth and economic development have been quite limited. As pointed out by a recent world bank report devoted to this region, the evidence demonstrates that school systems in MENA are generally of low quality, thereby promoting the accumulation of low quality human capital (World Bank, 2013). This means that we can safely impose that \( \chi \) is low, and lower than \( \rho \).

Let us now discuss the predictions of our model concerning the Lipsetian links between human capital, education, and democratization. The proposition establishes the incompatibility between permanent dictatorship and education (i). From the perspective of the elite, human capital has two implications: one the one hand, it increases the consumption aspiration and wages of the workers; on the other hand, it increases the return of the elite at the time of democratization. Permanent dictatorship excludes the second implication. As for the first one, since transferring \( \Theta \) to the workers is free, the elite finds it more effective to export all resources, bring the workers’ human capital to zero, and meet their “no-revolt constraint” with transfers \( \Theta \). When transfers are costly, some positive level of human capital may be consistent with permanent dictatorship.

The necessary existence condition in (8) states that resource windfalls net of the intrinsic subsistence consumption level should be larger than the level of consumption the elite just enjoy at the date dictatorship ceases, \( C(T) \). Finally, a sufficient condition for existence requires that the returns to human capital be higher than a threshold \( \chi \), defined by (9). High enough returns to human capital logically guarantee that it is worthwhile for the elite to engage in the path of education and sustained capital accumulation.

Under \( \rho > \chi \), the time path of consumption is decreasing whereas the stock of human capital is increasing. The intuition runs as follows. For the elite to find it optimal to democratize they should be able to accumulate a sufficient amount of human capital, which will directly affect the wealth they will hold in the post-dictatorship regime, and will also guarantee that they can enjoy their wealth in a peaceful environment. Thus investment in human capital should be favored over consumption. Moreover, by investing significantly in human capital, the elite foster the development of citizens’ claims for a freer system through the increasing awareness mechanism. In order to delay the political regime change the elite have no other option but to transfer more and more resources to the citizens, even if this comes at the expense of their own consumption.

Note that under the conditions of Proposition 1, solutions that combine positive education and a revolution in finite time exist for any \( H_D \leq H_T \). In other words, the stated conditions guarantee the existence of a solution with education-driven institutional change independently of the initial endowment in human capital. This is a reasonable feature of our model: it would otherwise be difficult to explain why some countries are doomed to dictatorial regimes exclusively based on their initial stock of human capital and would also raise the issue of identifying this initial period (of the development process). Importantly, this doesn’t mean that the initial stock of human capital is irrelevant to our analysis. As far as the optimality analysis is concerned, this variable will be crucial to determine which one of the optimality candidates yields the optimum.

In addition, it is worth emphasizing some other interesting features of the first optimality candidates. They are summarized in the next two corollaries.

Corollary 1. The solution with education-driven institutional change is possible only if, ceteris paribus,

(i) Resource wealth, \( p^sR \), is large enough.

(ii) Elite’s incentives to democratize, that are provided by the share of wealth accruing to the elite after they give up power, \( \pi \), are important enough.

(iii) The effectiveness of the education, \( h \), needs to be important enough too. But, in contrast to resource wealth and the sharing rule, it should not take an excessive value since the instantaneous returns to education, \( \chi \), cannot be too high.
These properties are in line with Lipset’s theory in two essential aspects: the link between democratization and education; and the link between resources (or income) and democratization. First of all, the model predicts that a large amount of resources (or of their export price) is a precondition for the emergence of a non-dictatorial regime through human capital accumulation. However, the resource wealth of a country (measured by \( p^*R \)) is not the unique relevant determinant of democratization. Two further factors matter: a sufficient return to investment in education, \( h \), and a sufficient reward for the elite at the time of institutional change, \( \pi \). Democratization may not occur under large resource revenues because one of the two latter parameters is too small (leading to violating conditions (15)). Importantly, the interaction between the resource wealth and these factors is likely to be responsible for the mixed support for the natural resource curse hypothesis (see the debate opposing proponents of this hypothesis, Ross (2001), and Tsui (2011), and detractors, Alexeev and Conrad (2009), and Haber and Menaldo (2011)) and is in line with the empirical studies pointing at the mis-management of education in several oil-exporting countries (see Gylfason, 2001). Finally, notice that the role of the return to education is tricky: it should be high enough ceteris paribus for democratization via education to arise, but it should not be too high as the induced wealth in the hands of the elite in such a case could be sufficient to compensate for the larger awareness of the workers. In this case, a developing dictatorship could be sustained, although no equilibrium paths exist (see the interpretation of (8)).

Next, we highlight how the time-to-democratization is affected by the parameters of the model (see the comparative statics exercise at the end of Appendix A.2).

**Corollary 2.** The optimal time for institutional change, \( T = T(H_0, R, \pi) \), is decreasing in both the initial endowment in human capital, \( H_0 \), the resource windfall, \( R \), and the sharing rule parameter, \( \pi \).

The first two features strengthen the correlation between wealth and democratization discussed before. The larger is the initial stock of human capital (another possible measure of human wealth) or the windfall of resources, the quicker are the elite in driving the country into an institutional change. While the larger windfall is also associated to a larger level of human capital at the time of institutional change, such an effect is absent for the initial human capital level. Finally, the optimal time-to-democratization is decreasing in \( \pi \). The elite compensate a less favorable sharing rule by increasing the human capital of the country at the institutional change, \( H_T \). This requires a longer period of investment in education.

To end up this discussion, it is important to measure the elite’s payoff associated with the solution with education-driven institutional change. Let such optimality candidate be referred to as regime 1; then the present value (for the elite) of following this regime is given by (hereafter, the optimal time for institutional change is expressed in terms of \( H_0 \) only):

\[
V^1(H_0) = e^{-\rho T(H_0)} \left( \frac{\gamma (h\pi)^{\chi}}{(1 - \gamma)(\rho - \chi (1 - \gamma))} \left( e^{\frac{(\rho - \chi (1 - \gamma))(h\pi)}{\rho}} - 1 \right) + \pi H_T \right).
\]

The typical dynamics corresponding to this first possible solution is depicted in Fig. 3.

In the next section the other optimality candidates are briefly reviewed.

---

\[\text{Note that they are a direct consequence of the second necessary condition in (8). Also note that the parameters } R \text{ and } \pi \text{ do not show up in the sufficient condition } \chi > \chi \text{ since they don’t enter into the expressions of } \Omega. \text{ In contrast, the parameters } h \text{ and } p^* \text{ enter this condition through } \Omega.\]
4.2. No-education and permanent dictatorship

The general solution corresponding to no investments in education, i.e. \( E = 0 \), is given by (the superscript 2 refers to the regime with no education):

\[
\begin{align*}
C^2(t) &= p^2 R - s + \Omega H^2(t) \\
H^2(t) &= e^{-\delta t} \\
\lambda^2(t) &= e^{(\rho+\delta) t} \left( L - \int \Omega C^2(u) e^{-\delta (\rho+\delta) u} du \right)
\end{align*}
\]

with \( H \) and \( L \) constants to be determined. We establish the following result:

**Proposition 2.**

(i) There always exists a solution combining permanent dictatorship with no investment in education.

(ii) There may be solutions alternating periods of investment in education with periods of no investment, but these never provide a candidate for optimality.

The solution with permanent dictatorship and no education is characterized by a decreasing flow of consumption and a decreasing stock of human capital. Consumption asymptotically converges toward \( C^2(\infty) = p^2 R - s \), while the stock of human capital vanishes. This is the path taken by the elite that considers investment in human capital too costly in terms of resources needed by educated workers and in terms of the risk of being overthrown. Solutions with no education exist for any level of the stock of human capital. The value function corresponding to the optimality candidate with no education is given by:

\[
V^2(H_0) = \int_0^\infty \frac{1}{1 - \gamma} \left( p^2 R - s + \Omega H_0 e^{-\delta t} \right)^{1-\gamma} e^{-\rho t} dt.
\]

Finally, solutions featuring a regime change from positive to zero education can be disregarded because these are always dominated by other solutions (see Appendix A.3).

At this stage of the analysis, we are left with two optimality candidates, which makes the options available to the elite very clear. Either they choose to rely on resource wealth and not to invest in education in order to keep the labor force uneducated and docile. But this requires sacrificing education-driven economic growth. Or, the elite engage in a policy of sustained investment in education, which promotes the accumulation of human capital at the cost of giving up political power in finite time because of the development of citizens’ claims for democracy. As expected, variables like the returns to education (and human capital), the initial stock of human capital, the discount rate but also the share of wealth accruing to the elite after a revolution will play a central role in explaining what is the elite’s best option.

Before determining the optimal choice of the elite, we compare regime 1 and 2 in terms of their implications for the link between inequality and institutional change.

4.3. Implications for inequality

So far, we have addressed the links income-institutional change and human capital-institutional change. In this respect, we have shown that the predictions of the model are consistent with Lipset’s theory. It remains to study the link inequalities-institutional change.

