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An eclectic approach

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Nominal and real interest rates in OECD countries

An eclectic approach

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Abstract. This paper revisits the economic theory of the interest rates and presents some estimation results obtained on annual macroeconomic data for a panel of OECD countries. The conventional macroeconomic theory based on the saving investment balance falls short of producing a consistent picture for the medium-term trends of the interest rates and related variables. The main reason lies in the fact that the usual propensity to save specification does not account for the observed saving behavior, and it turns out that an intertemporal approach works better. Some other extensions of the basic model are also discussed to account for the external influence and the risk premia on public bonds. Based on this theoretical discussion, an econometric relation is estimated on annual data for a panel of 19 OECD countries. The tests confirm the influence of the factors suggested by the theory and, in particular, the link between the fall in the interest rate and the economic slowdown. The role of the inflation expectations, of the real exchange rate and of the risk premia on public debt is also discussed and clarified.

Keywords, Interest rate, real interest rate, risk premia macroeconomic policy, potential growth, stagnation, panel data.

Resumé. Cet article revient sur la théorie économique des taux d'intérêt et présente quelques résultats d'estimations économétriques sur données annuelles pour un panel de pays de l'OCDE. La théorie macroéconomique conventionnelle fondée sur l'équilibre épargne-investissement ne rend pas compte de manière cohérente des évolutions à moyen terme des taux d'intérêt et des variables macroéconomiques associées. La principale raison réside dans le fait que la notion keynésienne de propension à épargner ne permet pas de comprendre les comportements d'épargne observés. Pour cela, une approche intertemporelle apparaît mieux appropriée. On discute également d'autres extensions du modèle de base pour tenir compte des influences extérieures et des primes de risque sur les obligations publiques. Sur la base de cette discussion théorique, on estime une relation économétrique sur des données annuelles pour un panel de 19 pays de l'OCDE. Les tests confirment l'influence des facteurs suggérés par la théorie et, en particulier, le lien entre la baisse du taux d'intérêt et le ralentissement économique. Le rôle de l'inflation anticipée, du taux de change réel et des primes de risque est aussi discuté et clarifié.

Mots clés : Taux d'intérêt, taux d'intérêt réel, primes de risque, politique macroéconomique, croissance potentielle, stagnation, économétrie de panels.

Introduction

Conventional macroeconomic theory falls short of explaining one of the major characteristics of the recent two decades namely the declining trend on real interest rate to historical lows. Many papers including Blanchard (2014), Summers (2014), Bernanke (2015), Bean et al. (2015), and others, have addressed the question: why are the interest rates so low? and, should we add, for how long?¹ an issue which is far from being totally clarified. Oddly enough, the real interest rate had already put our discipline in a hard test in older times, in the eighties, as interest rates developments were at the exact opposite². Beyond different historical situations, understanding the real interest (not the least of our macro variables) seems to remain a challenge. The simplest view of the interest rate as the variable which varies to adjust saving to investment appears either trivial or logically fragile and do not fit to the facts.

In this paper we first try to understand what is unsatisfactory or simply wrong with the conventional approach, then we propose an alternative eclectic approach. In section I we start from a variant the AS-AD model and we clarify the general notion of equilibrium rate of interest. We discussed some extension of this simple model to account for the external influence and the risk premia on public bonds. Then, in section II, we introduce a dynamic framework which allows a consistent treatment of inflation expectations, a crucial point for analyzing the real interest rate. Finally, we leave the Keynesian propensity to save assumption for a more appropriate intertemporal approach. This latter modification provides a plausible explanation of the empirical evidence of falling real interest rates together with potential growth slowdown. Based on this theoretical discussion, in section III, we specify and estimate an econometric relation on annual data for a panel of 19 OECD countries. The tests confirm the influence of the factors suggested by the theory and, in particular, the medium-term relation between interest rate and potential growth. The role inflation expectations, the real exchange rate and the risk premia on public debt is also discussed and clarified.

I The equilibrium real interest rates

In this section we first clarify the notion of the equilibrium real interest rate in a basic static framework. Based on a conventional macro-model (AS-AD) a comparative static analysis gives indications on the plausible impacts of macroeconomic demand and supply shocks on the real interest rate. However, when confronted to reality this model's predictions are not fully consistent of with some major observed facts. We extend the model in order to include important elements concerning the external openness, and the risk elements. Though some gains on the predictive power are obtained the basic static framework is still unsatisfactory.

¹ The persistence of low interest rates in relation with the secular stagnation hypothesis is another issue that have been discussed by Blanchard, Bean et al. Rachel and Smith, (2017) Rachel, Summers, (2019) and many others.

². The title of a Ben Bernanke's article "Why are the interest rates so high?" posted on March the 30th 2015 on the author's page in the Brookings site This title strangely echoes a NBER Working paper by Bodie, Kane and Mc Donald published in 1983, and intitled "Why are the interest rates so high?" Blanchard published (with Summers) "Perspectives on High World real Interest Rates" in 1984, forty years before his contribution (with others co-authors) intitled: "A prolonged period of low real interest rates?" in Baldwin and Teulings (2014), See also Atkinson Chouraqui (1985), and Bismut (1993)

1/ A basic model

We first start from a very simple static textbook model of a closed economy to clarify the notion of an **equilibrium interest rate**. This notion is often defined as the interest rate which equalizes supply and demand of loanable funds, or somewhat better, the rate which equalizes saving and investment. This definition, as we shall see, is not satisfactory because they rely on a partial equilibrium. A rigorous approach would be to define the equilibrium interest rate as the value of the interest rate that prevails when the economy reaches a general equilibrium. This requires the use of a macroeconomic model that involves the interactions between, at least, three markets: the goods market, the money market and the security market. The core of this model includes three equations that determine simultaneously the production level, the price level and the interest rate.

Equilibrium on the goods market requires that production Y is equal to private absorption H , which includes households' consumption C and firms' investment I , plus public expenditures G , thus: $Y = H + G$. If we define national saving as: $S = Y - C - G$, then $S = I$. Saving equals investment is strictly equivalent to : production is equal to total absorption.

Households' consumption C is an increasing function of real disposable income $Y - T$, and a decreasing function of the **real** interest rate \hat{r} : $C = C(Y - T, \hat{r})$. Investment I is a decreasing function of the **real** interest rate \hat{r} , and also possibly, an increasing function of total demand : $I = I(Y, \hat{r})$. Then equilibrium in the goods market implies:

$$(1) Y = H(Y, \hat{r}, T) + G, \quad \text{with } 0 \leq H_Y \leq 1, H_r < 0, H_T > 0, \text{ or equivalently:}$$

$$(1') S(Y, \hat{r}, G, T) = I(Y, \hat{r}).^3 \quad \text{with } S_Y > 0, S_r > 0, S_G < 0, S_T > 0, S_Y > I_Y > 0, I_r < 0.$$

Equilibrium on the money market requires that money demand is equal to money supply. We will consider that the money supply M is controlled by the central bank⁴ and that real money balances L are an increasing function of production and a decreasing function of the **nominal** interest rate r , thus:

$$(2) M/P = L(Y, r)$$

As for the supply side we retain for the discussion a fairly general assumption of an aggregate supply curve (AS) which encompasses two smooth-pasting limiting cases : the Keynesian regime where $P = \bar{P}$, the Classical regime where $Y = \bar{Y}$, and an Intermediate regime where the general price is an increasing function of output, (see figure 1 in Appendix 1).

$$(3) P = f^s(Y), \quad f_Y^s \geq 0, Y_{kn} \geq Y \geq \bar{Y}; \quad P = \bar{P} \quad \text{when } Y \leq Y_{kn}; \quad P \geq f^s(\bar{Y}).$$

At this stage, it is worth noting that we have introduced a distinction between the real and the nominal interest rates even though it is hardly justified, in a static framework. The variable that appears in the demand for money, is the nominal rate of interest (not the real) as the key determinant of the share between money and bonds. The notion of real interest requires some correction for inflation. Unfortunately, there is nothing like a rate of inflation in a static model and for the moment, we will ignore the difficulty and we will assimilate the comparative static variation to a time motion (which is notoriously incorrect, but with no consequence for our purpose). Accordingly, we will tentatively refer to the rate of inflation \dot{p} as defined by $P/P_0 - 1$

³ Note that $S = [(Y-T) - C] + (T-G)$. National saving is the sum of private and public savings.

⁴ This does not mean that the central bank controls directly the money aggregate. This simplified version of the model contains no detailed description of the monetary policy transmission mechanisms.

and the expected rate of inflation \tilde{p} as defined by $\tilde{P}/P_0 - 1$.

By definition, the real interest rate (or *ex ante* real interest rate) is the nominal interest rate corrected from **expected** inflation. Thus:

$$(4) \quad \hat{r}_t = r_t - \tilde{p}_t$$

This is, conceptually, (one of) the true determinant(s) of investment and saving decisions and should not be substituted by the nominal interest rate or by the so called “*ex post*” real interest rate, defined as the rate of interest less (effective) inflation⁵. The *ex post* real interest rate is relevant for evaluating the effective real return on investment or the real effective cost of debt. The possible gap between the *ex-post* and the (*ex-ante*) real interest rate is equal to the expectation error on the rate of inflation. This means that the effective outcome of private agents’ decisions may not necessarily coincide with their expectations.

As pointed above, the static framework is irrelevant for analyzing expectations. In this framework, the only consistent assumption is, precisely, the static expectation: $\tilde{p} = 0$. Under this assumption the real interest rate is always equal to the nominal interest rate and the model above merges with the standard AS-AD textbook model. The comparative static proprieties of the model remain unchanged if we set \tilde{p} as an exogenous (possibly non-zero) variable, the assumption we will retain at this point. Then, the resolution of the model is straightforward. (See appendix 1)

Unsurprisingly, we check the well-known property of the classical regime when the economy operates at full employment ($Y = \bar{Y}$), namely, the **real** interest rate is determined solely by the equilibrium in the goods market and money is neutral. The equilibrium interest rate is affected by the shocks that could modify the conditions of the goods market, typically fiscal shocks or shifts in the saving or investment schedules, including exogenous shocks on full employment production. More precisely a higher production capacity lowers the interest rate. However, the real interest rate is unaffected by monetary policy shocks or shifts in the money demand curve. In turn the nominal interest rate is determined by equation (4) and is not affected by the money market conditions provided inflation expectations \tilde{p} are insensitive to effective inflation. The model determines an **equilibrium real interest rate** (consistent with a general equilibrium of our model) but should be labeled in this regime **classical** or **market clearing real interest rate**, but not “natural”, as full employment here is *ad hoc* and do not refer explicitly to natural unemployment.

