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On the Modeling and Testing of Groundwater Resource Models

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Abstract

Economists have been attempting to take on the optimal management of groundwater for many decades, initially through static models, and since the 1970's through a dynamic framework. Since then, several attempts have been made to test dynamic models through laboratory experiments. Yet formulating and testing these models raises several challenges that we attempt to tackle in this study by testing a very simple dynamic groundwater extraction model in a laboratory experiment. We propose a full characterization of the theoretical solutions, taking into account economic constraints. In the experiment we mimic continuous time by allowing subjects to make their extraction decisions whenever they wish, with an actualization and updating the data (resource and payoffs) every second. The infinite horizon is simulated through the computation of payoffs, as if time were endless. To get around the weaknesses of the widely used Mean Squared Deviation (MSD) statistic and classify individual behavior as myopic, feedback or optimal, we combine the MSD with Ordinary Least Squares (OLS) regressions and time series treatments. Results show that a significant percentage of agents are able to adopt an optimal extraction path, that few agents should be considered truly myopic, and that using the MSD alone to classify agents would be misleading for about half of the study participants.

Keywords: Renewable Resources; Continuous Time; Dynamic Optimization; Differential Games; Experimental Economics; Applied Econometrics.

JEL Codes: C01; C73; C90; C91; C92; Q20; Q25.

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1 Introduction

Water is an essential resource for the survival of all living species. For his daily needs, man uses not only surface water, but also groundwater, which represents 98% of the planet's unfrozen fresh water reserves (UNESCO). Over the last 50 years groundwater use has increased, especially in the agricultural sector. Moreover, this resource faces new threats as pesticides used by the agricultural sector pollute groundwaters. Climate change is also having a negative effect, as it leads to reduced rainfall, which notably forces farmers to switch to groundwater to irrigate their crops, leading to the depletion of several aquifers. The WorldBank estimates that groundwater is being depleted faster than it is being replenished, so that in 2025, almost 1.8 billion people could experience an absolute water shortage. The overexploitation of groundwater could be explained by the phenomenon Hardin (1968) called the tragedy of the commons. Given their non-excludability and rivalrous nature, groundwaters are common resources which are overexploited without regulation. In the light of the urgency of adopting measures for the efficient management of common-pool resources (CPRs), it is essential to have acute knowledge of the different decision-making processes of individuals in order to better anticipate the effects of any measures intended to reduce groundwater exploitation.

To understand individual behaviors, economists have formulated mathematical models to represent the problem faced by the extracting agent. Until the 1970's, these models were static (Gordon, 1954), but the need to follow both the evolution in behavior and the studied resource over time, as well as to study the interactions among agents, has led to the use of a dynamic framework. Moreover, lab experiments on common resources have evolved from a static to a dynamic framework in order to make the experiments more realistic (Gisser & Sanchez, 1980; Feinerman & Knapp, 1983; Clark, 1990; Dasgupta & Heal, 1979). The baseline model has been altered in many ways, both theoretically and experimentally, to study the role of hydrological characteristics of groundwater (Gisser & Sanchez, 1980; Feinerman & Knapp, 1983; Rubio & Casino, 2003; Suter et al., 2012), to account for the presence of externalities (Herr et al., 1997; Gardner et al., 1997), to study the role of information (Hey et al., 2009), and even to study the choice between linear and non linear strategies (Tsutsui & Mino, 1990; Rowat, 2007; Colombo & Labrecciosa, 2015; Tasneem et al., 2017), to cite but a few examples. While the empirical testing of these models is essential to understanding agents' behavior, experimental procedures and empirical strategies implemented vary from one study to another, and some of the strategies used can reveal inappropriate.

In this paper, we use a simple dynamic groundwater extraction model and characterize the behavior of laboratory participants. We compute the myopic, feedback and social optimum solutions and voluntarily stay with a very simple model and classic predictions in order to propose methodological contributions that can be used by a wide range of studies in the field. Our study possesses four main features. First, we propose a full characterization of the cost function, paying particular attention to the positivity of marginal costs, which leads us to use a "LambertW" specification in the derivation of the feedback solution. Next, in the experiment we mimic continuous time by allowing subjects to make their extraction decisions whenever they wish, with an actualization of the data (resource and payoffs) every second. Third, the infinite horizon is simulated through the computation of payoff as if time were endless. Finally, in order to classify the individual as exhibiting myopic, feedback or optimal behavior and to get around the weaknesses of the widely used Mean Squared Deviation (MSD) statistic, we combine it with Ordinary Least Squares (OLS) regressions and time series treatments. The full characterization of the cost function and the combination of MSD and OLS are contributions of the paper. The representation of continuous time and infinite payoff are features borrowed from recent papers in the literature. These four features form what we argue to be good practices when studying dynamic games of common-pool resources in a laboratory experiment. To our knowledge, we are the first to conduct an experimental study on differential games comparing the behavior of experimental subjects according to theoretical predictions, as well as combining a single agent and a multiple agent treatment in the same experiment. Results show that a significant percentage of agents are able to adopt the optimal extraction path, that few agents should be considered truly myopic, and that using the MSD alone to classify agents would be misleading for about half of the study participants.

The remainder of the paper is organized in the following manner: Section 2 gives an overview of the literature related to the research question; Section 3 presents the theoretical predictions, both for a single agent and multiple agents; Section 4 describes the experiment; Section 5 shows the methodology followed to analyze the experimental data; Section 6 gives the results, and Section 7 provides a discussion and concluding remarks.

2 The Literature

The management of the commons has been the concern of many researchers for many decades, but studies were initially conducted through static models, which do not take into account the effect of resource evolution on individual behaviors. It was only in the $1970's$ that the transition to a dynamic framework took place. The interest of this framework lies in the existence of a set of state variables which are able, not only to describe the evolution of the system at any moment, but also to describe interactions occurring between individuals over time (Basar & Olsder, 1999; Dockner et al., 2000; Haurie & Zaccour, 2005; Engwerda, 2005; Van Long, 2010). Furthermore, several recent laboratory experiments rely on dynamic models for robust results. This paper is related to several strands of the literature which are discussed in this section. First, we review the initial experimental tests in discrete time. Second, we look at the first articles implementing continuous time in the lab. Next, we look at two recent articles that use differential games and take up the challenge of continuous time in the lab. Finally, we explain how our study relates to this literature.