As workers are a homogeneous mass of individuals, the only way to appraise inequalities in a direct and elementary way is by tracking the consumption of the elite vs. the consumption of workers. Although this is not completely in the spirit of Lipset’s theory concerning this aspect (see Jung and Sundt, 2014, for a tighter connection), this exercise turns out to be worthwhile. Recall that the workers’ income is entirely devoted to consumption. At any solution, we have \( C^i_w(t) = s + \phi H^i(t) \) for \( i = 1, 2 \). Let \( \bar{f}^i(t) \equiv \frac{C^i_w(t)}{C^i(t)} \) be the index of inequalities at solution \( i = 1, 2 \). Then, we can establish the following result.

**Proposition 3.** At the solution with education-driven institutional change, inequalities continuously shrink. At the solution with permanent dictatorship and no investment in education, the opposite result holds if:

\[
R > \frac{(1-\alpha)s}{\phi \alpha} \left( \frac{\alpha A}{p^2} \right)^{\frac{1}{\rho+\delta}}.
\]

Along the transition process to non-dictatorship, inequalities decrease. It is as if in order to prepare the ground for a democratic regime, the elite have to progressively reduce the income (consumption) gap between the two groups until the institutional change. Intuitively, since the elite invest in human capital along this path, growth is stimulated. But the positive growth effect is dominated by the negative effect due to increasing awareness and the elite have no option but to sacrifice part of their consumption to satisfy the no-revolt constraint and delay the date of leaving office.

Moreover and not surprisingly, permanent dictatorship implies a widening of inequalities if resource windfalls are high, the awareness cost is large, the international resource price is high, and the level of subsistence consumption is low. Under these conditions, the dictator is able to fill the revolt constraint at lower cost. By not investing in human capital, citizens are maintained under control while the elite become richer and richer relative to the workers.

The next section investigates the optimality of the above-identified solutions.
5. Optimality, poverty trap, and policy implications

The optimality analysis boils down to a study of the relative performance of the solution with education-driven institutional change vs. the solution with permanent dictatorship and no-education. To conduct this analysis, we compare the present values associated with our optimality candidates. We summarize the results as follows:

**Proposition 4.** Let $H_0 \in [0, H_T]$. The following cases can arise:

(i) The solution with permanent dictatorship and no-education is optimal for all $H_0$ iff $V^D(H_T) > V^I(H_T)$;
(ii) The solution with education-driven institutional change is optimal for all $H_0$ iff $V^I(0) > V^D(0)$;
(iii) Otherwise, a human capital poverty trap arises. There exists $H \in [0, H_T]$ such that the solution with education-driven institutional change is optimal iff $H_0 > H$.

Both the no-education regime with persistent dictatorship and the education regime with democratization can arise. Depending on the parameters, it might be possible that: (i) the first alternative is chosen independently of the initial stock of human capital; (ii) the second alternative is chosen independently of the initial stock of human capital; and (iii) the regime choice depends on the initial human capital stock, a low stock is associated to no-education investment and infinite horizon dictatorship while a large stock is associated to education investment and democratization in finite time.

This result sheds light on the relationship between education, development, and democratization. First, education is necessary for both development and democratization: it is the engine of economic growth; and, by increasing the workers awareness, it is also responsible for the institutional change. Second, education investments might be optimal for the ruling elite, despite it might lead to more democratic institutions, as their political power gets substituted by economic returns. Third, the existence of a poverty trap is particularly interesting for it teaches that development aid leading to “small” increases in human capital might not be sufficient for a regime switch and thus fails to have permanent effects on development and institutions of the recipient country. Indeed, our theory delivers much more in this respect, and we shall come back to this implication below.

The next result further emphasizes the conditions under which the elite find the democratization path optimal.

**Proposition 5.** The solution with education-driven institutional change is optimal for all $H_0 \in [0, H_T]$ if:

$$p^R R - s > e^{\pi c} (h_{\#})^{-\frac{1}{h}}.$$  \hspace{1cm} (11)

This sufficient condition can easily be interpreted once one observes that it is a stronger version of the second necessary existence condition (8). It confirms the previous intuition about which factors are crucial for the decision of the elite to educate the population and drive the country out of autocracy. Indeed, Proposition 5 illustrates that institutional change initiated by the elite is a matter of having the right conditions. A large stock of resources might not trigger education policies and democratization if the education sector doesn’t ensure sufficient economic returns to the elite. A permanent positive shock to international resource prices might give the elite the wealth needed to invest in education and human capital accumulation, but this opportunity will not be taken if the wealth prospects at the time of institutional change are not sufficiently compelling.

Last but not least, it is important to interpret the above results in light of education aid policies. Case iii) of Proposition 4 indicates that the model can deliver (optimal) poverty traps. Propositions 3 and 5—with the associated necessary and sufficient conditions (8) and (11)—teach us that a massive aid policy of education systems may temporarily increase human capital, i.e. by improving access to education and therefore raising the enrollment rates. Yet, this may not have a permanent economic and institutional effect. One reason is that education aid might be unable to improve sufficiently education systems, i.e. the parameter $h$ might not reach the level needed to escape the poverty trap. Another reason is that institutional conditions are not good enough (here for example, $\pi$ should be big enough). This is consistent with the question viewing the efficiency of large aid to the poorest countries (see Kraay and Raddatz, 2007, for example).

Thus, aid programs should target education efficiency at the same time as educational outcomes, especially in those economies where the resource wealth is limited. Our theory produces a clear hierarchy in this respect: if education efficiency (our parameter $h$) is above a certain threshold value for education, development and democratization will turn optimal *ceteris paribus* irrespective of the initial value of capital ($H_0$) and even though the country is run by an autocratic regime (as one can infer from Proposition 5). This result suggests a clear-cut way to settle the traditional tradeoff between expanding school enrollments versus improving school quality faced by development agencies.

6. Extension: Non-linear and endogenous scrap value

In this Section, we go one step further in the description of what is going on at the date of the regime change. Assume that a political transition to democracy arises at a date $T$. Then, the newly created democratic government has to decide how to share the natural resource windfall between the elite and the workers.\(^{15}\) There are two possible formulations of such a

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\(^{15}\) It is worth noting that there is nothing more to bargain over than windfall resources, $p^R$: the final good sector is competitive, resulting in zero profits at the equilibrium, and we can easily think of the public education sector as a non-profit organization.
problem in the literature: lobbying models (see for example, Grossman and Helpman, 1994, or Esteban and Ray, 1999) and Nash bargaining models (see Binmore, 2011). While the two modeling strategies are not strictly equivalent as very recently outlined by Voss and Schopf (2018), they share essentially the same implications when it comes to policy choice. Here, we assume that the sharing rule results from Nash bargaining solution, where the outside option is normalized to zero.\(^{16}\) Consistent with the Lipsetian theory, the negotiation power of the elite is increasing with respect to the workers’ human capital \(H_T\) (the elite can expect a larger part of the cake by interacting with more educated people). Let \(\varepsilon \in (0, 1)\) be the share of resources the elite is assigned and let \(\sigma(H(T)) \in (0, 1)\) be the elite bargaining power, where \(\sigma'(.) > 0\). The Nash bargaining solution is:

\[
\varepsilon^* = \arg \max \left\{ \varepsilon^{\sigma(H(T))} (1 - \varepsilon)^{1 - \sigma(H(T))} \right\} = \sigma(H(T)).
\]

When the bargaining power takes the linear form \(\sigma(H(T)) = \sigma H(T)\), assuming that the elite consume their entire revenue \(\sigma H(T)p^R\) from \(T\) on, we obtain the scrap value:\(^{17}\)

\[
S(H_T) = \frac{\pi \xi}{1 - \gamma} (H(T))^{1 - \gamma} \text{ with } \xi = \frac{(\sigma p^R)^{1 - \gamma}}{\pi \rho}.
\]

Using this alternative formulation, we can establish the counterpart of the main existence and optimality results as stated in Propositions 1 and 5. Importantly, the main conclusions remain qualitatively the same (see the corresponding proofs in the Appendix B.2). The next proposition deals with the existence issue in this extended framework.

**Proposition 6.**

(i) There is no solution combining permanent dictatorship and positive education.

(ii) There may be a solution combining institutional change and positive education.