In the fix price Keynesian regime ($P = \bar{P}$), the real interest rate and the real output level are determined simultaneously by the two equations (1)-(2) of the IS-LM system. The classical recursive causality, as well as the neutrality of money, do not hold anymore. The equilibrium of the goods market only determines a relationship between production and the real interest rate, namely the IS curve. In this regime, the real interest rate is determined not only by real, but also by monetary factors including money supply and shifts in real money demand. The (non-market clearing) equilibrium real interest rate in the Keynesian IS-LM regime is not associated to full employment. In this case one can talk about a **Keynesian** or **non-market clearing real interest**

⁵ The quote marks express our discomfort with this usual terminology. Strictly speaking the *ex-post* interest is the interest rate corrected by the inflation drift from the issuance to the maturity. It is often used for the more than one-year interest rate corrected by the current annual rate of inflation. In the latter case we prefer using the term “Inflation corrected interest rate”.

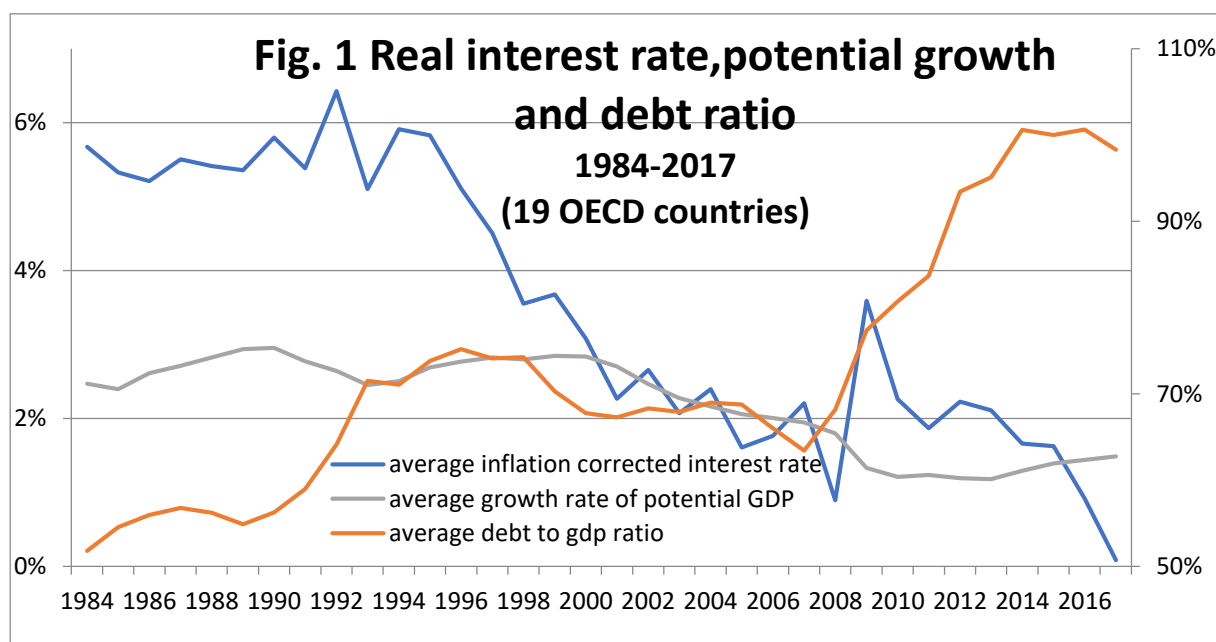
rate. The graphical discussion of static comparative effects using the saving and investment schedules becomes quite uneasy as fiscal and monetary shocks affect the production and the real interest rate simultaneously. Thus, resorting on the graphical treatment based on the I / S schedules can be misleading (see Appendix 1).

Unsurprisingly again, we find the usual properties of a Keynesian regime: fiscal and monetary policy are effective. A debt financed increase in public expenditure raises the (real) interest rate, but an expansionary monetary policy reduces the nominal interest rate and the real interest rate, because expected inflation as well as effective inflation are assumed to stay unchanged. (see Appendix 1).

The intermediate regimes (AS-AD) stand between the two polar Classical and Keynesian cases. The main difference with the Keynesian regime is that fiscal and monetary policy affects the general price level. Compared to the Keynesian regime, an expansionary demand policy has an effect somewhat weakened on real activity but pushes the rate of inflation up. We also find that a fiscal expansion increases the interest rate and an expansionary monetary policy lowers the interest rate as well as in the Keynesian regime. The effect on nominal versus real interest rates are the same, due to the assumed static expectations. This means implicitly that the inflationary consequences of fiscal and monetary policy always come as a surprise, a drawback that we will overcome in section II below, while introducing a dynamic framework. Finally, an adverse supply shock or a reduction of production capacity increases the interest rate.

2/ Confronting the basic model to reality

The confrontation of the model's prediction to the observed facts ⁶ brings out two puzzles. First, as we strongly argued in Bismut & Ramajo (2019), the long-term decline of the real interest rate is associated to a long-term slowdown of real activity, (Figure 1) whereas the conventional wisdom suggests, and here the model predicts, that a decline in activity would produce an increase in the real interest rate. It is noteworthy that this theoretical prediction holds in the AS-AD model irrespective to the regime (see appendix1). In particular, It holds in the Classical



⁶ See Bismut and Ramajo (2019) for a detailed presentation of the main trends and stylized facts.

regime as well. In other words, the model predicts a negative correlation between the interest rate and the level of activity, even at full employment, and a fall in productivity would lead to an increase in the real interest rate contrary to what we see.

More generally, the influence of the labor force operates through the labor demand (by firms). If the economy is at full employment the level of production is tightly linked to the level of the labor input which in turn depends on demographic factors, labor participation, work duration and the technical progress. Consistently with the logic of absorption in the model we use, a reduction of anyone of these factors affects negatively the production capacity and raises the real interest rate. However, if the shock is seen as durable, households may reduce consumption and increase savings which will neutralize the effect on the real interest rate. In addition, the government may also react to an adverse shock on production capacity by cutting public expenditures. These two last mechanisms are not included in the model and should be considered for explaining the falling interest rates.

A second puzzle is that, for many years, there is no evidence that the upward trend of public debt has led, on average to an increase in the real interest rate, while the model predicts an increase. One has to go back to the nineties to find such evidence (see Tanzi and Fanizza (1995)). However, recent literature (Ardagna et al. (2007), S Baldacci et al. (2010) and others) have emphasized increasing default risks on sovereign bonds when government are overindebted. We believe that, although there is some influence of public indebtedness on the rate of interest, the relationship is elusive and other factors operate at the same time in a different direction ⁷. In the logic of the model developed above, Summers (2015) argued that monetary policy would have been subject to an expansionary bias over a long period thus over-offsetting the influence of fiscal policy. Evidences are mixed on Summers analysis. If the empirical relation between the interest rate and the debt ratio remains fragile, there is a reasonable evidence of a relation between the risk as estimated by the rating agencies and the debt ratio (Appendix 5).

Despite these two unsolved puzzles the model we have introduced in this section, certainly helps clarifying the notion of equilibrium interest rate and the relation between saving, investment and the interest rate in the different regimes: classical, Keynesian and intermediate. The graphical analysis based on the I/S schedules suggests that the root cause of a fall in interest rates are to look for in a swelling of savings or a contraction of investment. At this point we only have introduced the usual instruments of fiscal and monetary policy and the production capacity for analyzing the shifts of the two schedules and we ended up with paradoxes.

Other factors may have played a role in explaining the shifts of the I/S schedules. Many authors have conducted such an analysis including Summers (2014, 2015), Blanchard, Furceri, and Pescatori (2014). Bean et al. (2015). Among the reasons that have been put forward: on the saving side, increased longevity and lower fertility to save, the overabundant reserves of the emerging countries and the increase of perceived risks, may have increased the propensity to save. On the investment side, the slowdown of the labor force and the decline of the relative prices of capital goods may have reduced investment. Some of these factors could be introduced as additional exogenous factors. Others can be considered through appropriate extensions of the model.

⁷ See Haugh, D., Ollivaud, P. and Turner, D. 2009. for a detailed investigation.

3 Extensions of the basic model.

Some useful extensions may be introduced without questioning the essentials of the basic macro-model discussed above. This concerns several additional exogenous variables or mechanisms which could be introduced in the goods market equilibrium relationship in order to account for possible shifts of the saving and/or investment schedules.

3.1 Introduction of a risky asset

Increasing financial instability on stocks and private bonds markets may push risk adverse investors toward the safest assets, typically government bonds, thereby pulling down interest rates. Government bonds themselves are not free of risks and in an area of high mobility there is evidence that, if an increase of sovereign risk is perceived in a given country, investors would charge a risk premium and/or would shift toward the highest rated government bonds. This mechanism can be considered without changing radically the logic of our basic model. It requires however the introduction of a third asset supposedly risky, such as stocks, in addition to money and bonds. One possible specification would be assuming that the risky asset, say A , would depend negatively on the interest rate on government bonds, and positively on its rate of return r^e corrected for the risk premium ε . The demand for money and bonds would be modified accordingly:

$$(2.1) \quad M/P = L(Y, r, r^e - \varepsilon) ; (2.2) \quad B/P = B(Y, r, r^e - \varepsilon) ; (2.3) \quad A/P = A(Y, r, r^e - \varepsilon)$$

The introduction of a risky asset also requires some changes in the goods market equilibrium. It is reasonable to assume that households' consumption would depend not only on the interest rate on government bonds but also on the rate of the risky asset, corrected for the risk premium. As for firms, the cost of funds would include the risk premium and that investment would depend on r^e . (see Appendix 2 for further details)

This extension of the model could be helpful to investigate the consequences of perceived increasing risks of holding A , in explaining a fall in the real interest rate. An increase in r^e resulting from financial instability or raising risk premia would shift some private savings towards the safer assets thus lowering the interest rate r . However, if such an extension of the model could help describe the developments since the last financial crisis it is unlikely to explain a downward long-term trend in real interest rates.

3.2 Imperfect capital mobility.

Many authors including Bernanke (2005) and Summers (2015) have suggested that the “saving glut” has been partly due to an unusually large accumulation of foreign reserves by emerging and developing countries. This is quite plausible as unwarranted large current account deficits have characterized the external sector of developed countries⁸ for decades. One reason for this configuration is that, while OECD countries are widely open on their capital account, emerging countries are not, and capital mobility remains low within the less developed and emerging area. Accumulating foreign reserve remains a strategic priority for non-OECD countries' government even though this is hardly advisable from a pure economic point of view. Whatever the reason, for analytical purposes, it is still possible to introduce as an exogenous variable the amount of

⁸ This refers to the Lucas paradox (1990). The savings of the developed countries should massively move to the less developed countries because the profitability of investment projects is, on average, much larger in those countries than in the developed countries.

foreign savings invested in the OECD countries, which creates the conditions of excess savings thus pushing down interest rates.