Using a discrete time model, Hey et al. (2009) studied with a one-player finite horizon model, the role of information (the number of fish units and their growth function) in the management of fisheries. They found that subjects under-harvested the resource when no information was given to them. Using a finite horizon n players model, Herr et al. (1997)'s work on dynamic and static externalities resulting from the use of groundwater revealed that most of the time subjects adopted more myopic behavior in the dynamic setting than in the static setting. Another study on externalities is that of Gardner et al. (1997). The authors tried to analyze the impact of three property rights regimes (no restriction, entry restriction and individual quota, respectively) in the mitigation of strategic, stock and congestion externalities resulting from the use of groundwater in the western United States. Using a finite horizon n players model, they found an improvement in efficiency, both in the entry restriction and the individual quota. Based on the same type of experiment, Suter et al. (2012) analyzed both a one-player (optimal control) and n-player (game) behaviors. In an infinite horizon framework, they studied the effect of taking into account hydrological characteristics of groundwater on subjects' pumping rates. Their main result is that, contrary to a simple groundwater model, considering a spatially explicit model can reduce both myopic behavior and pumping rates.

The first papers that dealt with continuous time models in the lab were applied to extensive form games à la Simon & Stinchcombe (1989).[1] The authors were interested in comparing continuous and discrete time in both public goods and minimum effort games, as well as in testing the importance of communication in these games (Oprea et al., 2014; Leng et al., 2018). They were also interested in testing the relationship between the time horizon and agent cooperation (Bigoni et al., 2015). Their results show that continuous time without communication does not perform better than discrete time and that a deterministic time horizon promotes cooperation. Although these are important explorations of continuous time in the lab, they do not take into account the evolution of a state variable. Other articles which have tried to bring more realism to the implementation of continuous time in the lab without using a model are Janssen et al. (2010) and Cerutti (2017). These authors have introduced spatial and temporal dimensions in the study of renewable resources and done it in real time to simulate the

[1] Simon & Stinchcombe (1989) defined in a time interval [0,1] a finite set of agents and imposed some limitations in the actions players could change, which allowed them to play games in continuous time, in the limit as the interval approached zero. In this paragraph dedicated to quasi-continuous time, all expressions that referred to «continuous time» actually referred to «quasi-continuous time».

real-life conditions of ecological systems.

Two studies have been conducted in continuous time, using differential games and taking up the challenge of infinite horizon. Using a linear quadratic model and focusing on feedback behavior, Tasneem et al. (2017) consider a simultaneous exploitation of a common renewable fishery by two identical actors in order to observe whether they adopt a linear or a nonlinear strategy. The results suggest that most players employ nonlinear reasoning. Moreover, the authors find that different initial extraction rates do not affect the subjects' behavior, whereas different initial stock do, causing overextraction when the initial stock is low. Tasneem et al. (2019) use the same model as Tasneem et al. (2017), in order to determine whether a single player is able to sustainably and efficiently manage a renewable fishery. Their results suggest suboptimal behavior due to initial overextraction of the resource because of the tradeoff between instantaneous payoff and the future sum of payoffs. However, contrary to Tasneem et al. (2017), the authors find that a linear model explains the observed extraction rates better than a non linear model. Although these are the first papers exploring differential games in a dynamic framework, they focus on the study of behavior inside a specific state-dependent strategy, also called the Markovian strategy.[2]

Our study builds on previous literature, such as Rubio & Casino (2003) for the theoretical model and Tasneem et al. (2017, 2019) for the experiment, but to our knowledge we are the first to compare the behavior of experimental subjects according to theoretical predictions. Following Tasneem et al. (2017, 2019), we have combined in a dynamic framework theoretical models and laboratory experiments in continuous time and infinite horizon, despite the challenges that their implementation in the laboratory pose. In fact, it is important to focus on continuous time and infinite horizon to replicate the evolution of real-world resources. Indeed, groundwaters are dynamic resources which continuously renew, nevertheless with a potential risk of exhaustion (Koundouri, 2004). The advantage of the continuous time dynamic framework is that it allows for the possibility of experimental subjects making decisions at any time and seeing the consequences of these decisions in real time. Second, in line with Suter et al. (2012), in the same experiment we combined optimal control (single agent) and game (multiple agents) treatments. However, we did this in a within subject design, whereas the authors used a between subject design. The participants of our experiment played the single-agent treatment first and the two-players game second. We proceeded this way for two reasons. Firstly, we believe that before we can successfully understand the behavior of a group of individuals who interact with each other, it is essential to understand how they behave individually. Secondly, given the complexity of our theoretical model, the dynamic environment involving both continuous time and the infinite horizon perceived through the computa-

[2]See Tasneem & Benchekroun (2020) for "A review of experiments on dynamic games in environmental and resource economic".

tion of payoffs and the piecewise cost function, it was relevant to run the study first with a single agent to ensure that he fully understand all the underlying mechanisms before introducing a second agent into the resource extraction game. Finally, we carried out a simple analysis without institutional arrangements such as communication or punishment, so that our study could serve as a benchmark for future, more complex studies on the optimal management of renewable resources.

3 The Model

We consider a simple continuous time linear quadratic model in which farmers harvest a renewable resource that can be assimilated to a groundwater.

The model is based on that of Rubio & Casino (2003), however, with an extension on the cost function. Water is the only input in the production process, and in the interest of simplification, the aquifer is assumed to have parallel sides and a flat bottom.[3] At a given time t, the extraction done by farmers gives them a benefit $B(w)$ depending only on the extraction rate w, according to the following equation[4]:

$$B(w) = aw - \frac{b}{2}w^2, \tag{1}$$

with a and b positive parameters. Figure 10 on the left in Appendix D shows a farmer's benefit function. However, farmers also incur harvesting costs, so that the total harvesting cost $C(H, w)$, positively depends on the extraction rate w and negatively depends on the level of the groundwater H, as shown by equation (2). In other words, H is the elevation of the water table above the bottom of the aquifer, so that c_0 is the maximum average cost. Thus:

$$C(H, w) = \overbrace{\max(0,\ c_0 - c_1 H)}^{\textit{marginal cost } (c(H))} w, \tag{2}$$

where c_0 and c_1 are positive parameters. We also pay attention to the positivity of the marginal or unitary cost $c(H)$, while theoretical models usually avoid this constraint and suppose that parameters are such that marginal cost is positive at the optimum and equilibria. This gives us a piecewise marginal cost function (3), allowing us to study all the different types of regimes, in addition to the steady state regime.

$$c(H) = \begin{cases} (c_0 - c_1 H) & \text{if } 0 \leq H < \dfrac{c_0}{c_1} \\ 0 & \text{if } \quad H \geq \dfrac{c_0}{c_1} \end{cases} \tag{3}$$

[3]We use a simple "Bathtub" model to illustrate the groundwater extraction.