(a) This solution is characterized by accumulation of human capital, institutional change in finite time \(T = T(H_0, R, \pi)\), and a corresponding end-point stock of human capital:

\[
H_T = \frac{h(1 - \gamma)(p^R - s)}{\rho - \chi (1 - \gamma) - \gamma h(\pi \xi)^{1 + \xi}}.
\]

(b) Necessary conditions for the existence of a solution for all \(H_0 \in [0, H_T]\) are:

\[
\begin{cases} 
\rho > \chi \\
p^R - s > h(\pi \xi)^{1 + \xi} H_T \iff (\rho - \chi (1 - \gamma) - \gamma h(\pi \xi)^{1 + \xi}) > h(h\pi)^{1 + \xi}.
\end{cases}
\]

(c) Sufficient conditions for the existence of a unique solution are (13) and \(\chi > \chi_*\), with \(\chi_* \in (0, \rho)\) the unique solution of:

\[
e^{\frac{\pi \xi}{\rho - \chi}} = \frac{\rho}{\rho - \chi}.
\]

As far as optimality is concerned, we can show the following:

**Proposition 7.** The solution with education-driven institutional change is optimal for all \(H_0 \in [0, H_T]\) if:

\[
p^R - s > (\pi \xi)^{1 + \xi} H_T e^{\frac{\pi \xi}{\rho - \chi}} \iff (\rho - \chi (1 - \gamma) - \gamma h(\pi \xi)^{1 + \xi}) > h(h\pi)^{1 + \xi} (1 + \gamma) e^{\frac{\pi \xi}{\rho - \chi}}.
\]

Importantly, these results highlight that: (i) the existence and optimality analyses can easily be extended to the (more general) non-linear scrap value function; and (ii) the existence and optimality conditions have the same shape and interpretation as the ones we get with a linear scrap value. Finally, the results corresponding to Propositions 2, 3, and 4 remain unchanged (see Appendix B.2).

7. Conclusion

We have proposed a Lipsetian theory of voluntary power handover. The democratization is in particular triggered by the tradeoff between economic returns to citizens’ education and their increasing claims to consumption. A high level of human capital is both a prerequisite and a consequence of institutional change. The negative correlation between inequality and democratization follows from the citizens becoming more aware and demanding as their human capital increases. Furthermore, consistently with Lipset’s theory, we show that higher resource wealth is (weakly) favorable to democratization. Indeed, resource wealth cannot alone trigger democratization in our setting: democratization may be suboptimal and

\(^{16}\) As opportunely mentioned in Voss and Schopf (2018), while the Nash bargaining solution may be questionable as a positive theory, it is used as such in dozens of articles. There is an underlying evolution-based justification for that emphasized by Binmore (2011): evolution has provided humans with fairness norms which favor cooperation even in situations where actual bargaining cannot take place. Therefore, if the Nash bargaining solution is plausible as a fairness norm, it can be considered as a positive model.

\(^{17}\) Note that the parameter \(\pi\) now has a different interpretation. Indeed, we may argue that even if the elite deliberately choose to leave power, the transition can be fierce and they can be killed with probability \(1 - \pi\). So the regime shift is peaceful and the bargaining stage takes place only with probability \(\pi\).
dominated by permanent dictatorship even when resources are abundant. The main engine of institutional change remains education: if the education sector is not effective enough, resource abundance may not lead to autocracies’ disruption. Finally, our theory brings out a new ingredients, which we believe essentially Lipsetian. No elite would hand out the power voluntarily if they expect significant punishment and confiscations after retiring from office, and such a risk is decreasing with the achieved level of education. This mechanism is modeled in an elementary way in our work, and it seems to be a decisive ingredient.

An important policy implication can be drawn in terms of aid programs intended to promote development. Education support may trigger the accumulation of human capital, sustained growth, and, as a side-product, democratization. However, its effectiveness requires: i) that development programs be large enough; and ii) that these be directed to the improvement of the education system (i.e. quality of teaching and school infrastructure), rather than to the achievement of specific education achievements (i.e. a certain level of alphabetization).

**Appendix A. Technical appendix**

**A.1. Optimality conditions**

The necessary optimality conditions (NOCs) of our problem are:

\[
(A\alpha(\frac{Q}{H})^{\alpha-1} - p^x)C^{-\gamma} - \mu_Q = 0,
\]

\[
-C^{-\gamma}h\lambda + \mu_E = 0,
\]

\[
\dot{\lambda} = (\rho + \delta)\lambda - (A(1 - \alpha)(\frac{Q}{H})^\alpha - \phi)C^{-\gamma},
\]

\[
\dot{H} = hE - \delta H.
\]

with \( \lambda \) the shadow value of the stock of human capital, \( \mu_E, \mu_Q \geq 0 \) the Lagrange multipliers associated respectively with the constraints \( E \geq 0 \) and \( R \leq Q \), and \( C + E = p(R - Q) + AQ^\alpha H^{1-\alpha} - s - \phi H \). Slackness conditions require:

\[
\mu_E \geq 0, \quad \mu_E E = 0 \quad \text{and} \quad \mu_Q \geq 0, \quad \mu_Q (R - Q) = 0.
\]  
(15)

This problem produces in general four different regimes, depending on whether \( E \geq 0 \) and \( R \leq Q \). We want to get rid of the regimes with \( R = Q \) since they are not interesting from an economic point of view and not realistic as well. It is straightforward to show that a regime with both \( E = 0 \) and \( R = Q \) cannot take place. So we only have to study the situation where \( E > 0 \) and \( R = Q \). This regime can hold (if and) only if

\[
\mu_Q \geq 0 \iff H \geq \left( \frac{p^x}{A\alpha} \right)^{\frac{1}{1-\gamma}} R = H.
\]

which sets a lower bound on the domain of variation of \( H \).

Evaluated at any potential steady state, the NOCs reduce to:

\[
\dot{h}\lambda = C^{-\gamma},
\]

\[
\rho + \delta = h(A(1 - \alpha)(\frac{Q}{H})^\alpha - \phi),
\]

\[
hE = \delta H.
\]

with \( C + E = (A(1 - \alpha)(\frac{Q}{H})^\alpha - \phi)H - s \). From the second equation, we get a necessary condition (on the marginal productivity of human capital) for the existence of a solution:

\[
A(1 - \alpha)\left( \frac{R}{H} \right)^\alpha > \phi \iff H < \left( \frac{A(1 - \alpha)}{\phi} \right)^\frac{1}{\alpha} R = \overline{H},
\]

which defines an upper bound on the domain of variation of \( H \).

Now, for the corner regime to take place and be permanent, it must hold that the interval \([H, \overline{H}]\) is non-empty, otherwise either it is not optimal to settle in this regime and/or it doesn’t host a steady state.

So, if we impose

\[
\overline{H} < H \iff 1 > \left( \frac{A(1 - \alpha)}{\phi} \right)^\frac{1}{\alpha} \left( \frac{A\alpha}{p^x} \right)^{\frac{1}{1-\gamma}}
\]

then, this corner regime is not relevant for the analysis. Note that this inequality is more likely to hold when \( A \) is small, \( p^x \) is high and so is \( \phi \).

When \( R > Q \), maximizing the income with respect to national resource supply \( Q \), requires that national prices equalize international ones, i.e. \( p = p^x \), and sets the optimal ratio between resources and human capital as follows:

\[
\frac{Q}{H} = \left( \frac{\alpha A}{p^x} \right)^{\frac{1}{\alpha}}.
\]  
(16)
Then, the set of optimality conditions are:

\[
\begin{align*}
C^<\gamma &= h\lambda + \mu E \\
\dot{\lambda} &= (\rho + \delta)\lambda - \Omega C^<\gamma \\
\dot{H} &= hE - \delta H
\end{align*}
\]

with \(\Omega\) defined in the main text.

According to the first equation above, the optimal investment in education is such that the marginal benefit from education equals the marginal cost of investing in education (in terms of foregone consumption).

The optimal time \(T < \infty\) for binding the no-revolt constraint, and inducing a regime change, is such that the current value of the Lagrangian be equal to the value of the salvage function; i.e.:

\[
u(C(T)) + \lambda[hE(T) - \delta H(T)] = \rho\pi H(T)
\]

Finally, the transversality condition requires that:

\[
\lambda(T) = \frac{\partial S(H)}{\partial H}\big|_{H=H(T)} = \pi
\]

Convexity of the problem with respect to optimal education investment \(E\) and internally supplied resources \(Q\) guarantees that the corresponding second order conditions are always satisfied. The second order conditions for the optimal stopping problem are not necessarily met.

A.2. Proof of proposition 1

A.2.1. Item (i)

From the system of optimality conditions, a steady state in the interior regime, \(E > 0, R > Q\), exists if and only if \(\rho = \chi\), which is a knife-edge situation. Moreover, we know that if \(\rho > \chi\), then we break the non-negativity condition on \(H\). And if \(\rho < \chi\), then we break the condition \(Q < R\), as long as \(R < \infty\) and \(X \geq 0\).