Considering these international mechanisms requires, however, some adjustments of the model. This involves a reformulation of the goods market equilibrium in an open economy. Avoiding any sophistication, if we denote by *Exp* and *Imp* for export and import, *CA* for the current account, and *X* for the net external position one could write:

$$Y + \hat{r}X = C(Y - T, \hat{r}) + I(Y, \hat{r}) + G + CA \quad \text{with} \quad CA = Exp - Imp + \hat{r}X$$

Recall that in an open economy, the $S = I$ relationship does not hold. Instead we get:

$$[Y + \hat{r}X - C(Y, \hat{r}) - G] - I(Y, \hat{r}) = S(Y, T, X, G, \hat{r}) - I(Y, \hat{r}) = CA$$

Solving for \hat{r} would yields an expression of the form: $\hat{r} = \varphi(\bar{Y}, X, T, G, CA, \dots)$.

This suggests, for instance, the introduction of the current account and/or the external position as explanatory variables for the domestic interest rate.

The two extensions should be considered in empirical attempts and would certainly improve our understanding of past and present developments on interest rates. In the present paper we will take the first one, namely the influence of risk variations. The second, concerning the imperfect capital mobility, would require a certain amount of additional data search and processing, as non-OECD countries are concerned, and we decided to leave it for further investigations.

Conversely, we decided to address other serious limitations of the theoretical approach used in this section. All the discussions, so far, have been developed using a static model under the assumption of an exogenous level of full employment production and exogenous inflation expectations. This means that attempts to estimate an equilibrium interest rate would be subject to somewhat arbitrary assumptions. In the next section we seek to overcome these limitations.

II The real rate of interest dynamics.

In the preceding we have claimed that a static framework is inappropriate for analyzing the real rate of interest as expected inflation is involved. On the one hand, inflation is essentially a dynamic process and cannot be reduced to a simple change between an initial and a final equilibrium level of the general price index. On the other hand, in a static framework there is no way of introducing a relevant expectation process, as static expectations (the only consistent assumption) implies that inflation is, in fact, totally unpredictable.

In the present section we move to a dynamic approach of the interest rate and we combine a short run dynamic model of inflation and a long run determination of the real interest rate borrowed to optimal growth models. This makes it possible to isolate a nominal component: or a Fisher effect, and a real component which includes the natural rate and other components such as risk premia. All these elements pave the way to a testable equation.

1. Expected inflation and the Fisher effect

Among the building blocks of short run dynamic models, the paradigm of the natural unemployment rate certainly stands in pole position. We will start from this conventional paradigm based on the assumption that there exists a short run inflation / unemployment trade-

off, but only one rate of unemployment that stabilizes inflation and no trade-off in the long term: the so-called natural rate of unemployment. Despite serious criticisms this approach has been overwhelmingly used because it makes it possible to define a theoretically consistent and empirically measurable notion of potential output. The idea is fairly simple: in a market economy, unexpected real shock requires price corrections and the equilibrium rate of unemployment is associated to a situation in which expected inflation errors are purely random, as rational expectations eliminate systematic errors. One important particular case is the NAIRU for which the rate of inflation follows a random walk. This means that an equilibrium (natural) rate of unemployment requires the rate of inflation be stationary.

Here, we will assume that the deviation of the inflation rate from its expected value, namely: the inflation surprise, is an increasing (linear) function of the deviation of the (log) effective production to its (log) potential level: the output gap:

$$(5) \quad \dot{p}_t - \tilde{p}_t = \mu(y_t - \bar{y}_t) \quad , \text{ with } \mu > 0 \quad ,$$

a relation known as the Lucas supply curve. This relation implicitly includes the natural unemployment rate hypothesis (see appendix 3). The implications of relation (5) are well known: the potential output is the only level of production consistent with stable inflation. This production level defined as the potential output is the one obtained at a level of employment which corresponds to the natural rate of unemployment. Any attempt to maintain production larger than potential output would accelerate inflation indefinitely. In other words, the natural rate is an equilibrium rate of unemployment derived from a stationary condition. On the contrary there is not one unique equilibrium inflation rate, but a multiplicity of equilibria and the effective inflation rate will emerge as a result of many influences among which monetary policy. These settings have quite interesting empirical implications.

The relation between the inflation surprise and the output gap can be used empirically to build a consistent estimate of the expected rate of inflation and gives the way to a consistent and theoretically funded estimate of the real interest rate. Starting from an estimated Lucas supply curve (5) one can easily calculate the expected inflation using the following formula:

$$(6') \quad \tilde{p}_t = \dot{p}_t - \mu(y_t - \bar{y}_t) \quad .$$

This formula, however, is somewhat restrictive in that sense that the unexpected inflation is solely linked to excess demand (output gap). Many other hardly predictable factors may also affect the rate inflation.

One possible source of persistent uncertainty concerning inflation lies in external factors, which are beyond governments control and prediction. It is important to keep in mind that in many open economies, households' consumption includes a sizable proportion of imports and that prices of imported goods enter as components of the CPI and other domestic demand price indices.⁹ Then, denoting by π_t the (log) real exchange rate and $\dot{\pi}_t$ its change over time, the growth of the CPI can be better approximated by $\dot{p}_t^c = \dot{p}_t + (1 - \lambda)\dot{\pi}_t$, where λ is the geometric weight of the domestic goods and (6') should be replaced by :

$$(6) \quad \dot{p}_t^c - \tilde{p}_t^c = \mu(y_t - \bar{y}_t) + (1 - \lambda)(\dot{\pi}_t - \tilde{\pi}_t) \quad , \text{ with } \mu > 0 \text{ and } 0 < \lambda < 1 \quad ,$$

⁹ See Bismut and Ramajo (2018)

were $\tilde{\pi}_t$ is the expected change of π_t . The transition from the previous model is straightforward if \bar{y}_t is understood as the level of production consistent with natural unemployment.

2 The natural rate of interest

It is now possible to define rigorously the notion of **the natural rate of interest**: it is the rate of interest that would equalize saving and investment if production is at potential (as defined by the production consistent with the natural rate of unemployment). However, two additional remarks should be added concerning the two extensions retained for the model. As for the exchange rate, which plays a part into the inflation dynamics, we must qualify a stationary condition for this variable. Since we have already retained the stability of the rate of inflation as a stationary condition, the stability of the real exchange rate turns out to be the consistent assumption, because it corresponds to the notion that domestic prices and import prices in domestic currency would move at the same pace.

Other factors may affect the rate of interest, such as the risk on public debt. We collect those factors in one unique variable z_t and modify relation (4) accordingly.

$$(4a) \quad r_t = \hat{r}_t + \tilde{p}_t^c + z_t$$

In principle, appropriate stationary conditions should be clarified for each element entering in z_t in order to extend the notion natural interest rate.¹⁰ For instance, a stable and low (negligible) risk premium sounds as a reasonable stationary condition.

If there is no inflation surprise, then $\tilde{p}_t^c = \dot{p}_t^c$ full employment is achieved, and the real interest rate is equal to the natural interest rate. Another implication is that *ex-post* and *ex ante* interest are equal. In other cases, the deviation of the *ex-post* interest rate from the equilibrium real interest rate depends on the output gap, on unexpected change of the real exchange rate surprise and possibly other factors.

$$(7) \quad r_t - \dot{p}_t^c = \hat{r}_t - \mu(y_t - \bar{y}_t) + (1 - \lambda)(\dot{\pi}_t - \tilde{\pi}_t) + z_t$$

At this point the model (1,2,5,4a) determines the production level the real interest rate, and the effective and expected rates of inflation. Compared to the model of section 1, by introducing some short run dynamic features via the Lucas supply curve, we have now a more satisfactory approach to expectations, with a true, although simplistic, expectation behavior which allows a distinction between expected and unexpected inflation. As a result, we have a consistent and computable notion of the real interest rate. This model, however, still fails to account for the decline of both the real interest rate and the real rate of growth and to overcome this difficulty we will have to move to another theoretical framework.

3 The Optimal growth (intertemporal)

The model of section I, though extended and completed by an expectation mechanism, does not really explain the observed developments in the interest rate for an obvious reason. There is no doubt that an increase in savings and/or a fall in investment would lead to a fall of the real interest rate but the model predicts that a slowdown of real growth would lead to an increase in the real interest rate because it would reduce (not increase) excess savings over investment. This effect is a straightforward implication of the saving and investment behaviors specified in the model

¹⁰ This question was raised in Hamilton et al. (2017) but not discussed in depth.

and the question is: why do we have excess savings? In fact, the implications of the model lie on the propensity to consume (and to save) assumption. As already said, we can imagine many exogenous reasons for an increase in excess saving, but these explanations would lie outside the model. What we are going to try now is to re-specify the model in order to include some plausible endogenous mechanisms consistent with the empirical evidence.

One alternative to the conventional propensity to consume theory: the intertemporal approach (or permanent income), could possibly reconcile the theory with the facts. This approach is nothing like a theoretical curiosum and covers a large variety of models including the optimal growth theory à la Lucas-Ramsey-Cass. It sets that saving is the consequence of the choice of an optimal consumption overtime. One important implication of the intertemporal approach to consumption and saving is that if the income perspective is revised downward, household may react by reducing consumption and increase savings in order to smooth future consumption and this will drive the interest rate down.

If we believe in the message of the optimal growth model, there is no difficulty in recognizing that there might be, a relation between the interest rate r and the growth of real GDP \dot{y} along the optimal path. Take, for instance, the optimal growth model of a decentralized economy (Cass (1965)), it is well known that in this model the optimal path state is characterized by the following relation:

$$(8) \quad \hat{r} = \delta + \frac{1}{\sigma} \dot{y} - \dot{n}$$

Where δ is the time preference, σ is the temporal elasticity of substitution, and \dot{n} the rate of growth of the population (see appendix 4).

The link between this relation and the models of the preceding sections is easy to establish. Flexible prices and perfect foresight are assumed in the optimal growth model we refer to, and therefore, the economy never depart from the classical regime. Full employment is achieved at any time and production is always equal to potential production. Then the interest rate is always equal to the natural rate. Therefore, merging the two approaches could lead to the following equation for the nominal interest rate. Using our notation and inserting (6) and (8) in (4a) we get:

$$(9) \quad r_t = \delta + \frac{1}{\sigma} \bar{y} - \dot{n} + \dot{p}_t - \mu(y_t - \bar{y}_t) + (1 - \lambda)\dot{\pi}_t + z_t \quad ,$$

a relation that could serve as a starting point for our econometric investigation.