[4]We omit the subindex t when not necessary.

Instantaneous payoff is the difference between benefit and total costs. Figure 10 on the right in Appendix D shows the marginal cost function.

3.1 The Case of a Single Agent: Optimal Control

In the single agent problem, the evolution of the water table is the following:

$$\dot{H} = R - \alpha w, \quad H(0) = H_0, \quad H_0 \quad \text{given},$$

where R is the natural recharge, namely the rain that we assume to be constant and α the constant return flow coefficient. The farmer's problem is to choose at time t, for all $t \in [0, \infty]$, the extraction rate $w(t)$. For experimental purpose and in order to study up to what point subjects take into account the evolution of the resource, we consider two extreme types of behavior: a forward-looking farmer who maximizes the discounted sum of his instantaneous payoffs over time, taking into account the evolution of the dynamics; and a myopic farmer who maximizes his instantaneous payoffs.[5]

3.1.1 The Social Optimum Solution

A farmer adopts a social optimum behavior when his extraction decision allows him to maximize his discounted net payoff in order to keep the resource at an efficient level. His maximization problem, where r is the discount factor is then:

$$\max_{w(t)} \int_0^{\infty} e^{-rt} \left[aw(t) - \frac{b}{2} w(t)^2 - \max(0,\ c_0 - c_1 H(t)) w(t) \right] dt \tag{4}$$

s.t

$$\begin{cases} \dot{H}(t) = R - \alpha w(t) \\ H(0) = H_0 \geq 0,\ H_0\ \ given \\ H(t) \geq 0 \\ w(t) \geq 0 \end{cases}$$

Condition 1 *: We suppose that:*

$$\frac{R}{\alpha} < \frac{a}{b}, \quad \frac{R\alpha c_1 + Rbr - a\alpha r + \alpha c_0 r}{\alpha c_1 r} > \frac{c_0}{c_1}$$

This condition is given to ensure that the steady state of the optimal solution is:

$$H^{\infty} = \frac{c_0}{c_1}$$

This will allow us to better differentiate the two types of behavior. In fact, as we will see, when the resource is less than $\frac{c_0}{c_1}$ and not so small, the optimal level of the water

[5] We give the steps to follow for the proof of the different solutions in the Appendix A and B.

table increases to $\dfrac{c_0}{c_1}$, while the myopic solution causes the water table to go down to its steady state, which is smaller than $\dfrac{c_0}{c_1}$.

Theorem 1 *: Under condition 1 the steady state of the optimal solution is:*

$$H_{op}^{\infty} = \frac{c_0}{c_1}, \quad w_{op}^{\infty} = \frac{R}{\alpha}$$

The optimal groundwater path has two regimes: it increases to this steady state when $H_0 < \dfrac{c_0}{c_1}$ (decreases when $H_0 > \dfrac{c_0}{c_1}$) till a certain time T where $H(t) = \dfrac{c_0}{c_1}$ for all $t \geq T$. The optimal extraction rate follows the same trajectory towards its steady state. It can be preceded by a regime with null extraction.

3.1.2 The Constrained Myopic Solution

In theory, the myopic solution is given by a situation where the farmer is only interested in the maximization of his current payoff. The constrained myopic problem faced by the farmer is:

$$\max_{w(t)} \left[aw(t) - \frac{b}{2} w(t)^2 - \max(0,\ c_0 - c_1 H) w(t) \right] \tag{5}$$

for each level of the water table. This maximization problem provides a feedback representation[6] of the solution $w(H)$, constrained to:

$$\begin{cases} \dot{H}(t) = R - \alpha w(H(t)) \\ H(0) = H_0 \geq 0,\ H_0 \ \textit{given} \\ H(t) \geq 0 \\ w(t) \geq 0 \end{cases}$$

Condition 2 *: We suppose that*

$$a > c_0, \quad \frac{R}{\alpha} - \frac{a - c_0}{b} > 0$$

This condition is to ensure the positivity of the steady state and the extraction of the constrained myopic solution.

Theorem 2 *: Under condition 2, the steady state of the constrained myopic problem is:*

$$H_{my}^{\infty} = \frac{b}{c_1} \left(\frac{R}{\alpha} - \frac{a - c_0}{b} \right), \quad w_{my}^{\infty} = \frac{R}{\alpha}$$

[6] The feedback representation is obtained when the solution is written according to the state variable, instead of according to time.

When $H_0 > H^{\infty}_{my}$, the constrained myopic path decreases to the steady state. Moreover, the constrained myopic extraction is given by:

$$w_{my}(H) = \begin{cases} \dfrac{a}{b} & \textit{if } H > \dfrac{c_0}{c_1} \\ \\ \dfrac{a - c_0 + c_1 H}{b} & \textit{if } 0 \leq H \leq \dfrac{c_0}{c_1} \end{cases}$$

Note that condition 1 implies that $H^{\infty}_{my} < H^{\infty}_{op}$. In our simulation the difference $H^{\infty}_{op} - H^{\infty}_{my}$ will be large enough to differentiate the two behaviors.

3.2 The Case of Multiple Agents: Game

We consider now that two identical and symmetrical farmers exploit the groundwater. Payoff and cost are the same as those of the single farmer, but now the evolution of the water table is the following:

$$\dot{H} = R - \alpha(w_1 + w_2), \quad H(0) = H_0, \quad H_0 \quad \text{given.}$$

As before, the farmer's problem is to choose at time t, for all $t \in [0, \infty]$, the extraction rate $w_i(t)$. We considered two types of individual behavior: feedback and myopic. In the feedback equilibrium, a forward-looking farmer maximizes the discounted sum of his instantaneous payoffs over time, taking into account the dynamics of the groundwater, while the myopic farmer maximizes his instantaneous payoffs. For sake of comparison we consider the joint maximization problem, also known as the cooperative solution (or the social optimum solution). We did this because we wanted to know if some kind of "tacit" cooperation could emerge without negotiation, because we thought that having first played the single agent dynamic problem, agents who behaved optimally would behave the same way in the game, because they already had information about optimal behavior.[7]