A.2.2. Item (ii)

The optimality condition (18), for the stopping time \(T\), implies (index 1 stands for the interior regime):

\[
C^1_0 = (h\pi)^{-\frac{1}{1-\gamma}} e^{-\frac{1-\gamma}{\gamma} T}.
\]

Substituting in (5), we obtain:

\[
\begin{align*}
C^1(t) &= (h\pi)^{-\frac{1}{1-\gamma}} e^{\frac{1}{\gamma}(\rho - \chi)(t-T)} \\
H^1(t) &= \varphi(T)e^{\gamma T} + \frac{h(p(R-s))}{\rho - \chi(1-\gamma)} e^{\frac{(\rho - \chi)T}{\gamma}} - \frac{h(p(R-s))}{\rho - \chi(1-\gamma)}
\end{align*}
\]

where \(\varphi(T) \equiv (H_0 + \frac{h(p(R-s))}{\rho - \chi(1-\gamma)} e^{\frac{(\rho - \chi)T}{\gamma}} - \frac{h(p(R-s))}{\rho - \chi(1-\gamma)})\). The optimal stopping condition (17) can be rewritten as:

\[
\frac{\gamma(h\pi)^{-\frac{1}{1-\gamma}}}{1-\gamma} + h\pi(p^*R - s) + \chi\pi H^1(T) = \rho\pi H^1(T)
\]

where the LHS is the marginal benefit of waiting, while the RHS is the marginal cost of waiting. Since \(\frac{\gamma(h\pi)^{-\frac{1}{1-\gamma}}}{1-\gamma} + h\pi(p^*R - s) > 0, \rho \leq \chi\) implies that LHS > RHS for each level of human capital.

When \(\rho > \chi\), the RHS increases faster than the LHS. Then, there exists an optimal end-point \(H^1(T) = H_f\), which is given by:

\[
H_f = \frac{h}{\rho - \chi} \left( \frac{\gamma(h\pi)^{-\frac{1}{1-\gamma}}}{1-\gamma} + p^*R - s \right) > 0.
\]

The second order condition (SOC) for the optimal stopping problem is satisfied iff \((\chi - \rho)H^1(T) < 0\), which requires \(H^1(T) > 0\). From the continuity of the state variable, we have:

\[
H_f = \varphi(T)e^{\gamma T} + \frac{\gamma h(h\pi)^{-\frac{1}{1-\gamma}}}{\rho - \chi(1-\gamma)} - \frac{h(p^*R - s)}{\chi}
\]

Rearranging, we obtain:

\[
\varphi(T)e^{\gamma T} = \frac{\rho h}{\rho - \chi} \left( \frac{p^*R - s}{\chi} + \frac{\gamma^2(h\pi)^{-\frac{1}{1-\gamma}}}{(1-\gamma)(\rho - \chi)(1-\gamma)} \right)
\]
Let \( F(T) \) be the LHS and \( G > 0 \) the RHS. By the monotonicity of the path \( \{H^1(t)\}_{t=0}^{T} \) (see the next item) and the SOC, \( H_0 \leq H_T \). Assume \( H_0 < H_T \). This is equivalent to \( F(0) < G \). Moreover, \( F(\infty) = -\infty \). The sign of the derivative of \( F(T) \):

\[
F'(T) = e^{\frac{T}{\gamma}} \left( \chi H_0 + h(p^sR - s) - h(h\pi) \frac{1}{\gamma} e^{-\frac{\rho \chi}{\gamma}} \right).
\]

For the existence of an optimal interior \( T \), we need that \( F'(0) > 0 \), which is equivalent to:

\[
H_0 > H_0 = \frac{h(h\pi) \frac{1}{\gamma} - h(p^sR - s)}{\chi}.
\]  

(23)

and for the interval \( (H_0, H_T) \) to be non-empty, we must impose:

\[
p^sR - s + \frac{(h\pi) \frac{1}{\gamma} (\chi - \rho(1 - \gamma))}{\rho(1 - \gamma)} > 0;
\]  

(24)

which is equivalent to the SOC.

We tackle existence by checking whether the optimality candidate exists for any \( H_0 \in (0, H_T] \). Substitute first \( H_0 = 0 \) in all the expressions above. Then, \( F'(0) > 0 \) simplifies to

\[
p^sR - s > (h\pi) \frac{1}{\gamma},
\]  

(25)

and under this condition the SOC is satisfied.

Next, let \( T^* \) be the time that maximizes \( F(T) \). It satisfies:

\[
\dot{T} = -\frac{\gamma}{\rho - \chi} \ln \left( \frac{p^sR - s}{(h\pi) \frac{1}{\gamma}} \right),
\]  

(26)

and for the existence of (at most two) \( T^* > 0 \) that solve(s) (22), it must hold that \( F(T^*) > G \), or, equivalently:

\[
\frac{(\rho - \chi)^2}{\rho(\rho - \chi(1 - \gamma))} e^{\frac{T^*}{\gamma}} \left[ \frac{p^sR - s}{(h\pi) \frac{1}{\gamma}} \right]^2 > \frac{p^sR - s}{(h\pi) \frac{1}{\gamma}} + \frac{\gamma^2 \chi}{(1 - \gamma)(\rho - \chi(1 - \gamma))},
\]

which is a simple polynomial of degree 2 in \( \frac{p^sR - s}{(h\pi) \frac{1}{\gamma}} \). Now, under (25), this ratio is larger than 1. So, a sufficient condition for \( \dot{T} > G \) is:

\[
e^{\frac{T^*}{\gamma}} > \rho(\rho - \chi(1 - \gamma)) + \gamma^2 \chi
\]

\[
(1 - \gamma)(\rho - \chi)^2
\]

(27)

where both terms of the inequality above are greater than 1. A quick inspection of the properties of the LHS and RHS of (27) as functions of parameter \( \chi \), reveals that there exists a unique threshold \( \chi \in (0, \rho) \) such that (27) holds iff \( \chi < \chi \).

The reasoning above is also valid for any \( H_0 > 0 \). Actually, the existence of a solution for the particular value \( H_0 = 0 \) implies that such a solution exists for any \( H_0 > 0 \). In general, the optimal stopping time can be expressed as a function \( H_0 \):

\[
T^* = T(H_0) \text{ that satisfies, by differentiating (22):}
\]

\[
\frac{\partial T}{\partial H_0} = -\frac{1}{\chi \psi(T(H_0)) + \psi'(T(H_0))}.
\]

Given that we want \( \frac{\partial T}{\partial H_0} < 0 \) (uniqueness of the optimal trajectory), only the solution corresponding the increasing part of \( F(T) \) is relevant, i.e. one has \( F'(T) > 0 \), which is equivalent to:

\[
p^sR - s > e^{\frac{(\rho - \chi)T}{(h\pi) \frac{1}{\gamma}}}
\]  

(28)

Comparative statics on \( T \) that solves (22), given that this equation can be rewritten as \( F(T, R, \pi) = G(R, \pi) \). By the implicit function theorem, we have:

\[
\frac{\partial T}{\partial R} = \frac{\frac{\partial G}{\partial R} - \frac{\partial F}{\partial R}}{\frac{\partial F}{\partial T}} \quad \text{and} \quad \frac{\partial T}{\partial \pi} = \frac{\frac{\partial G}{\partial \pi} - \frac{\partial F}{\partial \pi}}{\frac{\partial F}{\partial T}}.
\]

given that \( \frac{\partial G}{\partial R} - \frac{\partial F}{\partial R} = \frac{h^2(\rho - \chi)}{\chi} < 0 \) and \( \frac{\partial G}{\partial \pi} - \frac{\partial F}{\partial \pi} = \frac{h^2(\rho - \chi)}{\chi} \left( \frac{\rho - \chi}{\rho - \chi(1 - \gamma)} \right) e^{\frac{(\rho - \chi)T}{(h\pi) \frac{1}{\gamma}}} + \frac{(\rho - \chi)T}{(h\pi) \frac{1}{\gamma}} < 0 \). Finally, since the solution satisfies \( \frac{\partial \pi}{\partial T} > 0 \) (because we want \( T(H_0) < 0 \) for all \( H_0 > 0 \)), we can conclude that \( T^* \) is decreasing w.r.t both \( R \) and \( \pi \).

A.2.3. Monotonicity of trajectories

The value function at any \( H^1(t_i) = H_i \) taken on the optimal path is given by:

\[
V^1(H_i) = e^{-\rho \theta(H_i)} \left( \frac{\gamma (h\pi) \frac{1}{\gamma} \theta(H_i)}{(1 - \gamma)(\rho - \chi(1 - \gamma))} \left[ e^{\frac{(\rho - \chi(1 - \gamma))}{\gamma} \theta(H_i) - 1} + \pi H_T \right] \right.
\]
with $\theta(H_1) = T(H_1) = t_1$, the optimal time-to-go before stopping, which doesn’t depend on $t_2$.

If there exists a non monotone optimal trajectory, it must be true that $H$ is first decreasing and then increasing. This implies that there exists $(t_1, t_2)$, with $t_1 < t_2$, such that: $H^1(t_1) = H^2(t_2)$. Thus, we have $\theta(H_1) = \theta(H_2)$. The time that elapses between $t_1$ and $T(H_1)$ must be the same as the one between $t_2$ and $T(H_2)$, which yields a contradiction because the optimal trajectory is uniquely defined ($H_T$ is invariant) and (initial) consumptions at $t_1$ and $t_2$ necessarily differ.