III Empirical application

Our objective is to achieve a better understanding of the evolution of worldwide interest rates. In restraining our investigation to the OECD countries, we inevitably miss an important side of the reality, in particular the role of the emergent countries. We believe that, at this stage, discussing about the natural interest rate in regions where the economy is characterized by substantial market imperfections is unwarranted.

On the contrary it is important to take advantage from a diversity of country cases within the developed area and for this reason we used panel data econometrics. Most available empirical studies use time-series analysis applied to a single country (essentially the United States) and put the stress on the dynamic process. We adopted a more medium term and structural approach

based on panel data which allows two possible interpretations. We are first concerned by testing a theoretical model and, in this perspective, the cross-country dimension helps testing a general model. We are also concerned by characterizing the development of the OECD area, and moreover global developments, and in this case a country data should be weighted according to its size for testing a kind of aggregate model for the OECD area. We have conducted the analysis by testing systematically the unweighted (cross-country or international) and the weighted (aggregate or global) model ¹¹.

Our estimation strategy is based on a single equation econometric model respectful to the theory but it also reflects some eclecticism. The ten-year nominal interest rate appears as the dependent variable. It could be additively decomposed in an expected inflation component (the Fischer effect) and a real interest component. To establish the link with the theoretical model of section II, we assume that the real interest rate is equal to the natural rate plus a risk premium and possibly a white noise reflecting other determinants ignored in our simplified specification. The natural rate of interest depends on potential growth and, fiscal variables: the deficit and debt ratios. The risk premium is proxied as a function of the rating indicator (Appendix 5 and Ramajo (2013))

$$(10) \quad r = \beta_0 + \beta_1 \tilde{p} + \beta_2 \dot{y} + \beta_3 sb + \beta_4 rp + \beta_5 (d/y) + \varepsilon$$

Problems concerning the variables remains to be discussed before getting a testable equation.

The **expected rate of inflation** \tilde{p}_t^c is an unobserved variable, and it stands here as an attempt to capture the Fisher effect while not imposing the theoretical unit value. What we tried to do is to avoid the awkward use of replacing the true variable: expected inflation, by effective inflation. This is not only for obvious theoretical consistence: such a proxy would imply perfect foresight and thus no inflation surprise, but also for an empirical reason. In fact, there is a serious risk that this supposedly benign specification error would introduce a spurious correlation between the real rate of interest and some explanatory variables that have no reason to play a role, leading to biased estimates and misinterpretation as we will see below.

As for expected inflation we finally decided to use the implicit estimates of the variable extracted from our own estimation of the Lucas supply curve (Bismut & Ramajo, 2018). The idea is straightforward. The Lucas supply curve is a relation between the inflation surprise and the output gap, and more generally other unexpected shocks. We calculated the expected inflation as the difference between the observed inflation and the estimated unexpected inflation calculated from the Lucas supply curve.

$$(11) \quad \tilde{p}_t^c = \dot{p}_t^c - \{\mu(y_t - \bar{y}_t) + (1 - \lambda)\dot{\pi}_t\}$$

This method is not beyond criticisms, but the benefit is that, by construction, expected inflation is uncorrelated to the output gap and others unexpected shocks.¹²

As for the **natural real interest rate**, we found arguments to retain two main factors: the potential rate of growth and the government balance ratio. We used the OECD data for these two variables. The influence we would like to capture is not the automatic stabilizer effect but rather

¹¹ See appendix 6 for technical details

¹² We can retain many different alternative hypotheses on the inflation surprise, such as: the rate of inflation follows a random walk, so the expected inflation is equal to the lagged inflation rate. Another option can be to use official survey establish by IMF or OECD. The method used here is introduced in Bismut and Ramajo (2018). Updated estimations reported in appendix 7, include 2017 and to cover the 19 countries in the regression (2019).

a permanent effect due to a possible Ricardian non neutrality. For this reason, we found appropriate to use the structural balance (Cyclically Adjusted Balance). One other reason of a possible influence of these variables is that they can be interpreted as indicators of sustainability.

The **risk premium** could be measured directly using a rating indicator or indirectly as a function of the debt ratio. We tested the two options. A crude indicator was derived from the Moody's rating and introduced as an explanatory variable on the regressions. We have also tried to introduce the debt ratio as a regressor although this raises problems as we will see further. A rough test of the relation between the risk indicator and the debt ratio was done by regressing the interest rate on the debt ratio. The test confirms a significant but low correlation $R^2=0,28$. This is essentially due to the absence of perceived risks for a number of countries and years. The same test applied to the sub sample of countries that faced critical episodes (the GIIPS) yield a much stronger correlation ($R^2=0,56$) (see appendix 5).

The econometric analysis based on equation (10) was conducted on a panel of 19 advanced economies, over 56 years, using 4 sub-samples, according to the availability of data

Sample 1: Largest group of countries (19) and the shorter period (1984-2016)

Sample 2: Small group (G7+Belgium and Sweden) and the longest period (1970-2016)

Sample 3: GIPS (Greece, Italy, Portugal and Spain) Most risky countries (1984-2026)

Sample 4: The less risky countries (15) non GIIPS (1984-2026)

Each estimation was performed on unweighted and GDP weighted data. A large number of specific results can be found in appendix 7,

Table I. below contains the main results obtained on unweighted data using panel econometric analysis with individual (country) fixed effects. We now discuss our main results based from our selected best equations.

Table 1 Econometric estimation of model (10) : bests selected results.

Dependent variable Ten-year interest rate	All countries	G7+Belgium and Sweden	GIPS	Non GIPS
Time period	1984-2016	1970-2016	1984-2026	1984-2026
Expected inflation	0.965*** (0.0333)	0.513*** (0.0310)	0.920*** (0.0527)	1.074*** (0.0514)
Potential growth	1.682*** (0.0995)	1.213*** (0.140)	1.694*** (0.292)	1.782*** (0.0992)
Risk indicator	0.155*** (0.0109)	0.00314 (0.0232)	0.200*** (0.0230)	0.0408** (0.0187)
Structural balance	-0.339*** (0.0360)	-0.335*** (0.0423)	-0.517*** (0.0844)	-0.321*** (0.0396)
Constant	-0.0236*** (0.00312)	0.00788* (0.00406)	-0.0589*** (0.0116)	-0.0192*** (0.00280)
Nb of observations	633	426	133	500
Number of countries	19	9	4	15
R-squared	0.718	0.644	0.792	0.691
Adjusted R-squared	0.708	0.634	0.780	0.680

Standard errors between brackets.

*** p<0.01, ** p<0.05, * p<0.1

The Fisher effect, as measured by the coefficient β_1 of the expected rate of inflation, is clearly confirmed. The estimated coefficient is close to one and significantly different from zero when estimated on the full sample. It is closer to 0.5 and significantly different from 1 when estimated on the G7+ sample and significantly different from 1. This result may come from an oversimplified specification of the anticipation, especially in the time structure.¹³ However, the coefficient has the expected sign and the positive empirical relation between nominal interest rate and expected inflation rate is established. The coefficients obtained in the weighted estimations (see table 1 in appendix 7) are quite similar, which means that the explanatory power of expected inflation rate is almost identical for all countries, regardless of their size.

The relation between potential growth and the natural rate of interest turns out to be particularly significant in our econometric investigation and comforts the results obtained on time-series by Laubach-Williams (2016), Hamilton et al (2016) and others. Coefficient β_2 is highly significant in all regressions and tends to be larger than one, except in the case of the GIPS for which econometric results should be interpreted with caution, given the peculiarity of this small group. Based on the theoretical interpretation a figure of 1.2 to 1.7 sounds quite plausible as it would correspond to an intertemporal elasticity of 0.8 or 0.6 a range often retained in calibrations. This effect highly contributes to explain the declining trend in interest rate in OECD countries.

The cyclically adjusted fiscal balance has been retained among the plausible determinants of the rate of interest to make allowance for some Keynesian and/or non-Ricardian features. There is, actually, a clear statistical evidence of an effect of this variable on the real interest rate consistent with a number of empirical findings that has been published for more than four decades (see: Baldacci and Kumar (2010), Faini (2006), Haugh, et al (2009), Tanzi and Fanizza (1995)). The estimated coefficient β_3 is significantly negative, consistently with the theory, and equal to -0.34 when estimated on the full on the G7+ samples and larger and equal to -0.52 when estimated on the GIPS suggesting some sort of non-linearity. This means that a deterioration of the (structural) fiscal deficit by 1% of GDP would lead to an increase of the rate of interest by 0.3 percentage points for the countries that do not suffer from fiscal difficulties. However, the theoretical interpretation is not easy. One important point to note is that during the last recession soaring fiscal deficit did not raise the interest rate that much, although a negative effect is identified econometrically. It is noteworthy that the weighted estimation lead to smaller values of β_3 coefficients except for the GIPS. It may be the case that other factors such as a potential slowdown have offset the positive effect of fiscal policies, but the results are still puzzling.

The risk factor has been tested throughout the introduction a measure based on the ratings but there the evidences are mixed. The estimated coefficients β_4 are of the correct positive sign (an increasing risk lead to a rise of the interest rate), not significant in the G7+ sample because ratings are high and invariant in those countries. On the contrary the coefficient is larger and significant for the full sample through which the risk indicator varies. Furthermore, the risk indicator is poor and factors explaining the risk are correlated with other variables, in particular fiscal variables. We believe that the fiscal balance captures in part a risk effect and the debt ratio is likely to be an even better indicator (see Corsetti et al. 2012).

The introduction of the debt ratio brings counterintuitive outcome (table 2 of appendix 7) with an estimated coefficient β_5 stubbornly of the wrong signs and significantly. This result is of

¹³ Additional tests tend to support this guess (see Appendix 7). This point deserves further investigation.

course unacceptable, and we have tried to understand why this was so. The first reason we found is that there is an important collinearity between the debt ratio and other variables noticeable the fiscal balance. This is apparent because when the debt ratio is introduced the other coefficients change but the R^2 does not change that much. But there is another deeper reason. We think that the debt ratio cannot be treated as an exogenous variable because decision concerning debt level is largely (a decreasing) function of the rate of interest and the negative sign of β_5 reflect the reversed causality : a low interest rate makes the ease the increase of the government debt. In fact, the interest rate and the debt ratio are jointly determined, and a single equation method of estimation may lead to simultaneity biases.¹⁴

To assess the robustness and the adequacy of the specifications derived from the theory we have tested unconstrained (less constrained) specifications. One important point to check to that respect is the choice we have made to use the rate of expected inflation derived from the Lucas supply curve. For this purpose, we have tested a specification of our model (10) but replacing the expected inflation rate we have calculated by its determinants: the output gap and the real exchange rate. The coefficients have the expected signs but the gain in terms of R^2 is negligible. We have also tested a model including both expected and unexpected inflation (not reported here). Expected inflation remained always the most robust and significant variable but we found the inflation surprise sometimes significant. This suggests that predictive capacities of private agents might be underestimated. However, the major findings are not put into question and we conclude that our estimated model is fairly robust.