3.2.1 The Social Optimum Solution

Farmers adopt a cooperative behavior when extraction decisions maximize the joint discounted net payoff in order to keep the resource at an efficient level. The maximization problem, where r is the discount factor is then:

$$V(H_0) = \max_{w_1(t), w_2(t)} \int_0^{\infty} e^{-rt} \sum_{i=1}^{2} \left[a w_i(t) - \frac{b}{2} w_i(t)^2 - \max(0,\ c_0 - c_1 H(t)) w_i(t) \right] dt \qquad (6)$$

[7] We give the step to follow for the proof of the different solutions in Appendix C.

s.t

$$\begin{cases} \dot{H}(t) = R - \alpha(w_1(t) + w_2(t)) \\ H(0) = H_0 \geq 0,\ H_0 \ \textit{given} \\ H(t) \geq 0 \\ w_i(t) \geq 0 \end{cases}$$

Condition 3 *: We suppose that:*

$$\frac{R}{\alpha} < \frac{a}{b}, \quad \frac{2R\alpha c_1 + Rbr - 2a\alpha r + 2\alpha c_0 r}{2\alpha c_1 r} > \frac{c_0}{c_1}$$

As in the single agent case, this condition is designed to ensure that the steady state of the optimal solution is:

$$H^\infty = \frac{c_0}{c_1}$$

Theorem 3 *: Under condition 3 the steady state of the optimal solution is:*

$$H^\infty_{op} = \frac{c_0}{c_1}, \quad w^\infty_{i,\,op} = \frac{R}{2\alpha}$$

The optimal resource path has two regimes: it increases to this steady state when $H_0 < \frac{c_0}{c_1}$ (decreases when $H_0 > \frac{c_0}{c_1}$) till a certain time T where $H(t) = \frac{c_0}{c_1}$ for all $t \geq T$. The optimal extraction rate follows the same trajectory towards its steady state. It can be preceded by a regime with null extraction.

The proof of this theorem follows the same structure as that of the single agent.

3.2.2 The Nash Feedback Solution

Now we consider a scenario in which farmers adopt non-cooperative behavior, maximizing their own net payoffs and taking into account the evolution of the groundwater. For each farmer, the maximization problem, where r is the discount factor is then:

$$\max_{w_i(t)} \int_0^\infty e^{-rt} \left[a w_i(t) - \frac{b}{2} w_i(t)^2 - \max(0,\ c_0 - c_1 H(t)) w_i(t) \right] dt \tag{7}$$

s.t

$$\begin{cases} \dot{H}(t) = R - \alpha(w_1(t) + w_2(t)) \\ H(0) = H_0 \geq 0,\ H_0 \ \textit{given} \\ H(t) \geq 0 \\ w_i(t) \geq 0 \end{cases}$$

Condition 4 *: We suppose that:*

$$Rb + 2\alpha^2 A_2 - 2\alpha(a - c_0) > 0, \quad \frac{Rb + 2\alpha^2 A_2 - 2\alpha(a - c_0)}{2\alpha(c_1 - \alpha A_3)} < \frac{c_0}{c_1}, \quad a - c_0 - \alpha A_2 > 0,$$

Where,

$$A_2 = \frac{(a-c_0)(-c_1+2\alpha A_3) - RbA_3}{-rb - 2c_1\alpha + 3A_3\alpha^2},$$

and A_3 is the solution of:

$$-\frac{3\alpha^2}{2b}A_3^2 + \frac{rb+4c_1\alpha}{2b}A_3 - \frac{c_1^2}{2b} = 0,$$

with $-c_1 + \alpha A_3 < 0$

Conditions $Rb + 2\alpha^2 A_2 - 2\alpha(a-c_0) > 0$ and $\frac{Rb+2\alpha^2A_2-2\alpha(a-c_0)}{2\alpha(c_1-\alpha A_3)} < \frac{c_0}{c_1}$ ensure that the steady state of the feedback path is positive and in the regime where cost is positive. Condition $a - c_0 - \alpha A_2 > 0$ ensures that extraction is always positive.

Theorem 4 *: Under condition 4 the steady state of the feedback equilibrium is:*

$$H_f^\infty = \frac{Rb+2\alpha^2A_2-2\alpha(a-c_0)}{2\alpha(c_1-\alpha A_3)}, \quad w_{i,\,f}^\infty = \frac{R}{2\alpha}$$

Groundwater increases to this steady state when $H_0 < H_f^\infty$ (decreases when $H_0 > H_f^\infty$). The extraction rate follows the same trajectory towards its steady state.

3.2.3 The Constrained Myopic Solution

As in the single agent model, the myopic solution is given by a situation where each farmer is only interested in the maximization of his current payoff. The constrained myopic problem faced by the farmer is:

$$\max_{w_i(t)} \left[aw_i(t) - \frac{b}{2}w_i(t)^2 - \max(0,\ c_0 - c_1H)w_i(t) \right] \tag{8}$$

for each level of the water table. This maximization problem provides a feedback representation of the solution $w_i(H)$, constrained to the evolution of the water table exploited by the two symmetrical farmers:

$$\begin{cases} \dot{H}(t) = R - 2\alpha w(H(t)) \\ H(0) = H_0 \geq 0,\ H_0 \textit{ given} \\ H(t) \geq 0 \\ w_i(t) \geq 0 \end{cases}$$

Condition 5 *: We suppose that:*

$$a > c_0, \quad \frac{R}{2\alpha} - \frac{a-c_0}{b} > 0.$$

This condition is to ensure the positivity of the steady state and the extraction of the constrained myopic solution.

Theorem 5 *: Under condition 5 the steady state of the constrained myopic problem is:*

$$H_{my}^{\infty} = \frac{b}{c_1}\left(\frac{R}{2\alpha} - \frac{a - c_0}{b}\right), \quad w_{i,\,my}^{\infty} = \frac{R}{\alpha}$$

When $H_0 > H_{my}^{\infty}$ the constrained myopic path decreases to the steady state. Moreover, the constrained myopic extraction is given by:

$$w_{i,\,my}(H) = \begin{cases} \dfrac{a}{b} & H > \dfrac{c_0}{c_1} \\[2ex] \dfrac{a - c_0 + c_1 H}{b} & 0 \leq H \leq \dfrac{c_0}{c_1} \end{cases}$$

By condition 3, we know that $H_f^{\infty} < \frac{c_0}{c_1} = H_{op}^{\infty}$, and by condition 4, we also know that $H_f^{\infty} > H_{my}^{\infty}$, so we can conclude that:

$$H_{my}^{\infty} < H_f^{\infty} < H_{op}^{\infty} \tag{9}$$

4 The Experiment

4.1 Experimental Design

The experiment took place at the Experimental Economics Laboratory of Montpellier (LEEM), during 6 sessions in the second half of 2018. A total of 70 students from the University of Montpellier, randomly drawn from a pool of volunteers, participated in the experiment.[8]