Finally note that the solution with positive education and a revolution in finite time yields the following present value to the elite:

$$V^1(H_0) = e^{-\rho T(H_0)}\left(\frac{\gamma(1+\rho)}{(1-\gamma)(\rho - \chi(1-\gamma))} \right)\left(e^{\lambda^1} - 1\right) + \pi H_T$$

(29)

This completes the proof of Proposition 1.

A.3. Proof of proposition 2

A.3.1. Item (i)

If regime R2 is permanent, then from the transversality condition $L = 0$, and the solution reduces to (using the superscript 2):

$$H^2(t) = H_0 e^{-\lambda t}$$

$$C^2(t) = p^2 R - s + \Omega H^2(t)$$

$$\lambda^2(t) = -e^{(\rho + \delta)u} \int_0^1 \Omega C(u)^{-\gamma} e^{-(\rho + \delta)u} du < 0.$$ 

The value function is given by:

$$V^2(H_0) = \int_0^\infty \frac{1}{1-\gamma} (p^2 R - s + \Omega H_0 e^{-\lambda t})^{-1-\gamma} e^{-\rho t} dt.\quad (30)$$

A.3.2. Item (ii)

Consider a trajectory $([H^1], [C^1])$ that reaches the locus $E = 0$ at date $t_1$ for some stock $H^1(t_1) = \tilde{H}$ and consumption $C^1(t_1) = \tilde{C}$.

From the dynamical system, both $H^1$ and $C^1$ are all decreasing w.r.t. time. The approach is to consider a solution with permanent $E = 0$ as a limit case of the solution with a regime change from $E > 0$ to $E = 0$. Let’s work with the general solution obtained by combining regimes 1 and 2. For the time being, let $t_1$ be given. Recall that the general solution in each regime is:

$$C^1(t) = C_0 e^{\frac{(x-p)\mu}{\nu}}$$

$$H^1(t) = \left(\frac{H_0 + \frac{h(p^R - s)}{\nu - \chi (1 - \gamma)}}{\nu - \chi (1 - \gamma)}\right) e^{\frac{\lambda t}{1-\gamma}} + \frac{\gamma h C_0}{\nu - \chi (1 - \gamma)} e^{\frac{\lambda t}{1-\gamma}} - \frac{h(p^R - s)}{\nu}$$

and,

$$H^2(t) = \tilde{H} e^{-\lambda (t-t_1)}$$

$$C^2(t) = p^2 R - s + \Omega H^2(t)$$

From the continuity of consumption at $t_1$, we obtain: $C_0 = (p^2 R - s + \Omega \tilde{H}) e^{\frac{(x-p)\mu}{\nu} (t_1 - t)}$ and $C^1(t) = (p^2 R - s + \Omega \tilde{H}) e^{\frac{(x-p)\mu}{\nu} (t_1 - t)}$.

From the continuity of the state variable at $t_1$, $\tilde{H}$ can be expressed as a function of $t_1$: $\tilde{H} = \tilde{C}(t_1)$ with:

$$\tilde{C}(t_1) = \frac{\rho - \chi (1 - \gamma)}{\rho - \chi (1 - \gamma) + \gamma \Omega h} \left( H_0 + \frac{h(p^R - s)}{\nu - \chi (1 - \gamma)} e^{\frac{\lambda t}{1-\gamma}} - \frac{h(p^R - s)}{\nu} \right)$$

$$\rho - \chi (1 - \gamma) + \gamma \Omega h \left( e^{\frac{(x-p)\mu}{\nu} (t_1 - t)} - 1 \right)$$

So the value corresponding to this trajectory can be written as:

$$V(t_1) = \frac{1}{1-\gamma} \left[ \int_0^{t_1} (p^2 R - s + \Omega \tilde{C}(t_1))^{-1-\gamma} e^{\frac{(x-p)\mu}{\nu} (t-t_1)} e^{-\rho t} dt + \int_1^\infty (p^2 R - s + \Omega \tilde{C}(t_1)) e^{-\lambda (t-t_1)} \right]^{-1-\gamma} e^{-\rho t} dt$$

Taking the derivative w.r.t $t_1$ yields:

$$\frac{\partial V}{\partial t_1} = \frac{1}{1-\gamma} (p^2 R - s + \Omega \tilde{C}(t_1))^{-1-\gamma} e^{-\rho t_1}$$

$$+ \frac{1}{1-\gamma} \int_0^{t_1} e^{\frac{(x-p)\mu}{\nu} (t-t_1)} e^{\frac{(x-p)\mu}{\nu} (t_1 - t)} (p^2 R - s + \Omega \tilde{C}(t_1))^{-\gamma} \left[ (1-\gamma) \Omega \tilde{C}''(t_1) \right. $$

$$+ \left. (1-\gamma) \rho - \chi (p^2 R - s + \Omega \tilde{C}(t_1)) \right] - \frac{1}{1-\gamma} (p^2 R - s + \Omega \tilde{C}(t_1))^{-1-\gamma} e^{-\rho t_1}$$
Taking the limit when $t_1 \to 0$, we obtain:

$$\lim_{t_1 \to 0} \frac{\partial V}{\partial t_1} = \int_0^\infty \Omega(\zeta(t_1)) e^{-\delta(t-t_1)}(p^R - s + \Omega \zeta(t_1) e^{-\delta(t-t_1)})^{-\gamma} e^{-\rho t} dt.$$  

The sign of the limit is determined by the sign of $\zeta'(0) + \delta \zeta(0)$. Direct computations yield: $\zeta(0) = H_0$ and the derivative of $\zeta(t_1)$ evaluated at $t_1 = 0$ is given by: $\zeta'(0) = -\delta H_0$. Thus, $\zeta'(0) + \delta \zeta(0) = 0$. Applying multi-stage optimal control theory (interpreting the change from $E > 0$ to $E = 0$ as a regime switching problem), a necessary condition for an immediate switch $t_1 = 0$ is $\lim_{t_1 \to 0} \frac{\partial V}{\partial t_1} \leq 0$ (see Amit, 1986, Theorem 1). Thus trajectories of the 1–2 type are always dominated by the ones associated with permanent $E = 0$.

The last possibility is a regime change from 2 to 1. Suppose the economy starts in regime $E = 0$ and enters the region with positive education at $t_1 < \infty$. Two options emerge. First, the economy stays in region 1 until the institutional change. This would imply the crossing of the locus $\hat{H} = 0$ in finite time. Yet, this is excluded since the trajectory $\{H(t)\}$ must be monotonous (see Appendix A.2). Second, the economy stays for a while in regime 1 before going back in regime 2. This is not optimal by the reasoning developed just above: the elite prefers instead to directly settle on the locus $E = 0$.

This completes the proof of Proposition 2.

A.4. Proof of proposition 3

At solution 1 (democratization), the index of inequalities is:

$$l^1(t) = \frac{(s - \phi h(p^R - s))}{h \pi} = \frac{1}{h \pi} e^{-\rho t} e^{-\delta t} + \phi \varphi(T) \chi^{-\rho} + \frac{\phi \gamma h}{\rho - \chi(1 - \gamma)}.$$

Take the derivative w.r.t time:

$$\dot{l}^1(t) = \frac{h \pi}{\gamma} e^{-\rho t} e^{-\delta t} [\rho - \chi(s - \phi h(p^R - s)) + \phi (\rho - \chi(1 - \gamma) \psi(T)).$$

The sign of the derivative is given by the sign of the term, denoted $\Psi$, in squared brackets. Evaluating $\Psi$ at $t = 0$, gives:

$$\Psi = (\rho - \chi)s + \phi (\rho - \chi(1 - \gamma)) H_0 + \gamma \phi h (p^R - s - (h \pi)^{\frac{1}{\gamma}} e^{-\delta t})$$

which is positive due to (28). Thus $\dot{l}^1(t) > 0$ for all $t \in [0, T]$.

At solution 2 (permanent dictatorship), the inequality index is:

$$l^2(t) = \frac{s + \phi H^2(t)}{p^R - s + \Omega H^2(t)}.$$  

with derivative:

$$\dot{l}^2(t) = \frac{p^R \dot{H}}{(p^R - s + \Omega H^2(t))} \left[ \phi \gamma - \frac{1 - \alpha}{\alpha} \left( \frac{\alpha A}{p^R} \right)^\frac{1}{\gamma} s \right].$$

Thus,

$$R > \frac{(1 - \alpha)s}{\phi \alpha} \left( \frac{\alpha A}{p^R} \right)^\frac{1}{\gamma}$$

is sufficient to conclude that $\dot{l}^2(t) < 0$ for all $t \in [0, \infty)$.

A.5. Proof of proposition 4

The proof of the first item relies on a time consistency requirement for optimal trajectories. By contradiction, assume that $V^1(H_T) < V^2(H_T)$. Two cases are possible.