Overall, the weighted estimations are close to the unweighted ones indicating that OECD countries are structurally similar (see Appendix 7). We note that the estimation coefficient of the potential growth is much larger in weighed than in unweighted estimations and we have not found any reason for explaining such a difference. The largest figures are not implausible, and we may simply say the role of potential growth in explaining the decline of the interest rates in the OECD as a region is confirmed.

Concluding remarks.

In this paper, we have tried to clarify some aspects of the debate concerning the medium-term evolution of the interest rates. Conventional macroeconomic models, although useful, fall short of explaining some important observed facts in particular the downward trends in interest rates together with the slowdown of real activity across the developed countries. Macro-models based on intertemporal microeconomic behavior perform somewhat better. The plausible explanation that comes out of these models is that household have reacted to a permanent slowdown of potential by increasing their savings thereby pushing interest down. However, this story, in line with the recent literature on the secular stagnation remain to be confirmed and refined. For that purpose, we have developed some empirical works based on an eclectic approach, leaving allowance to alternative pieces of explanation. We used panel data econometrics for testing a number to ideas. The result obtained at this stage are encouraging but a lot of work remain to do that we plane to develop in further researches on the four following points.

¹⁴ In a recent paper, Blanchard (2019) has argued that governments should take advantage of historically low interest rates to expand public expenditures in order to sustain flattering activity. Our results suggest that governments could have anticipated this explicit advice well before. This point will be investigated in further research.

- One useful complement would be to extend our database to include emerging and less developed countries.
- Based on some preliminary tests it would be important to refine the dynamics of the tested equations, even though the panel econometrics applied on annual data does not offer favorable conditions for that.
- The fiscal determinants need deeper investigations. As indicated above, we suspect a serious simultaneity problem between the interest rate and the level of public debt. Needless to insist on the relevance of this interaction for the fiscal policy in the coming years.
- The empirical relation between the risk premia and level of debt and possibly other determinants requires some additional research. We have identified non-linearity problems that could be treated properly.

The above list is of course not exhaustive, and we will keep on contributing in further research.

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Appendix 1 Basic model (AS-AD)

1/ Resolution of the AS AD model

We set $\tilde{p} = 0$ so that (4) implies $\hat{r}_t = r_t$ and we differentiate equation (1) and (2):

$$(1) (1 - H_Y)dY + H_Y dT - H_r dr - dG = 0$$

$$(2) dM - PL_Y dY - PL_r dr - LdP = 0 \quad \text{or alternatively} \quad L \left[\frac{dM}{M} - \frac{dP}{P} \right] - L_Y dY - L_r dr = 0$$

- The classical regime (flexible prices) is obtained by setting $Y = \bar{Y}$ exogenous in (1),(2).

The endogenous variable are r et P . We find the typical causal structure of the classical model: 1/ dr determined by the goods market, 2/ dP determined by the money market. The multipliers are:

$$\begin{aligned} \frac{\partial r}{\partial G} &= -\frac{1}{H_r} > 0; \quad \frac{\partial r}{\partial T} = \frac{H_Y}{H_r} < 0; \quad \frac{\partial r}{\partial M} = 0; \quad \frac{\partial r}{\partial \bar{Y}} = \frac{1-H_Y}{H_r} < 0 \\ \frac{\partial P}{\partial G} \frac{1}{P} &= \frac{L_r}{H_r} > 0; \quad \frac{\partial P}{\partial T} \frac{1}{P} = -\frac{H_Y L_r}{H_r} < 0; \quad \frac{\partial P}{\partial M} \frac{M}{P} = 1; \quad \frac{\partial P}{\partial \bar{Y}} \frac{1}{P} = -\frac{L_Y}{L} \left[1 + \frac{(1-H_Y)L_r}{H_r L_Y} \right] < 0 \end{aligned}$$

- Keynesian regime (fix prices). Here, P is assumed to be exogenous.

Denoting the slopes of the IS and LM curve: $\eta_{LM} = -\frac{L_Y}{L_r}$, $\eta_{IS} = \frac{(1-H_Y)}{H_r}$, it is useful to set:

$$D = (1 - H_Y) + \frac{H_r L_Y}{L_r} = (1 - H_Y) \left[1 - \frac{\eta_{LM}}{\eta_{IS}} \right]$$

Solving (1) and (2) for dY and dr We get the following multipliers:

$$\begin{aligned} \frac{\partial Y}{\partial G} &= \frac{1}{D} > 0; \quad \frac{\partial Y}{\partial T} = -\frac{H_Y}{D} < 0; \quad \frac{\partial Y}{\partial M} = \frac{H_r}{DPL_r} > 0; \quad \frac{\partial Y}{\partial P} = -\frac{H_r}{DPL_r} \frac{M}{P} < 0 \\ \frac{\partial r}{\partial G} &= -\frac{L_Y}{D} > 0; \quad \frac{\partial r}{\partial T} = \frac{H_Y L_Y}{D} < 0; \quad \frac{\partial r}{\partial M} = \frac{1-H_Y}{DPL_r} < 0; \quad \frac{\partial r}{\partial P} = -\frac{1-H_Y}{DPL_r} \frac{M}{P} > 0 \end{aligned}$$

- Intermediate regimes (AS-AD),

By eliminating dr in (1) and (2) we get the aggregate demand, a relation between dY and dP .

$$(AD) \quad DdY = \frac{LH_r}{L_r} \left[\frac{dM}{M} - \frac{dP}{P} \right] + dG - H_Y dT = 0$$

We add the linearized aggregate supply curve (3). Denoting the elasticity of f : $\varepsilon_{AS} = \frac{f'}{f} Y$:

$$(AS) \quad \frac{dP}{P} = \left(\frac{f'}{f} \right) dY \quad \text{or} \quad \frac{dP}{P} = \varepsilon_{AS} \frac{dY}{Y}$$

Denoting the elasticity of the demand for money with respect to Y : $\varepsilon_{LY} = \frac{L_Y Y}{L}$ and setting:

$$\Delta = D + \left(\frac{f'}{f} \right) \frac{H_r M}{PL_r} = (1 - H_Y) \left[1 - \frac{\eta_{LM}}{\eta_{IS}} \left(1 + \frac{\varepsilon_{AS}}{\varepsilon_{LY}} \right) \right]$$

Solving for dY is then trivial.

$$\Delta dY = dG - H_Y dT + \frac{H_r}{PL_r} dM$$

We then take equation (2), we replace dY and dP and we solve for dr and we get finally:

$$\Delta dr = -\frac{L_Y}{L_r} \left[\left(1 + \frac{\varepsilon_{AS}}{\varepsilon_{LY}} \right) \right] \{dG - H_Y dT\} + \frac{(1 - H_Y)}{PL_r} dM$$

It is easy to establish the table 3x3 multipliers of which none have an ambiguous sign.

$\frac{\partial Y}{\partial G} = \frac{1}{\Delta} > 0$	$\frac{\partial Y}{\partial T} = -\frac{H_Y}{\Delta} < 0$	$\frac{\partial Y}{\partial M} = \frac{H_r}{\Delta PL_r} > 0$
$\frac{\partial P}{\partial G} \frac{1}{P} = \frac{(f/f')}{\Delta} > 0$	$\frac{\partial P}{\partial T} \frac{1}{P} = -\frac{\varepsilon_{AS} H_Y}{\Delta} < 0$	$\frac{\partial P}{\partial M} \frac{1}{P} = \frac{(f/f') H_r}{\Delta PL_r} > 0$
$\frac{\partial r}{\partial G} = \frac{-\frac{L_Y}{L_r} \left[\left(1 + \frac{\varepsilon_{AS}}{\varepsilon_{LY}} \right) \right]}{\Delta} > 0$	$\frac{\partial r}{\partial T} = \frac{H_Y \frac{L_Y}{L_r} \left[\left(1 + \frac{\varepsilon_{AS}}{\varepsilon_{LY}} \right) \right]}{\Delta} < 0$	$\frac{\partial r}{\partial M} = \frac{(1 - H_Y)}{\Delta PL_r} < 0$

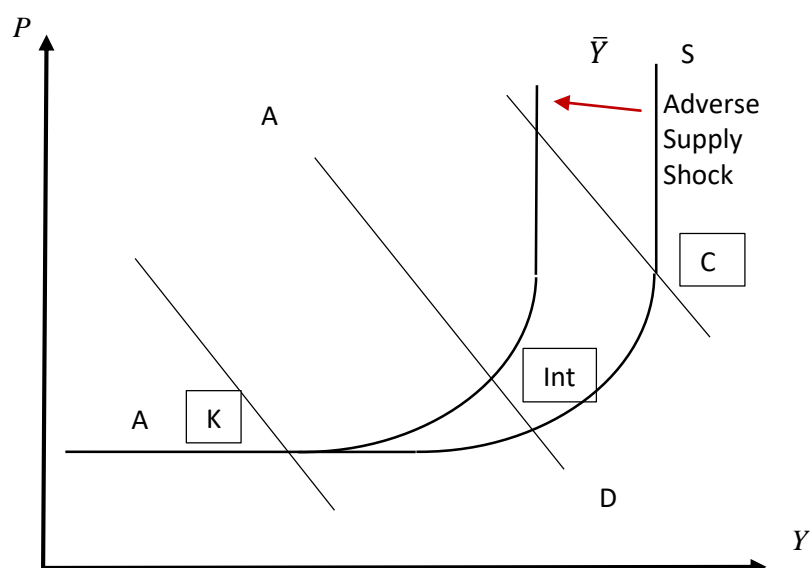
The bottom line of multipliers is the less familiar but at the same time the most important for our purpose. These multipliers indicate the response of the interest rate to fiscal and monetary policy in the AS-AD (and in the Keynesian and Classical regime as particular cases).

Note: $\varepsilon_{AS} = 0$ correspond to the Keynesian regime and $\varepsilon_{AS} \rightarrow \infty$ to the Classical regime.