This non-contextualized experiment was divided into two parts: in the first part subjects played alone, as described in subsection 3.1, and the second part was played by groups of two, as described in subsection 3.2.[9] In each part, subjects played two five-minute training phases and one additional five-minute effective phase, that counted for the experiment payoff.[10] Upon arriving in the lab, subjects read the instructions of Part 1.[11] These instructions specified that there were two independent paid parts, and that subjects would receive the instructions from Part 2 when Part 1 was completed. Subjects accumulated units that were converted into cash payment with the conversion

[8]Given the complexity of the experiment, we restricted the pool of volunteers to students from disciplines commonly employing complex calculations (physics, mathematics, economics, biology, medicine and computer science). This can be easily done with ORSEE (Greiner, 2015), the software used by the LEEM to manage the subject pool. The software we used for the experiment was LE2M.

[9]We decided not to contextualize the experiment 1) to avoid framing effects and 2) to have a more general experimental framework which could serve as a benchmark to different types of renewable common-pool resources.

[10]The experiment payoff was composed of the payoffs in Part 1 and Part 2, which were paid in cash at the end of the experiment. Subjects had information about the composition of their payment.

[11]Available upon request.

rate of 10 units (ECU) equal to 0.5 euro.[12] Each experimental session lasted around 90 minutes.[13]

4.1.1 Part 1 - The Case of a Single Agent: Optimal Control

The instructions explained the evolution of the resource, the decision to be taken (a level of extraction) and its consequences on the level of the resource, the cost of extraction and the payoff. After time for a silent individual reading was given, an experimenter read the instructions aloud. Subjects then answered a computerized comprehension questionnaire to ensure that they understood the dynamics of the resource and the calculation of payoffs. Subjects were also allowed to ask questions if any clarification was needed. The experiment began with two identical and successive training phases, followed by a third phase for pay. Each phase lasted 5 minutes (300 seconds). The purpose of the training phases was to familiarize subjects with the graphical interface, the continuous evolution of variables (resource, extraction and payoff) and the graphs shown on the screen.

Before starting the countdown, subjects had to choose an initial level of extraction between 0 and 2.8, by moving a cursor on a graduated slider, which allowed values with two decimals. We chose these values in order to have a positive benefit, given the quadratic nature of our benefit function. Figure 10 in Appendix D shows a concave benefit curve, where the maximum benefit is reached for an extraction rate of 1.4. The figure also shows the unitary cost function, which decreases as the level of the groundwater increases and becomes equal to zero as soon as the level of the groundwater reaches the steady state level 20.

Once the initial extraction was chosen, a new screen appeared and subjects were able to see the level of the resource and their payoff, which included the cumulative and continuation payoffs, all updated every second.[14] They also had the possibility to modify their extraction level at any moment by simply moving the cursor. The extraction level was updated as soon as the cursor was released and the new value was then used for the calculations (resource and payoff). Subjects had real-time information in graphical and textual form, which was updated every second. At the graphical level, a curve at the top left of the screen showed the subject's extraction level; a curve at the bottom left displayed the dynamics of the resource and a curve at the top right showed the subject's payoff for the part. At the bottom right of the screen the same information was displayed in text form. The start and finish of each phase were also synchronized, i.e., all the subjects in the room started and finished at the same time. A screenshot of the user's interface is given in Figure 11 in Appendix D.

[12] ECU means Experimental Currency Unit.

[13] The infinite horizon in our experiment was not implemented through the time of the experiment but rather through payoffs. For more details about infinite horizon, refer to subsection 4.2, paragraph "Infinite horizon".

[14] More details on the computation of the payoff is given at subsection 4.2, "Infinite horizon".

4.1.2 Part 2 - The Case of Multiple Agents: Game

In this part, new instructions were given to subjects, specifying that the environment remained the same as in Part 1, except that instead of extracting the resource individually they would now be doing so in pairs. This part also included two identical and successive training phases, followed by a third phase for pay. It was also common knowledge that the pairs were randomly reformed after each phase. The screen, given by Figure 12 in Appendix D was identical to that of Part 1, except that in the top left graphic two additional curves showed the extraction of the other player and the total extraction of the pair.

4.2 Continuous Time and Infinite Horizon

Continuous time

There are two main ways of implementing continuous time in the lab: via extensive form games and via differential games. Given the difficulty of having pure continuous time using differential games, most of the continuous time lab experimentations used extensive form games. In fact, continuous time extensive form games are defined by Simon & Stinchcombe (1989) as, “A discrete time model with an infinitely fine grid”. Therefore, articles using the method of Simon & Stinchcombe are referred to as quasi-continuous time articles (Friedman & Oprea, 2012; Oprea et al., 2014; Bigoni et al., 2015; Leng et al., 2018).[15]

In this article, we use differential games, which are dynamic games in continuous time. The implementation in the laboratory is very challenging because two important aspects should be taken into account. The first is that subjects were allowed to change their extraction level whenever they wished instead of per period. Secondly, their decisions applied continuously. More precisely, subjects moved a graduated slider to make their choice whenever they wanted, and once the slider was released, the level of extraction was sent to the server, which set the new extraction level to the chosen value. The time step in the experiment was set to one second, meaning that all information was updated every second.

Infinite horizon

We found two ways of implementing infinite horizon in lab experimentation. The first was to impose an exogenous probability of termination of the decision making round, or in other words, a random end, so that subjects do not know exactly when the experi-

[15]For Simon & Stinchcombe (1989), "When restricted to an arbitrary, increasingly fine sequence of discrete-time grids, any profile of strategies drawn from this class generates a convergent sequence of outcomes, whose limit is independent of the sequence of grids." The authors consider this limit to be a class of continuous time strategy.

mentation will end (Suter et al., 2012). However, this technique implies a different end for each player and is not necessarily interpreted as an infinite horizon but rather as an unknown end. The second way is like Tasneem et al. (2017, 2019) to add a continuation payoff, which computes the payoff subjects would have obtained if the experimentation were pursued indefinitely, supposing that the last action remains constant.