Case 1: $V^1(0) < V^2(0)$. If the curves $V^1(H_0)$ and $V^2(H_0)$ intersect, then the number of intersections must be even. For instance, consider two intersections at $\tilde{H}$ and $\hat{H}$, with $0 < \tilde{H} < \hat{H} < H_T$. By construction, $V^1(H_0) > V^2(H_0)$ for all $H_0 \in (\tilde{H}, \hat{H})$; $V^1(H_0) < V^2(H_0)$ for all $H_0 \in [0, \tilde{H}) \cup (\hat{H}, H_T]$; $V^1(\tilde{H}) = V^2(\hat{H})$; and $V^1(H_T) = V^2(H_T)$.

At $H_0 = \hat{H}$, there exist two optima, i.e. the elite are indifferent between following path 1 (with positive education) or path 2 (no education). If the economy settles on path 1, then human capital increases (see Appendix A.2). Yet, by construction again, for any $H$ in $(\tilde{H}, H_T]$, $V^1(H) < V^2(H)$; i.e. the elite prefer path 2 to path 1, implying that the solution considered is not time consistent. This yields a contradiction. If the elite chooses path 2, then human capital decreases monotonically (see Appendix A.3). Yet, $V^1(H) > V^2(H)$ for all $H \in (\tilde{H}, H_T)$; i.e. there is a (non-degenerate) interval of time during which the elite prefer regime 1. Again, this contradicts time consistency.
**Case 2:** \( V^1(0) > V^2(0) \). Then, the number of intersections between \( V^1(H_0) \) and \( V^2(H_0) \) (if any) is odd. Assume a unique intersection at \( \hat{H} \). Then, \( V^1(H_0) > V^2(H_0) \) for all \( H_0 \in [0, \hat{H}) \); \( V^1(H_0) < V^2(H_0) \) for all \( H_0 \in (\hat{H}, H_T) \); and \( V^1(\hat{H}) = V^2(\hat{H}) \). At \( H_0 = \hat{H} \), there is a multiplicity of optima. Either \( E > 0 \) and \( H > 0 \). But then, \( V^1(H) < V^2(H) \) for all \( H \in (\hat{H}, H_T) \) and a contradiction emerges. Or, \( E = 0 \) and \( H < 0 \) and, again, a contradiction emerges.

Proofs of the remaining items are left to the reader since they exactly follow the same line. In particular the reasoning of the second item is symmetric when one works with \( V^1(0) > V^2(0) \). As for the third item, assuming that \( V^1(0) \leq V^2(0) \) and \( V^1(H_T) \geq V^2(H_T) \) (with one strict inequality), it’s easy to show that there exists a unique intersection between the two value functions, for a critical initial stock of human capital \( \hat{H} \) such that \( V^1(H_0) \equiv V^2(H_0) \iff H_0 \geq \hat{H} \).

### A.6. Proof of proposition 5

For the sake of exposition, let \( T(0) \) be denoted by \( T \). Then, \( V^1(0) > V^2(0) \) if and only if:

\[
e^{-\rho T} \left( \frac{\gamma (h\pi)^{\frac{(1-\gamma)}{\rho}}}{(1-\gamma)(\rho - \chi(1-\gamma))} \left[ e^{\frac{(p^1 R - s)\gamma}{\rho (1-\gamma)}} - 1 \right] + \pi H_T \right) > \frac{(p^1 R - s)^{1-\gamma}}{\rho (1-\gamma)};
\]

which, by the definition of \( H_T \) in (20) and by (22), yields:

\[
\frac{(p^1 R - s)^{1-\gamma}}{\rho (1-\gamma)} < \frac{\gamma (h\pi)^{\frac{(1-\gamma)}{\rho}}}{(1-\gamma)(\rho - \chi(1-\gamma))} \left[ e^{\frac{(p^1 R - s)\gamma}{\rho (1-\gamma)}} + \frac{h\pi (p^1 R - s)}{\rho} e^{(x - \rho)T} \right).
\]

Denote the RHS of (31) by \( J(T) \). It follows that \( J(0) = \frac{h\pi}{\rho} \left( \frac{\gamma}{1-\gamma} h\pi \right)^{\frac{1-\gamma}{\rho}} + p^1 R - s \) > 0, \( \lim_{T \to \infty} J(T) = \infty \) and:

\[
J'(T) = \frac{h\pi (\rho - \chi)}{\rho} e^{(x - \rho)T} \left[ (h\pi)^{\frac{1}{\rho}} e^{\frac{(p^1 R - s)\gamma}{\rho (1-\gamma)}} - (p^1 R - s) \right].
\]

We observe that \( J'(T) \leq 0 \iff T \leq \tilde{T} \), where \( \tilde{T} \) has been defined in (26). Thus, imposing \( J(\tilde{T}) > \frac{(p^1 R - s)^{1-\gamma}}{\rho (1-\gamma)} \), which is equivalent to:

\[
p^1 R - s > e^{\frac{\tilde{T}}{\rho}} (h\pi)^{\frac{1}{\rho}},
\]

is sufficient to conclude that \( V^1(0) > V^2(0) \).

Moreover, whatever the regime, the value functions are strictly increasing in \( H_0 \), it’s clear that a sufficient condition for having \( V^2(H_0) > V^1(H_0) \) for all \( H_0 \in [0, H_T) \) is \( V^2(0) \geq V^1(H_T) \), which is equivalent to:

\[
\frac{(p^1 R - s)^{1-\gamma}}{\rho (1-\gamma)} \geq \pi H_T \iff \frac{(p^1 R - s)^{1-\gamma}}{\rho (1-\gamma)} \geq \frac{\gamma (h\pi)^{\frac{(1-\gamma)}{\rho}}}{(\rho - \chi)(1-\gamma)} + \frac{\pi h (p^1 R - s)}{\rho - \chi}.
\]

### Appendix B. extensions

#### B.1. Elite’s problem in a risky world

In democratization processes, stochastic factors are often determinant. These factors potentially concern the emergence of a revolt or the likelihood that the revolt is successful. Hereafter we show how these factors opportunely modeled can be accounted for in our simple framework.

Consider the following a stochastic version of the no-revolt constraint (3), where workers may revolt even if their consumption is large enough:

\[
\begin{align*}
\text{revolt} & \quad \text{if } wH + \Theta < s + \phi H \\
\text{revolt with probability } \nu' & < 1 \quad \text{otherwise}
\end{align*}
\]

Moreover, assume that only a certain proportion \( \nu'' \in [0, 1] \) of revolts are successful in terms of delivering a regime change. Then,

\[
\text{Prob(successful revolt)} = \begin{cases} 
1 & \text{if } wH + \Theta < s + \phi H \\
\nu'' & \text{otherwise}
\end{cases}
\]

This extension nests the previous model when \( \nu = 0 \). Now, the elite need to take into account that a successful revolt may occur with probability \( \nu' \) at any time, even if \( wH + \Theta \geq s + \phi H \). An immediate implication of \( \nu > 0 \) is that, even if an autocratic regime does not plan to give up power and independently of the education policies implemented, successful revolts might happen and would lead to a regime change. In this sense, it seems that our results will be severely affected. Yet, while changing quantitatively the incentives of the elite, the optimality candidates and the optimal education policies remain qualitatively unaffected, as we shall argue next.
Let $T$ denote the time (finite or not) until which the elite ensures that $wH + \Theta \geq s + \phi H$. Let $\tau$ denote the random variable capturing the date of an unexpected and uncontrolled revolt, which would cost $L > 0$ to the elite. The information regarding its occurrence is summarized by a probability distribution function $F(t) = \text{prob}(\tau < t)$, with density $f(t)$ and support $[0, \infty)$. The constant probability $\nu$ of a successful revolt constraint implies that $F(t) = 1 - e^{\nu t}$. Then, the expected intertemporal utility of the elite can be written as:

$$U^* = \int_{t=0}^{T} e^{-\rho'T}[u(c(t))(1 - F(t)) - Lf(t)]dt + e^{-\rho'T}(1 - F(T))\pi H(T), \quad (34)$$

where $\rho' > 0$ is the pure time discount factor and $L \geq 0$ is the loss the elite incurs due to the revolt. After straightforward manipulation, the objective of the elite reduces to:

$$\int_{t=0}^{T} e^{-(\rho'+\nu')t}[u(c(t)) - \nu L]dt + e^{-(\rho'+\nu')T}\pi H(T). \quad (35)$$

Defining $\rho = \rho' + \nu$, and further assuming that $L = 0$, our analysis fully applies.

### B.2. Non-linear scrap value

#### B.2.1. Proof of proposition 6

Once the Nash bargaining problem solved, we can compute the indirect utility of democratization at $T$ for the elite. In each period, the elite consume their entire revenue $\sigma(H_T)p^R$. This yields the following scrap value:

$$\tilde{S}(H(T)) = \int_{t=0}^{T} \frac{1}{1 - \gamma} (\sigma(H(T))p^R)^{1-\gamma}e^{-\rho(1-\gamma)t}dt$$

When the bargaining power takes the linear form $\sigma(H(T)) = \sigma H(T)$, we obtain:

$$S(H_T) = \pi \tilde{S}(H(T)) \Rightarrow S(H_T) = \frac{\pi \zeta}{1 - \gamma}H(T)^{1-\gamma} \text{with } \zeta = \frac{(\sigma p^R)^{1-\gamma}}{\pi \rho}.$$  

Note that for $\varepsilon$ to be in the desired interval, $\sigma$ needs to be low enough. Also, for the sake of comparability with the baseline model, we have kept the parameter $\pi$ in the formulation.