2/ The good use of the I/S schedules diagram

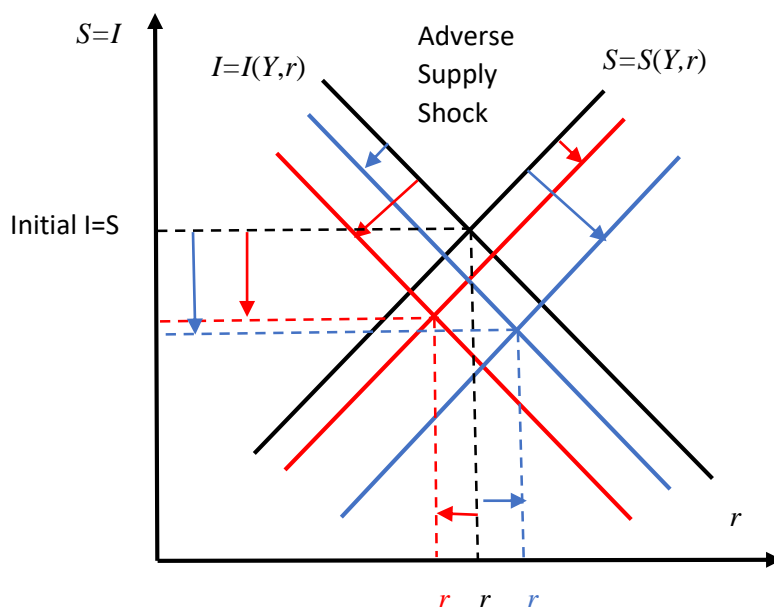
The AS-AD diagram (figure 1) helps understand the consequences of various shocks on output and the general price level, but the response of the interest rate is not directly visible. The I/S schedules diagram (figure 2) is often used to discuss the reaction of the rate of interest to shocks, but it turns out to be uneasy to use under non classical regimes. In Keynesian or intermediate regimes output reacts and will cause endogenous shifts to the saving and investment schedules. For example, a monetary expansion will raise output and will push the saving schedule upward thus causing a fall in the rate of interest, but at the same time, demand expansion will move the investment schedule upward thus pushing the interest rate up. In this case, a pure diagrammatic reasoning is inconclusive while an analytical resolution leads to the conclusion that the interest rate will fall unambiguously (see calculated multipliers). In addition, whatever the regime, reactions of the interest rate to supply shocks are uneasy to characterize. An adverse supply shock (a reduction of production capacity) will reduce Y and thus will push down savings and raise the interest rate but it will also reduce investment and thus will push interest rates downward. Drawing a conclusion about the resulting effect on the interest rate requires that we know the relative size of the shifts on saving and investment schedules, and not just their orientation (figure 2). Again, the diagrammatic approach is inconclusive while the analytical resolution concludes to an unambiguous increase of the interest rate.

Figure 1: The Three regimes of the AS-AD model.



Effects on the interest rate are not visible on the P - Y plan. The interest rate is on third dimension.

Figure 2 : The $I=S$ diagram : Two cases of adverse supply shocks ($dY < 0$)



The diagrammatic method is inconclusive while the multiplier analysis leads to rule out the red case and conclude to an increase in the interest rate as in the blue case.

Appendix 2 Introduction of a risky asset.

What difference it makes when a risky asset is introduced along supposedly non-risky government bonds? Clearly, the model should be modified but not to the extent that the general logic would be put into question. At minimum we will have to consider that, in addition to money and bonds, households have the option to put a part of their savings, for instance, in equities which returns are not guaranteed. We denote by A the amount of risky assets and by r^e the expected rate of return. Private agents are risk-averse and thus, will ask for a risk premium: ε , when putting their savings on the risky asset, which means that: $r^e - \varepsilon$ would be a certainty equivalent rate or return, comparable to r . Money, bonds and equities are gross substitutes and the two rates would affect the three assets demands. Households' consumption is likely to be affected by the two rates in real terms. As for firms it is reasonable to assume that investment would depend on the real risky rate \hat{r}^e as a measure of the cost of financing capital goods. Thus, the model should be re-specified as follow:

- (1) $Y = C(Y, \hat{r}, \hat{r}^e - \varepsilon) + I(Y, \hat{r}^e) + G$, or equivalently (1') $S(Y, \hat{r}, G) = I(Y, \hat{r})$.
 (2.1) $M/P = L(Y, r, r^e - \varepsilon)$; (2.2) $B/P = B(Y, r, r^e - \varepsilon)$; (2.3) $A/P = B(Y, r, r^e - \varepsilon)$
 (3) $P = f^s(Y)$
 (4.1) $\hat{r}_t = r - \tilde{p}_t$. (4.2) $\hat{r}^e = r^e - \tilde{p}_t$.

Although the rates of returns are not equal, they are related. From equations (2.2) and (2.3) we can describe private agents' bond/equity arbitrage by considering the ratio B/A :

$$\frac{B}{A} = \xi(Y, r, r^e - \varepsilon)$$

This equation contains an implicit relation between r^e and r .

$$r^e = \varphi\left(\frac{B}{A}, Y, r, \varepsilon\right)$$

If the ratio of debt to equity remains unchanged an increase in the rate of interest implies a rise in the rate of return of the risky asset. The increase in the share of government debt relative to firms' equities requires a raise in the interest rate on bonds or a reduction of the rate of return on equities. An increase in the risk premium requires an offsetting increase in the rate of return of the risky asset.

It is easy to check rapidly that the macroeconomic properties of the model remain mostly unchanged. If we eliminate r^e using φ we get a model very similar to model (1-4). If A/B is unchanged the multipliers have the same forms and the same signs. If the risk increases in the private sector, so that the risk premium widens, the model predicts a fall in the interest rate on government bonds.

Appendix 3 the Lucas supply curve and the natural rate of unemployment.

The Lucas supply curve can be derived from two assumptions

The natural rate of unemployment hypothesis. This means that although some inflation unemployment trade-off may exist in the short run, there is only one rate of unemployment \bar{U} , the natural rate of unemployment, consistent with the absence of inflation surprise in the long run. One general expression for his assumption is a relation between inflation surprise and the unemployment gap, the difference between the rate unemployment U and the natural rate \bar{U} :

$$(1) \quad \dot{p}_t - \tilde{p}_t = -\beta(U_t - \bar{U})$$

Where \dot{p}_t is the rate of inflation and its expected value \tilde{p}_t . Note that NAIRU (Non-Accelerating Inflation Rate of Unemployment) is the case for which $\tilde{p}_t = \dot{p}_{t-1}$

The Okun's law. It is the short run relation between the unemployment and the level of production. This can be written as a relation between the unemployment gap and the output gap.

$$(2) \quad U_t - \bar{U} = -\zeta(y_t - \bar{y})$$

The coefficient ζ , called Okun's coefficient, measures the reduction of the unemployment resulting from a one percent increase in production.

The Lucas supply curve is obtained by replacing the unemployment gap $U_t - \bar{U}$ in (1) by its expression taken from (2).

$$(3) \quad \dot{p}_t - \tilde{p}_t = \mu(y_t - \bar{y}) \quad \text{with } \mu = \beta\zeta$$

Therefore, assuming a Lucas supply curve implies the natural rate hypothesis.

Appendix 4. Intertemporal approach / decentralized models.

Equation (8) in the text: $\hat{r} = \delta + \frac{1}{\sigma}\dot{y} - \dot{n}$, where δ is the time preference, and σ is the intertemporal elasticity of substitution, and \dot{n} the rate of growth of the population, is said to be well known. It is nothing but the Ramsey rule transposed to the decentralized competitive economy (Cass, 1965), and assuming an isoelastic utility function:

$$U(t) = \int_{s=t}^{\infty} e^{-(\delta-\dot{n})s} \frac{c(s)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$$

where σ is the intertemporal elasticity of substitution.

These elements are usual in neoclassical growth theory. It is important to keep in mind that equation (8) is a property of the optimal path (not only the stationary state) generated by a flexible price model (full employment along the adjustment path).

The intertemporal approach is based on this type of model, but it focusses particularly on the response of the economy to different kinds of shocks, typically: permanent as opposed to transitory. This point of view is relevant for most of the questions raised in this paper.

Appendix 5: Relation between the debt ratio and the risk indicator

The notion that high public debt generates high interest rates is widely accepted although evidences are mixed. In dramatic cases of unsustainable indebtedness, things are rather clear but when considering intermediate situations often qualified as “excessive indebtedness” what has been found is that some limited risk premia arise (see Alesina et al. (1992)). In addition, other factors can affect the interest rate. What lies behind the relation between the interest rate and debt is simply that the private agents are inclined to see the level of the debt ratio as an indicator of default risk.

To test this assumption, we have built a (crude) risk indicator based on the Moody’s rating and we have run a panel regression for the four samples. The indicator varies from 0 (no risk) to 1 (default almost certain) at a pace of 0.5 between two successive value of Moody’s rating (Table1). The statistical relation between the risk indicator and the debt ratios is clearly confirmed. (Table 2).

Table 1 Moody’s rating and risk indicator

Moody’s rating	Risk indicator	Moody’s rating	Risk indicator	Moody’s rating	Risk indicator
Aaa	0	Baa1	0.35	B2	0.7
Aa1	0.05	Baa2	0.4	B3	0.75
Aa2	0.1	Baa3	0.45	Caa1	0.8
Aa3	0.15	Ba1	0.5	Caa2	0.85
A1	0.2	Ba2	0.55	Caa3	0.9
A2	0.25	Ba3	0.6	Ca	0.95
A3	0.3	B1	0.65	C	1

Table 2 Test of the relation between the Interest rate and debt ratio $rp = \beta_0 + \beta_5(d/y) + \varepsilon$

Endogenous variable: Risk indicator	All countries	G7+Belgium & Sweden	GIPS	Non-GIPS
Sample	(1)	(2)	(3)	(4)
Debt/GDP	0.203*** (0.0131)	0.0916*** (0.00724)	0.442*** (0.0346)	0.0749*** (0.00839)
Constant	-0.0678*** (0.00997)	-0.0413*** (0.00596)	-0.187*** (0.0345)	-0.0117** (0.00582)
Observations	646	432	136	510
R-squared	0.277	0.275	0.556	0.139
Number of countries	19	9	4	15

*** p<0.01, ** p<0.05, * p<0.1 / Standard errors in parentheses

Appendix 6: Sources and information on the database.

In this paper, we have conducted a statistical and econometrical analysis on nineteen selected OECD countries for which most macroeconomic annual data are available from 1970 to 2017. The list includes Australia, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, Korea, the Netherlands, New Zealand, Portugal, Spain, Sweden, Switzerland, the United Kingdom and the United States. From this database we extracted four different sub-samples (balance panel) for analytical purposes.