We followed the same procedure as these authors, using the mechanism of "scrap value", applied every second. The payoff in our experiment is then composed of two elements: a cumulative payoff from the first instant to the current one, and a continuation payoff from the current instant to infinity. To compute the later we supposed that the player keeps his extraction level unchanged. More precisely, for a given time t, the computer computes the cumulative payoff till $t = p$ and adds a continuation payoff, starting from $t = p$ until infinity, by assuming that the player's extraction remains at the same level.

4.3 Experimental settings

Table 1 reports the parameters used in both the theoretical model and the experiment, which have been determined by taking into account theoretical and experimental constraints. First, the speed of convergence to the steady state had to be reasonable, neither too short – a few seconds – nor too long – several minutes. In fact, the steady state can be interpreted as a static framework, which simplifies the experimentation and allows subjects to stabilize their extraction rate and pay attention to the sustainability of the resource. Given the infinite horizon, this required to set a small discount rate r.

Variable	Description	Value
a	Extraction parameter	2.5
b	Extraction parameter	1.8
c_0	Maximum average cost	2
c_1	Variable cost	0.1
$c_0 - c_1 H$	Marginal or unitary cost	$2 - 0.1H$
r	Discount rate	0.005
R	Natural recharge (rain)	0.56
α	Return flow coefficient	1
H_0	Initial resource level	15

Table 1 – Parameters for the experiment

Second, as the steady state extraction rate is the same for all types of behavior, we wanted to have a clear difference in the paths leading to the steady state groundwater level of social optimum, feedback and constrained myopic behaviors. More precisely, we chose these parameters to obtain a steady state of the social optimum leading to a high level of groundwater, while lowering the level of groundwater for the Nash feedback and constrained myopic. Third, for simplification we set α, the return flow coefficient equal to 1 and the natural recharge R a little smaller, to avoid floods and highlight the renewable nature of the resource.[16] Figures (1) and (2) below give the theoretical time path for the different types of behavior, both for the optimal control and for the game, according to the chosen parameters.

Finally, we established a rule for the possible situation where the extraction level chosen by players exceeds the available resource. In that case, we decided to force the extraction level to zero until either the quantity of the resource increases enough for this extraction or until players change their decisions.

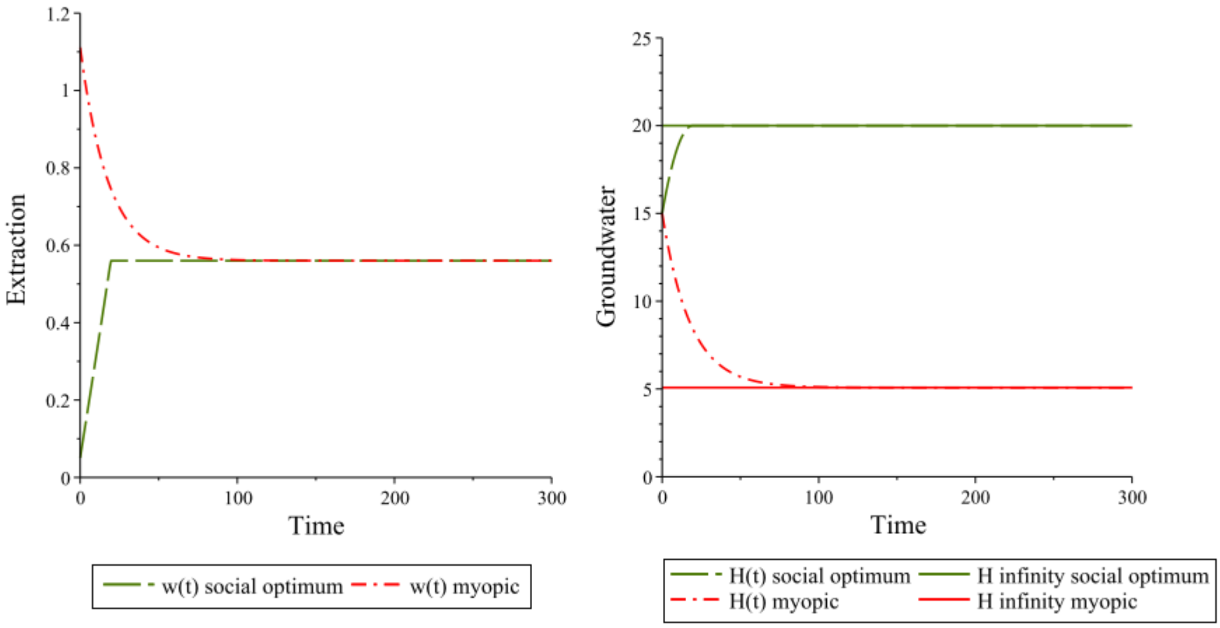

Figure 1 – Social optimum and myopic extraction and groundwater level for the optimal control

[16]The return flow coefficient is the quantity of water returning to the groundwater after each extraction.

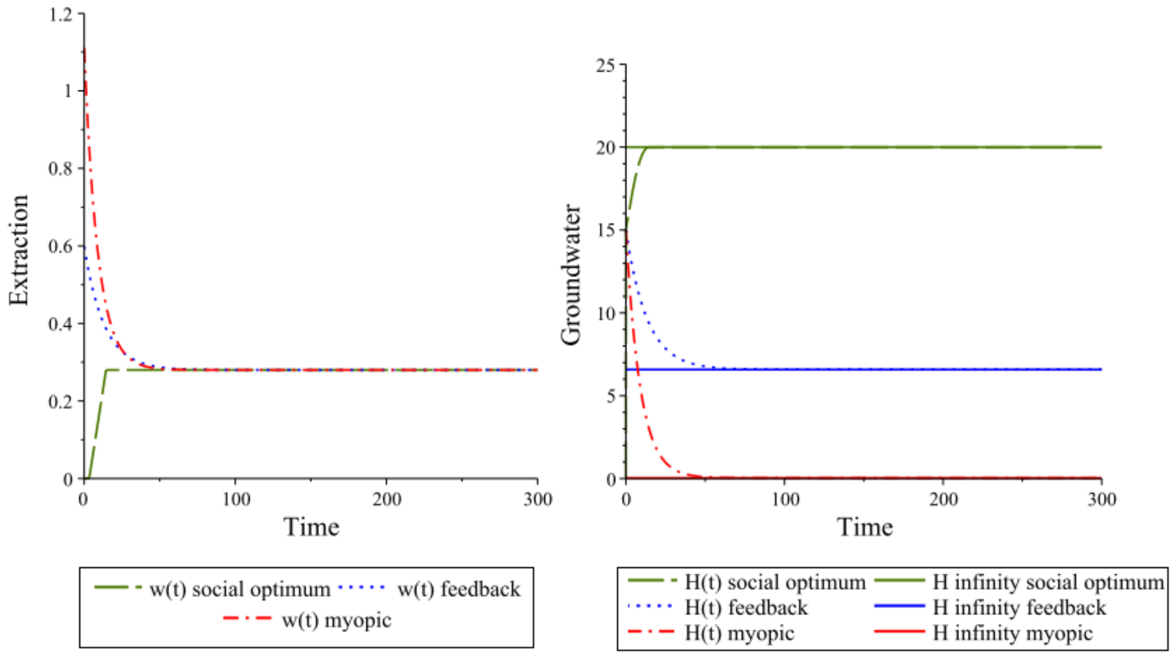

Figure 2 – Social optimum, feedback and myopic extraction and groundwater level for the game