Substituting this non-linear formulation of the scrap value in the optimization problem of the elite leads to the following optimality conditions for the stopping time:

$$\lambda(T) = \pi S'(H(T)) = \pi \zeta H(T)^{\gamma} \quad (37)$$

and,

$$u(C(T)) + \lambda(T)[hE(T) - \delta H(T)] = \rho \pi S(H(T)) \quad (38)$$

**Item (i)** The condition is not affected by the non-linearity of the scrap value. Thus, existence requires $\rho > \chi(1 - \gamma)$.

**Item (ii)** Let $H(T) \equiv H_T$. From the first optimality condition 37, for the stopping time $T$, we have:

$$C_0 = (h \pi \zeta)^{-\frac{1}{\gamma}}H_T e^{-\frac{h \pi \zeta}{\gamma}}.$$  

Substituting this expression in (5), we obtain:

$$C^1(T) = (h \pi \zeta)^{-\frac{1}{\gamma}}H_T e^{\frac{h \pi \zeta}{\gamma(1-\gamma)}(1-T)}$$

$$H^1(T) = \varphi(T)e^{\varphi T} + \frac{\gamma h(\pi \zeta)^{\frac{1}{\gamma}}}{\rho - \chi(1-\gamma)}H_T e^{\frac{h(p^R - s)}{\gamma}} - \frac{h(p^R - s)}{\gamma}$$

where $\varphi(T) = (H_0 + \frac{h(p^R - s)}{\gamma} - \frac{h(\pi \zeta)^{\frac{1}{\gamma}}}{\rho - \chi(1-\gamma)}H_T e^{\frac{h(p^R - s)}{\gamma}})$. The second optimal stopping condition (38) can be rewritten as:

$$\frac{\gamma (h \pi \zeta)^{\frac{1}{\gamma}}}{1 - \gamma}H_T^{1-\gamma} + \pi \zeta H_T^{1-\gamma}(\chi H_T + h(p^R - s)) = \frac{\rho \pi \zeta}{1 - \gamma}H_T^{1-\gamma} \quad (39)$$

Rearranging, we get the expression of $H_T$, the optimal end-point:

$$H_T = \frac{h(1-\gamma)(p^R - s)}{\rho - \chi(1-\gamma) - h(\pi \zeta)^{\frac{1}{\gamma}}}.$$  

So, a necessary and sufficient condition for the existence of a positive $H_T$ is:

$$\rho - \chi(1 - \gamma) > \gamma h(\pi \zeta)^{\frac{1}{\gamma}} \Rightarrow \frac{h(p^R - s)}{\gamma} > (h \pi \zeta)^{\frac{1}{\gamma}}.$$


It immediately follows that $\frac{\partial H_T}{\partial s} < 0$, $\frac{\partial H_T}{\partial T} < 0$ and

$$
\frac{\partial H_T}{\partial R} = \frac{h(1 - \gamma)p^R}{(\rho - \chi)(1 - \gamma) - \gamma h(h(\pi \xi)^{-\frac{1}{r}})^2} \left[ \rho - \chi(1 - \gamma) - h(h(\pi \xi)^{-\frac{1}{r}}) \left( \gamma + (1 - \gamma) \frac{p^R - s}{p^R} \right) \right].
$$

(42)

Compared to the corresponding formulation with linear scrap value, there is an additional effect channeling through $p^R$ that pushes in the opposition direction at the original one. However, the overall effect of an increase in $R$ on $H_T$ remains positive, as we shall discuss later.

The second order condition (SOC) for the optimal stopping problem is:

$$
-\pi H^1(T)S'(H_T) \left[ \rho - \chi(1 - \gamma) + \gamma \left( \frac{h(p^R - s)}{H_T} - h(h(\pi \xi)^{-\frac{1}{r}}) \right) \right] < 0.
$$

(43)

From the continuity of the state variable, we have:

$$
H_T = \varphi(T)e^{\tilde{T}T} + \frac{\gamma h(h(\pi \xi)^{-\frac{1}{r}})H_T}{\rho - \chi(1 - \gamma)} - \frac{h(p^R - s)}{\chi}.
$$

(44)

Rearranging we obtain:

$$
\varphi(T)e^{\tilde{T}T} = \frac{\rho h(p^R - s)}{\chi(\rho - \chi(1 - \gamma))}.
$$

(45)

Let $F(T)$ be the LHS and $G > 0$ the RHS.

Hereafter, let’s focus on an interior solution, i.e. $H_0 < H_T$. This is equivalent to $F(0) < G$. Moreover, $F(\infty) = -\infty$. The sign of the derivative of $F(T)$ is given by:

$$
F'(T) = e^{\tilde{T}T} \left( \chi H_0 + h(p^R - s) - h(h(\pi \xi)^{-\frac{1}{r}})H_T e^{-\frac{(p^R - s)}{\rho}} \right).
$$

For the existence of an optimal interior $T$ for democratization, it’s first necessary that $F'(0) > 0$, which is equivalent to:

$$
H_0 > H_0 = \frac{h(h(\pi \xi)^{-\frac{1}{r}})H_T - h(p^R - s)}{\chi}.
$$

(46)

One way to tackle (and simplify) the existence analysis is to make sure that the optimality candidate exists for any initial level of the stock of human capital $H_0 \in [0, H_T]$. We first scrutinize the conditions under which such a candidate exists for $H_0 = 0$. Substitute $H_0 = 0$ in all the expressions above. Then, $F'(0) > 0$ simplifies to

$$
\frac{h(p^R - s)}{\chi} > h(h(\pi \xi)^{-\frac{1}{r}})H_T
$$

$$
\Leftrightarrow \frac{\rho - \chi(1 - \gamma)}{\rho} > \frac{1}{\chi} \left( \frac{\sigma p^R}{\gamma} \right)^{\frac{1}{\gamma}} > (h(\pi))^{-\frac{1}{r}}.
$$

(47)

Note that this condition is stronger than (41). In addition, under this condition, the SOC is satisfied iff $H^1(T) > 0$. Remember that we focus on monotone paths for $H^1(t), \{H^1(t)\}_{t=0}^T$ (see the next item). Thus, using the SOC, $H_0 \leq H_T$ holds. This also proves that, under (47), $\frac{\partial H_T}{\partial T} > 0$.

Next we define the value that maximizes $F(T)$, say $\tilde{T}$, as:

$$
\tilde{T} = \frac{\gamma}{\rho - \chi} \ln \left( \frac{p^R - s}{h(\pi \xi)^{-\frac{1}{r}} H_T} \right).
$$

(48)

and for the existence of (at most two) $T^* > 0$ that solve(s) (45), it must hold that $F(\tilde{T}) > G$. This condition can be restated as a condition on the parameters of the technology and preferences. Indeed, $F(\tilde{T}) > G$ implies

$$
\frac{e^{\frac{\rho}{\rho - \chi}}}{p^R - s} > \frac{(h(\pi \xi)^{-\frac{1}{r}} H_T}{p^R - s}.
$$

(49)

A quick inspection of the properties of the LHS and RHS of (49), seen as functions of parameter $\chi$, reveals that the LHS is inverted U-shaped, takes value 1 at $\chi = 0$, and asymptotically goes to $\infty$. The RHS is monotonically increasing, starts at a level below 1, and reaches a finite value when $\chi$ approaches $\rho$. So, there are two possible cases: either the LHS is always above the RHS; or there exist $\chi, \frac{\pi}{\rho} \in (0, \rho)$ such that (49) holds iff $\chi \in [0, \chi) \cup (\frac{\pi}{\rho}, \rho)$.