The data come mostly from the OECD Economic Outlook database No 104-November 2018 (GDP in volume, CPI index, long-term interest rates on government bonds, general government gross financial liabilities in percent of GDP, general government net lending in percent of GDP). These data were completed using older issues of OECD Economic Outlook (mainly No 73-June 2003) for a limited number of countries in order to have the largest possible samples (data were adjusted for base years changes in particular for real growth, for which there are two base years). In some cases, as for the French output gap, we found substantial revisions since OECD-2003 (up to 2 percentage points of GDP for the year 1985). Thus, we found safer to replace the output gap data, for the years before 1985, taking the high frequencies component of an HP filter estimation, as a proxy for the output gap (see Baghli et al (2002) for a methodological discussion).

The OECD data for Unified Germany start in 1991 and in order to include Germany in our panel the data available for the Federal Republic of Germany (Western Germany) were used for the years before unification. The two sub periods were connected and the 1991/1990 rate of growth was proxied by the average rate of growth of GDPs 3 years before and 3 years after the reunification in order to avoid a jump in 1991 and smooth the series, as there is some overlap between the old national accounts for the FRG and new national account of the Unified Germany. This ad-hoc rule can be criticized but has a negligible impact on the econometric results.

In some cases, such as Australia and New Zealand, some annual data for the debt ratio are missing in the 80's. The missing value were generated using the data for the general government net lending as a percentage of GDP. Thanks to this approximation those countries could be included in our panel for econometric analysis.

The double dimension our database panel (years x countries) means that we have many observations for testing a theoretical assumption but if we want to draw conclusions for the OECD as an area, we need to proceed using weighted calculation. For some variables we computed a weighted average which tells something on the OECD as a whole and, a weighted standard deviation which gives a fair approximation on the cross-country dispersion within the area, as opposed to the unweighted dispersion which corresponds to the notion of variety of country cases.

The formulas for weighted average (1) and weighted standard deviation (2) are:

$$\bar{x}^* = \frac{1}{\sum_{i=1}^n w_i} \sum_{i=1}^n w_i \cdot x_i \quad (1) \quad S^* = \sqrt{\frac{1}{\sum_{i=1}^n w_i} \sum_{i=1}^n w_i (x_i - \bar{x})^2} \quad (2)$$

The weight w_i for each country is as calculated as the share of the GDP of country i in OECD GDP, in volume, as constant purchasing power parities in US dollar (2010, same base year for all countries, except for France, 2014) for the year 2010. The weight for a given country is the level of its national GDP divided by the sum of GDPs of our sample, detailed calculations in table 1 below.

Table 1 Country weights in total GDP of the 19 selected countries

Countries	GDP in volume at constant PPP 2010 (US dollar, 2010)	w_i
Australia	906 199 520 623,16	0,02405842
Belgium	436 790 169 722,09	0,01159621
Canada	1 361 085 791 459,75	0,03613506
Denmark	238 772 396 980,27	0,0063391
Finland	207 942 600 830,63	0,00552061
France	2 334 823 941 037,22	0,06198655
Germany	3 200 656 838 994,76	0,08497329
Greece	197 601 206 559,89	0,00524606
Italy	2 074 917 032 961,91	0,05508636
Japan	4 482 754 896 976,16	0,11901134
Korea	1 505 136 498 023,30	0,03995942
Netherlands	748 640 465 771,13	0,01987543
New Zealand	134 744 891 697,94	0,0035773
Portugal	289 013 472 066,47	0,00767293
Spain	1 488 327 970 362,84	0,03951318
Sweden	390 048 336 804,86	0,01035528
Switzerland	415 127 912 809,37	0,01102111
United Kingdom	2 262 551 679 151,65	0,06006782
United States	14 991 485 697 625,10	0,39800452
Total	37 666 621 320 458,50	1

The definitions of the variables used in our paper to describe the economies are the following:

- Real growth is the annual growth rate of Gross Domestic Product in volume at 2010 market prices.
- Inflation is the annual growth rate of the consumer price index (CPI).
- The nominal interest rate is the interest rate on the 10 years fix rate government bonds.
- The Debt/GDP ratio is the general gross financial liabilities as a percentage of GDP.
- The fiscal balance corresponds to the general government net lending as a percentage of GDP.
- The risk indicator based on ratings is defined in appendix 5.
- Potential growth is the growth rate of potential output in volume as estimated by the OECD.
- The output gap is the difference between GDP in volume and potential GDP, in percent of potential GDP.
- The real exchange rate is proxied as the ratio of the Consumer Price Index (CPI) and the deflator of GDP at power $(1-\lambda)$. The rate of growth of the real exchange rate is simply the difference between the inflation rate and the rate of growth of the GDP deflator. The deflator of GDP is used as a proxy for the producer price. It is calculated as the ratio of GDP at current prices to GDP in volume.
- The structural balance is the cyclically adjusted general net lending, as a percentage of potential GDP.
- Expected inflation estimated as effective inflation less the inflation surprise, as calculated from the estimated Lucas supply curve.

Appendix 7: Econometric results

The four tables of this appendix contain the econometric estimation of specification (10) under various conditions. Some restrictions were retained in the choice of our “best estimate” --we would rather say: our most sensible estimate-- The expected rate of inflation was derived from the Lucas supply curve and the debt ratio was ruled out because of suspected simultaneity. It is important however to know how the restricted estimations compare with unconstrained estimations. Table 1 contains the selected (restricted estimation) Table 2 contains unconstrained estimation, without the debt ratio.

Table 1 Constrained weighted and unweighted estimations (selected estimation):

	All countries		G7+Belg. & Sweden		GIPS		Non GIPS	
	unweighted	weighted	unweighted	weighted	unweighted	weighted	unweighted	weighted
Expected Inflation	0.965*** (0.0333)	1.076*** (0.0598)	0.513*** (0.0310)	0.516*** (0.0356)	0.920*** (0.0527)	0.859*** (0.0672)	1.074*** (0.0514)	1.107*** (0.0703)
Potential growth	1.682*** (0.0995)	2.665*** (0.0997)	1.213*** (0.140)	2.101*** (0.167)	1.694*** (0.292)	0.778*** (0.250)	1.782*** (0.0992)	2.694*** (0.112)
Risk indicator	0.155*** (0.0109)	0.0600*** (0.0188)	0.00314 (0.0232)	0.0632 (0.0413)	0.200*** (0.0230)	0.0428** (0.0166)	0.0408** (0.0187)	0.0372 (0.0278)
Structural balance	-0.339*** (0.0360)	-0.0737** (0.0293)	-0.335*** (0.0423)	-0.116** (0.0469)	-0.517*** (0.0844)	-0.514*** (0.0633)	-0.321*** (0.0396)	-0.0771** (0.0334)
Const.	-0.024*** (0.00312)	-0.002*** (0.000173)	0.00788* (0.00406)	-0.002** (0.000627)	-0.059*** (0.0116)	-0.001 (0.00157)	-0.019*** (0.00280)	-0.003*** (0.000242)
Nb Obs	633	633	426	426	133	133	500	500
Nb Ctr	19	19	9	9	4	4	15	15
R2	0.718	0.738	0.644	0.624	0.792	0.859	0.691	0.738
Adj R2	0.708	0.729	0.634	0.613	0.780	0.851	0.680	0.728

Table 2. Unconstrained weighted and unweighted estimations (selected estimation):

	All countries		G7+Belg. & Sweden		GIPS		Non GIPS	
	unweighted	weighted	unweighted	weighted	unweighted	weighted	unweighted	weighted
Inflation	0.994*** (0.0318)	1.005*** (0.0610)	0.506*** (0.0320)	0.426*** (0.0397)	0.945*** (0.0538)	1.042*** (0.0716)	1.063*** (0.0497)	1.004*** (0.0719)
Potential growth	1.616*** (0.0996)	2.395*** (0.110)	1.202*** (0.148)	2.136*** (0.173)	1.696*** (0.274)	0.938*** (0.237)	1.706*** (0.103)	2.430*** (0.125)
Output gap	-0.214*** (0.0367)	-0.225*** (0.0324)	-0.174*** (0.0502)	-0.276*** (0.0432)	-0.216** (0.0832)	-0.146** (0.0574)	-0.220*** (0.0444)	-0.216*** (0.0371)
Real exch rate	-0.225*** (0.0670)	-0.267*** (0.0866)	-0.0165 (0.0952)	-0.0714 (0.112)	-0.369** (0.152)	-0.223 (0.144)	-0.148** (0.0738)	-0.291*** (0.0994)
Risk indicator	0.137*** (0.0121)	0.0471** (0.0192)	0.00692 (0.0236)	0.0475 (0.0413)	0.176*** (0.0262)	0.0636*** (0.0177)	0.0443** (0.0190)	0.0124 (0.0284)
Structural balance	-0.338*** (0.0342)	-0.120*** (0.0294)	-0.337*** (0.0421)	-0.100** (0.0477)	-0.477*** (0.0815)	-0.378*** (0.0618)	-0.335*** (0.0396)	-0.134*** (0.0339)
Constant	-0.0216*** (0.00297)	-0.0017*** (0.000176)	0.00776* (0.00410)	-0.0015** (0.000627)	-0.0511*** (0.0109)	-0.00222 (0.00146)	-0.0168*** (0.00281)	-0.0024*** (0.000249)
Nb Obs.	633	633	426	426	133	133	500	500
Nb Cntr.	19	19	9	9	4	4	15	15
R2	0.746	0.740	0.648	0.628	0.822	0.886	0.704	0.729
Adj R2	0.736	0.732	0.636	0.615	0.809	0.877	0.691	0.718

Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

The selected estimated equation (Table 1) explains roughly 70% of the variability of the nominal interest rates as measured by the R^2 , for the larger sample and from 60% to 80% for the other samples. The unconstrained estimation (Table 2) consists essentially in replacing the expected inflation by its determinants and estimating their coefficients freely. The adjusted R^2 of the constrained estimations turn out to be very close to the unconstrained estimations indicating that the restrictions resulting from the Lucas supply curve assumption does not violate the data. The debt ratio was not included in the estimation to facilitate the interpretation and are introduced in table 3 and 4 below.

In all regressions, inflation rate and potential growth are highly significant, and of the right sign (positive). The coefficient of the expected inflation ranges between 0.51 and 1.06. It is not significantly different from one when estimated on full sample, but significantly less than one on the G7+ sample, probably due to the lack of time lags in the relation that generates expected inflation. Expected inflation is calculated using formula (11) as the effective inflation less inflation surprise, which depends on the output gap and on the real the exchange rate. In the unconstrained estimations (Table 2) expected inflation is replaced by its determinants and estimated without any restriction. The estimated coefficients of the rate of inflation, the output gap and the real exchange rate are highly significant in most cases and show the expected sign with no exception. There is no reason to fear that the method used has violated the data.