4.4 Predictions

The experiment aims to determine which type of behavior is the most commonly exhibited by subjects, given the environment (alone or in a group) in which they make their extraction decisions. Considering the experimental setting, as well as learning and mimicry effects that can occur, we formulate four main predictions.

Prediction* 1*: *The externality implied by the mutliplicity of extractors increases the number of myopic agents in the game.*

Most of the literature on common pool resources favors the emergence of myopic behavior when strategic interactions among players occur. Potential explanation is the fact that they engage in a race for the resource in order to exploit it at a lower cost while the resource level is still high (Herr et al., 1997; Leng et al., 2018; Tasneem et al., 2019).

Prediction* 2*: *An agent who behaved optimally in the optimal control is more likely to behave optimally in the game.*

To formulate this prediction, we relied on the analytical capabilities of individuals. We believe that having successfully understood the principle of the experiment by preserving the resource and ensuring high payoffs, the optimal agent will be more inclined to maintain the same tendency when playing in a group, by trying to anticipate the way to proceed in response to the behavior of the other player(s) in the group.

Prediction 3***:*** *When two agents who behaved optimally in the optimal control play together, they have a greater chance of again adopting optimal behavior in the game.*

This prediction supposes a learning effect, because subjects who have individually understood the problem when playing alone, with only a cost externality (costs negatively depend on the level of the resource), will be better able to manage the additional strategic interaction. Indeed, they are more likely, at the steady state, to take half the extraction they performed when they played alone, or to ensure that the sum of their extractions is equivalent at the steady state to the extraction they performed when they played alone, compared to an agent that was not able to determine the optimal extraction level when he played alone.

Prediction 4***:*** *When paired with agents that behaved optimally in the optimal control, other types of agents are more likely to behave optimally in the game.*

The idea is that, by being confronted with individuals who optimally manage the resource, other types of individuals may be positively influenced. Thus, by a mimicry effect they will be more likely to follow the behavior of optimal agents.

5 Empirical Strategy

70 agents participated in the one player (optimal control) and the multiple players (game) experiments. They took extraction decisions for 300 seconds in each part. Using the extraction decisions data, we intend to determine whether these agents demonstrated myopic or optimal behavior (or feedback behavior in the game). We start by examining the behavior of agents in the optimal control experiment because we first want to identify an agent's type without strategic interactions.

To identify which theoretical extraction pattern an agent's extraction comes closest to, common practice in experimental economics is to compute the mean squared deviations (MSD), e.g., Herr et al. (1997). The minimum MSD gives the agent's type. The MSDs are calculated for each individual such as:

$$\begin{aligned} MSD^{th}_{my} &= \frac{\sum_{t=1}^{T}\left(w(t) - w(t)^{th}_{my}\right)^2}{T} \\ MSD^{th}_{op} &= \frac{\sum_{t=1}^{T}\left(w(t) - w(t)^{th}_{op}\right)^2}{T} \end{aligned} \tag{10}$$

where $w(t)$ is the extraction of the agent at time t, $w(t)^{th}_{my}$ is the constrained myopic theoretical extraction at time t, and $w(t)^{th}_{op}$ is the optimal theoretical extraction at time t. In this case, agents would be classified as myopic or optimal, depending on which MSD, MSD^{th}_{my} or MSD^{th}_{op} is the smallest. Comparing extractions of the agent to the theoretical constrained myopic and optimal extraction in this way is imperfect since an individual can make mistakes and begin playing perfectly optimally after, say, 30 periods, while this will not be captured correctly by the method.

For instance, if an agent A under-extracts for the first 30 seconds, the optimal extraction at time 31, given the observed groundwater level H (called conditional, $w(31)^{c}_{op}$) will be greater than the optimal extraction at time 31 if the agent behaved perfectly optimally since time 0 ($w(31)^{th}_{op}$). Thus, in order to correctly identify an agent's behavior type - myopic or optimal -, for the rest of the paper, we compare observed extraction to conditional extractions. Conditional extractions are computed with respect to the $t-1$ groundwater level. The conditional groundwater level H^c is also computed, using an approximation involving the observed $t-1$ groundwater level in the experiment, the natural recharge, and the conditional extraction. Thus, we are interested in the following MSDs:

$$\begin{aligned} MSD^{c}_{my} &= \frac{\sum_{t=1}^{T}\left(w(t)-w(t)^{c}_{my}\right)^2}{T} \\ MSD^{c}_{op} &= \frac{\sum_{t=1}^{T}\left(w(t)-w(t)^{c}_{op}\right)^2}{T}, \end{aligned} \tag{11}$$

where $w(t)^{c}_{my}$ is the conditional constrained myopic extraction of the agent at each second, and $w(t)^{c}_{op}$ is the conditional optimal extraction of the agent at each second. Agents are classified as myopic or optimal depending on which MSD, MSD^{c}_{my} or MSD^{c}_{op} is the smallest.