All the above reasoning remains valid for any $H_0 > 0$. Actually, the existence of a solution for the particular value $H_0 = 0$ implies that such a solution exists for any $H_0 > 0$. In general, the optimal stopping time can be expressed as a function $H_0$: $T^* = T(H_0)$ that satisfies, by differentiating (45):

$$
\frac{\partial T^*}{\partial H_0} = -\frac{1}{\chi \varphi'(\tilde{T}(H_0)) + \varphi'(\tilde{T}(H_0))}.
$$

\[\text{Note that (47) implies (49) when the LHS is larger or equal to 1, which requires $\chi$ be above a boundary. In this case, the statement is very similar to what we have but this is only a sufficient condition.}\]
Given that we want $\frac{\partial T}{\partial H_0} < 0$ (uniqueness of the optimal trajectory), only the solution corresponding the increasing part of $F(T)$ is relevant, i.e. one has $F'(T) > 0$, in particular for $H_0 = 0$, which is equivalent to:

$$p^sR - s > (h\pi \xi)^{\frac{1}{2}} H_T e^{\frac{(p^sR - s)}{\gamma}}$$

(50)

Let use rewrite (45) as $F(T, R, \pi) = G(R)$ and let $T^*$ denote (one of) its solution. By the implicit function theorem, we have:

$$\frac{\partial T}{\partial R} = \frac{\frac{\partial G}{\partial R} - \frac{\partial F}{\partial R}}{\frac{\partial F}{\partial \pi} - \frac{\partial T}{\partial \pi}}$$

and

$$\frac{\partial T}{\partial \pi} = \frac{\frac{\partial G}{\partial \pi} - \frac{\partial F}{\partial \pi}}{\frac{\partial F}{\partial R} - \frac{\partial T}{\partial R}}$$

given that $\frac{\partial F}{\partial \pi} = \frac{\gamma h(h\pi \xi)^{\frac{1}{2}} H_T e^{\frac{(p^sR - s)}{\gamma}}}{\rho - \chi(1 - \gamma)} - \frac{\gamma h e^{\frac{(p^sR - s)}{\gamma}}}{\rho - \chi(1 - \gamma)} > 0$, $\frac{\partial G}{\partial R} = \frac{h(1 - \gamma)p^s}{h(1 - \gamma)}$ and

$$\frac{\partial F}{\partial \pi} = e^{T} \left[ \frac{h p^s}{\rho - \chi(1 - \gamma)} - \frac{\gamma h(h\pi \xi)^{\frac{1}{2}} H_T e^{\frac{(p^sR - s)}{\gamma}}}{\rho - \chi(1 - \gamma)} \left( \frac{\partial \zeta}{\partial R} + \frac{\partial H_T}{\partial R} \right) \right]$$

with

$$- \frac{\partial \zeta}{\partial R} + \frac{\partial H_T}{\partial R} = \frac{p^s}{\rho - \chi(1 - \gamma)}((1 - 2\gamma)p^sR - (1 - \gamma)s - \gamma^2 p^sR h(h\pi \xi^{\frac{1}{2}}) + \gamma^2 p^sR h(h\pi \xi^{\frac{1}{2}})).$$

Monotonicity of trajectories

The value function at any $H_1(t_1) = H_1$ taken on the optimal path is given by:

$$V^1(H_1) = e^{-\rho \theta(H_1)} \left( \frac{\gamma h h\pi \xi^{\frac{1}{2}} H_T e^{\frac{(p^sR - s)}{\gamma}}}{\rho - \chi(1 - \gamma)} (e^{\frac{(p^sR - s)}{\gamma}} H_T - 1) + \pi \zeta \right) \frac{H_T^{1 - \gamma}}{1 - \gamma}$$

with $\theta(H_1) = T(H_1) - t_1$, the optimal time-to-go before stopping, which doesn’t depend on $t_1$, and $S(H_T)$ defined in (36).

If there exists an optimal trajectory of type 1 from some $H_0$ with $H(t)$ non monotone, it must be true that $H$ is decreasing first, then increasing. This implies that there exists $(t_1, t_2)$, with $t_1 < t_2$, such that $H_1(t_1) = H_1(t_2)$. Thus, we have $\theta(H_1) = \theta(H_2)$. The time that elapses between $t_1$ and $T(H_1)$ must be the same as the one between $t_2$ and $T(H_2)$. This yields a contradiction because the optimal trajectory is uniquely defined ($H_T$ is invariant) and (initial) consumptions at $t_1$ and $t_2$ necessarily differ.

Finally note that the solution with positive education and a revolution in finite time yields the following present value to the elite:

$$V^1(H_0) = e^{-\rho \theta(H_0)} \left( \frac{\gamma h h\pi \xi^{\frac{1}{2}} H_T e^{\frac{(p^sR - s)}{\gamma}}}{\rho - \chi(1 - \gamma)} (e^{\frac{(p^sR - s)}{\gamma}} H_T - 1) + \pi \zeta \right) \frac{H_T^{1 - \gamma}}{1 - \gamma}$$

(51)

This completes the proof of Proposition 6.

B.2.2. Extended proof of proposition 2

Unchanged.

B.2.3. Extended proof of proposition 3

At solution 1 (democratization), the index of inequalities is:

$$l^1(t) = \frac{(s\chi + \phi h(p^sR - s))}{\chi} (h\pi \xi)^{\frac{1}{2}} H_T e^{\frac{(p^sR - s)}{\gamma}} + \phi \varphi(T)(h\pi \xi)\frac{1}{2} H_T e^{\frac{(p^sR - s)}{\gamma}} e^{\frac{(p^sR - s)}{\gamma}} + \frac{\phi \gamma h}{\rho - \chi(1 - \gamma)}$$

Take the derivative with respect to time:

$$i^1(t) = \frac{(h\pi \xi)^{\frac{1}{2}}}{\gamma} H_T e^{\frac{(p^sR - s)}{\gamma}} \left[ (\rho - \chi)(s\chi - \phi h(p^sR - s)) + \phi (\rho - \chi(1 - \gamma)) F(T) \right].$$

The sign of the derivative is given by the sign of the term, denoted $\Psi(T)$, between squared brackets. This function is increasing in $T$ at the optimal solution. Evaluating it at $T = 0$, we obtain:

$$\Psi(0) = (\rho - \chi)s + \phi (\rho - \chi(1 - \gamma)) H_0 + \gamma \phi h \left[ p^sR - s - (h\pi \xi)^{\frac{1}{2}} H_T \right],$$

which is positive according to (50). Thus $i^1(t) > 0$ for all $t \in [0, T]$.

At solution 2 (permanent dictatorship), the index is simply given by:

$$l^2(t) = \frac{s + \phi H^2(t)}{p^sR - s + \Omega H^2(t)}.$$
with derivative:

\[
\tilde{P}(t) = \frac{p^4H}{(p^3R - s + \Omega H^2(t))} \left[ \phi R - \frac{1 - \alpha}{\alpha} \left( \frac{\alpha A}{p^x} \right)^{\frac{1}{p^x}} s \right].
\]

Thus,

\[
R > \frac{(1 - \alpha)s}{\phi \alpha} \left( \frac{\alpha A}{p^x} \right)^{\frac{1}{p^x}}
\]

is sufficient to conclude that \( \tilde{P}(t) < 0 \) for all \( t \in [0, \infty). \)

B.2.4. Extended proof of proposition 4

Unchanged.

B.2.5. Proof of proposition 7: Optimality condition

For the sake of notational simplicity, let \( T(0) \) be renamed as \( T \). Then, \( V(0) > V^2(0) \) if and only if:

\[
e^{-\rho T} \left( \gamma (h \pi) \frac{(\rho^x)^T}{(1 - \gamma)(\rho - \chi (1 - \gamma))} \left[ e^{(\rho - \chi (1 - \gamma)T)} - 1 \right] + \pi H_T \right) > \frac{(p^3R - s)^{1-\gamma}}{\rho(1 - \gamma)} \tag{52}
\]

which, by the definition of \( H_T \) in (40) and by (45), yields:

\[
\frac{\pi \zeta H_T^{\gamma} e^{-(\rho - \chi)T}}{\rho(1 - \gamma)} \left[ (1 - \gamma) h(p^3R - s) + \gamma h(h \pi \xi)^{-\frac{1}{h}} H_T e^{(\rho - \chi)T} \right] > \frac{(p^3R - s)^{1-\gamma}}{\rho(1 - \gamma)} .
\]

Denote the RHS of (52) by \( J(T) \). It follows that \( J(0) = \frac{\pi \zeta H_T^{\gamma} e^{-(\rho - \chi)T}}{\rho(1 - \gamma)} \left[ (1 - \gamma) h(p^3R - s) + \gamma h(h \pi \xi)^{-\frac{1}{h}} H_T e^{(\rho - \chi)T} \right] > 0 \) and:

\[
J'(T) = -\frac{(\rho - \chi) \pi \zeta H_T^{\gamma} e^{-(\rho - \chi)T}}{\rho} \left[ h(p^3R - s) - h(h \pi \xi)^{-\frac{1}{h}} H_T e^{(\rho - \chi)T} \right].
\]

We observe that \( J'(T) \leq 0 \Leftrightarrow T \leq \tilde{T} \), where \( \tilde{T} \) has been defined in (48). Thus, \( J(\tilde{T}) = \frac{(p^3R - s)^{1-\gamma}}{\rho(1 - \gamma)} \) is equivalent to:

\[
h(p^3R - s) > h(h \pi \xi)^{-\frac{1}{h}} H_T e^{\frac{\gamma}{h}} \tag{53}
\]

which is equivalent to \( (\rho - \chi (1 - \gamma)) \xi^{\frac{1}{h}} > h(h \pi)^{\frac{1}{h}} (\gamma + (1 + \gamma)e^{\frac{\gamma}{h}}). \) This is sufficient to conclude that \( V(0) > V^2(0). \)

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.jebeo.2019.10.010.

References