The estimated coefficient of potential growth ranges between 1.2 and 1.71 depending on the samples, constantly with the empirical discussed in section I.2 of this paper and on Bismut & Ramajo (2019). Moreover, the estimates of this coefficient are highly significant and the implied values for the time elasticity of substitution σ , are respectively of 0.80 and 0.58, less than one. This comes in strong support in favor of the interpretation suggested for the positive relation between the rate of growth and the rate of interest (see appendix 4). The weighted estimations confirm these findings except in the case of the GIPS, but we should not give to much importance to this particular case given the limited number of countries in this sub-sample.

The estimated coefficient of the risk indicator is of the right sign (positive) though significant in three out of the four samples (not significant in the case of the G7+) and range between 0.04 and 0.18. This confirms that an increase in the perceive sovereign risk raises the nominal interest rate. Unsurprisingly these coefficients turn out to be particularly higher and significant in the in the case of the GIPS sample as default risk concentrates on these countries (especially in Greece with a default of payment in 2012). Conversely, the coefficient is small (but of the right sign) in the case of G7+, the countries of which are known to be creditworthy, and nonsignificant because of the absence of variance of the risk indicator for these countries.

The structural balance (the cyclically adjusted fiscal balance) turns out to be always significant and with the expected sign (negative) with estimated coefficients ranging between -0.48 and -0.34. One may say that this just confirms everyone knows, fiscal deficits push the interest rate up. In fact, here we talk about a non-cyclical effect of the fiscal balance on the interest rate. Introducing explicitly the output gap separately among the explanatory variables in the unconstrained estimation (Table 2) does not change that much the estimated coefficients for the cyclically adjusted balance. In addition, part of the fiscal balance effect operates through the modification of the perceived risk which is already captured by the risk indicator variable. The negative effect of fiscal balance is detected irrespective to the cycle and to the risk premium.

The same sets of regression were run by adding the debt/GDP ratio to the model. Results are respectively presented in, Table 3 for constrained and Table 4 for unconstrained estimations.

Table 3 Constrained weighted and unweighted model with debt ratio (main equation):

	All countries		G7+Belg. & Sweden		GIPS		Non GIPS	
	unweighted	weighted	unweighted	weighted	unweighted	weighted	unweight	weighted
Expected Inflation	0.857*** (0.0348)	1.109*** (0.0543)	0.455*** (0.0317)	0.428*** (0.0372)	0.819*** (0.0744)	0.750*** (0.0786)	0.980*** (0.0513)	1.162*** (0.0633)
Potential growth	1.439*** (0.100)	1.568*** (0.131)	0.667*** (0.167)	1.152*** (0.225)	1.622*** (0.292)	0.701*** (0.247)	1.531*** (0.103)	1.353*** (0.160)
Risk indicator	0.195*** (0.0117)	0.137*** (0.0183)	0.0512** (0.0240)	0.154*** (0.0424)	0.228*** (0.0273)	0.0893*** (0.0246)	0.0958*** (0.0198)	0.214*** (0.0298)
Structural balance	-0.403*** (0.0354)	-0.0465* (0.0266)	-0.399*** (0.0424)	-0.142*** (0.0452)	-0.483*** (0.0854)	-0.473*** (0.0640)	-0.397*** (0.0398)	-0.00763 (0.0307)
Debt/gdp ratio	-0.037*** (0.00481)	-0.0436*** (0.00378)	-0.033*** (0.00582)	-0.042*** (0.00697)	-0.035* (0.0182)	-0.0472* (0.0187)	-0.031*** (0.00472)	-0.050*** (0.00464)
Const.	0.00720 (0.00502)	0.00093*** (0.0004)	0.0441*** (0.00758)	0.00458*** (0.00118)	-0.0241 (0.0215)	0.0109** (0.00488)	0.00633 (0.00475)	0.0017*** (0.000475)
Nb Obs	633	633	426	426	133	133	500	500
Nb Ctr	19	19	9	9	4	4	15	15
R2	0.743	0.785	0.669	0.655	0.798	0.865	0.716	0.789
Adj R2	0.733	0.777	0.659	0.644	0.785	0.857	0.705	0.781

Table 4 Unconstrained weighted and unweighted model with debt ratio (main equation):

	All countries		G7+Belg. & Sweden		GIPS		Non GIPS	
	unweighted	weighted	unweighted	weighted	unweighted	weighted	unweighted	weighted
Inflation	0.892*** (0.0342)	0.990*** (0.0565)	0.451*** (0.0325)	0.310*** (0.0409)	0.846*** (0.0792)	0.949*** (0.100)	0.975*** (0.0501)	0.975*** (0.0663)
Potential growth	1.434*** (0.0997)	1.456*** (0.139)	0.659*** (0.174)	1.089*** (0.221)	1.652*** (0.274)	0.898*** (0.238)	1.487*** (0.106)	1.249*** (0.172)
Output gap	-0.223*** (0.0355)	-0.318*** (0.0314)	-0.158*** (0.0486)	-0.288*** (0.0409)	-0.238*** (0.0836)	-0.177*** (0.0618)	-0.214*** (0.0428)	-0.359*** (0.0375)
Realexch rate	-0.209*** (0.0646)	-0.354*** (0.0807)	-0.0465 (0.0921)	0.0446 (0.107)	-0.357** (0.151)	-0.253* (0.145)	-0.142** (0.0712)	-0.353*** (0.0919)
Risk indicator	0.169*** (0.0126)	0.116*** (0.0190)	0.0537** (0.0243)	0.151*** (0.0417)	0.196*** (0.0286)	0.0828*** (0.0228)	0.0937*** (0.0201)	0.192*** (0.0326)
Structural balance	-0.394*** (0.0340)	-0.0812*** (0.0275)	-0.400*** (0.0423)	-0.120*** (0.0452)	-0.461*** (0.0814)	-0.360*** (0.0632)	-0.403*** (0.0398)	-0.0429 (0.0328)
Debt/GDP	-0.0320*** (0.00470)	-0.0417*** (0.00415)	-0.0319*** (0.00583)	-0.0485*** (0.00689)	-0.0299* (0.0178)	-0.0282 (0.0213)	-0.0285*** (0.00469)	-0.0504*** (0.00545)
Constant	0.00573 (0.00482)	0.00110*** (0.000326)	0.0435*** (0.00763)	0.00562*** (0.00116)	-0.0213 (0.0207)	0.00513 (0.00575)	0.00656 (0.00470)	0.00209*** (0.000532)
Nb Obs	633	633	426	426	133	133	500	500
Nb Ctr	19	19	9	9	4	4	15	15
R2	0.764	0.772	0.672	0.668	0.827	0.887	0.725	0.770
Adj R2	0.755	0.763	0.660	0.656	0.812	0.878	0.713	0.760

Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

Attempts to introduce the debt/GDP ratio as an additional explanatory variable led to disappointing results indicating that the single equation approach of our data has reached its limits. The estimated coefficients of the debt/GDP ratio turn out to be systematically of the wrong sign (negative). On the other side the gains in terms of explained variance score are limited and some estimated coefficients are altered, reflecting problems of collinearity. More fundamentally the robustness of estimations seem to be deteriorated.

Appendix 8: An updated estimation of the two factors Lucas supply curve

The key idea behind the two factors Lucas supply curve is that inflation expectation errors could come not only from unexpected domestic real shocks (on the output gap) but also on unexpected external real shocks (the real exchange rate) as indicated in equation (6). This relation is discussed and estimated in Bismut and Ramajo (2018). The table below contains updated estimation of the two parameters of this relation that were used for calculating expected inflation.

Country	Sample	Output Gap	Real Exchange Rate	R^2 F-statistic	DW
Australia	1975-2017	0.37* (0.21)	0.16 (0.23)	0.07 1.58	2.30^{oo}
Belgium	1971-2017	0.36*** (0.11)	0.29 (0.24)	0.23*** 6.54	1.84^{oo}
Canada	1966-2017	0.32*** (0.10)	-0.01 (0.17)	0.22*** 6.74	1.97^{oo}
Denmark	1971-2017	0.23** (0.11)	0.54*** (0.19)	0.21*** 6.00	1.87^{oo}
Finland	1975-2017	0.18*** (0.06)	0.08 (0.16)	0.17** 4.23	1.43 ^o
France	1961-2017	0.35*** (0.11)	0.85*** (0.21)	0.35*** 14.48	1.43
Germany	1966-2017	0.37*** (0.05)	0.11* (0.06)	0.48*** 22.95	1.75 ^{oo}
Greece	1975-2017	0.13 (0.08)	0.41* (0.23)	0.12* 2.62	1.42 ^o
Ireland	1977-2017	0.14 (0.12)	0.22 (0.19)	0.07 1.38	1.57 ^o
Italy	1963-2017	0.54*** (0.12)	0.50 (0.32)	0.29*** 10.58	1.61 ^o
Japan	1970-2017	0.44*** (0.20)	-0.11 (0.43)	0.11* 2.75	2.29^{oo}
Korea	1975-2017	0.45** (0.19)	0.48 (0.29)	0.14* 3.22	1.97^{oo}
Netherlands	1972-2017	0.27*** (0.08)	0.46*** (0.16)	0.29*** 8.96	1.58^o
New Zealand	1980-2017	0.49* (0.26)	0.41 (0.32)	0.12 2.43	2.17^{oo}
Portugal	1971-2017	0.02 (0.09)	1.23*** (0.20)	0.51*** 22.85	2.36^{oo}
Spain	1979-2017	0.17*** (0.05)	0.70*** (0.22)	0.27*** 6.80	1.99^{oo}
Sweden	1967-2017	0.39*** (0.11)	0.53** (0.21)	0.26*** 8.52	2.31^{oo}
Switzerland	1978-2017	0.24*** (0.09)	0.47*** (0.18)	0.29*** 7.44	2.05^{oo}
United Kingdom	1970-2017	0.39** (0.18)	0.21 (0.36)	0.10* 2.49	2.11^{oo}
United States	1964-2017	0.32*** (0.07)	1.05*** (0.19)	0.53*** 28.73	1.35

Notes: Standard errors in parenthesis, below estimated coefficients. Stars indicate that a coefficient is significant at 10% (*), at 5% (**) or at 1% (***) . Based on the Durbin-Watson statistic, error autocorrelation is rejected at 5% (^{oo}), or test inconclusive (^o) , or not rejected (no indication).

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