The discommoding feature of a classification of agents based on the MSD alone, is that an agent will always be classified, even if he doesn't follow the theoretical pattern studied at all.[17] To overcome this flaw, we add a second criteria based on regression analysis. Supposing that for a given agent A, we have:

$$\begin{aligned} &w(t)^{c}_{my} < w(t)^{c}_{op}, \quad or \\ &w(t)^{c}_{my} > w(t)^{c}_{op}, \end{aligned} \tag{12}$$

then we run the following regression:

[17] To take a concrete example, instead of comparing the agent's extraction $w(t)$ to the conditional constrained myopic and conditional optimal extraction, $w(t)^{c}_{my}$ and $w(t)^{c}_{op}$, we could compare it to the temperature in Moscow and Istanbul, and we would find that our agent's extraction is closer to the temperature in Moscow or in Istanbul, because one MSD will always be smaller than the other, even if completely irrelevant.

$$
\begin{aligned}
w(t) &= \beta_0 + \beta_1 w(t)^c_{my} + \varepsilon_t, \quad or \\
w(t) &= \beta_0 + \beta_1 w(t)^c_{op} + \varepsilon_t.
\end{aligned} \tag{13}
$$

We consider an agent to be significantly myopic (or optimal) if β_1 is positive and significantly different from 0. This allows us to categorize the agents as: myopic, optimal, or undetermined.[18] Regarding the econometric time series treatments, we implement an augmented Dickey-Fuller test to detect the presence of unit roots in the series. In case of non-stationarity of the variables, we run our regressions on differentiated series. Serial correlation of the error terms is dealt with using Newey-West standard errors, and sensitivity tests using 1, 5, and 10 lags are implemented.[19] Finally, we follow exactly the same strategy to analyze experimental data for the game, but this time for the three instead of for the two predicted behaviors, namely: myopic, optimal, and feedback. An example of application of the methodology is given in Appendix E.

6 Results

6.1 The Case of a Single Agent: Optimal Control

Classifying agents in the optimal control experiment using the MSD leads us to find 65 optimal agents and 5 myopic agents. Figure 3 presents the location of agents with respect to the conditional optimal MSD (MSD^c_{op}) and the conditional constrained myopic MSD (MSD^c_{my}). The y axis on the figure shows MSD^c_{op}, while the x axis shows MSD^c_{my}.

[18] An alternative is proposed by Suter et al. (2012), who run a similar regression (without the constant term) and consider that an agent follows a given behavior if the coefficient is not significantly different from 1. A natural way to do this is to implement a Wald test with:

$$
\begin{cases} H_0 : \beta_1 = 1 \\ H_A : \beta_1 \neq 1, \end{cases} \quad and \;\; W = \frac{(\hat{\beta}_1 - 1)^2}{var(\hat{\beta}_1)} \rightarrow F_{(1,300)}
$$

In this case, a very imprecisely estimated coefficient β_1 (very large $var(\hat{\beta}_1)$) will lead us to reject H_A and classify the agent as myopic or optimal, while he follows neither an optimal or myopic path. This is the reason why we propose the aforementioned alternative rule for classification.

[19] We present regression results using 5 lags. Results using 1 and 10 lags are available upon request.

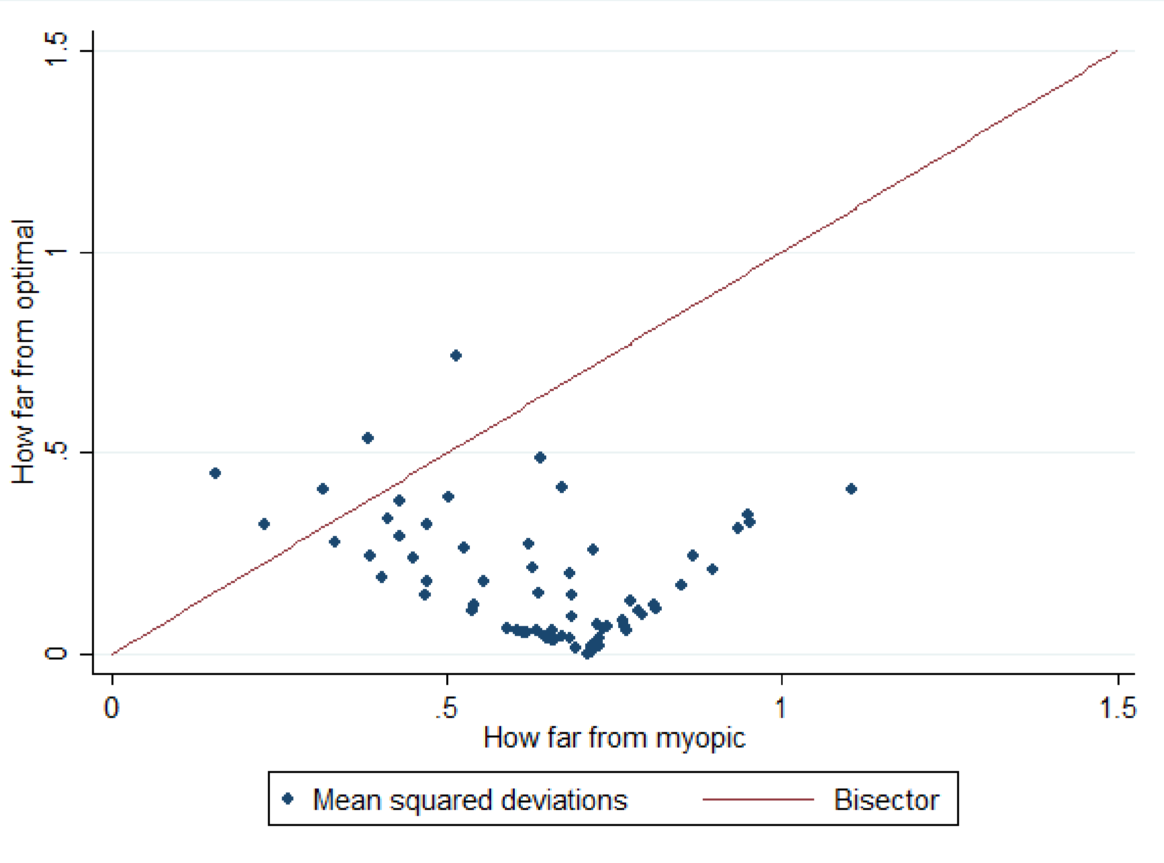

Figure 3 – MSD position of agents in the optimal control

Agents located above the bisector are considered as myopic ($MSD^c_{op} > MSD^c_{my}$) and reversely. Thus, according to the MSD criteria, 65 agents should be classified as optimal and 5 as myopic. As we explained in Section 5, this simple criteria is unsatisfactory, because we want to know if agents are significantly optimal or myopic. Applying the regression filter presented in the previous section leads us to find that 31 agents can be classified as significantly optimal and the rest are undetermined, meaning that 39 undetermined agents would have been incorrectly classified without the regression filter we propose. Figure 4 represents the 31 optimal agents. Compared to Figure 3, we can see that significantly optimal agents tend to be those with a small MSD^c_{op} and a relatively large MSD^c_{my}, which supports the accuracy of our empirical strategy.

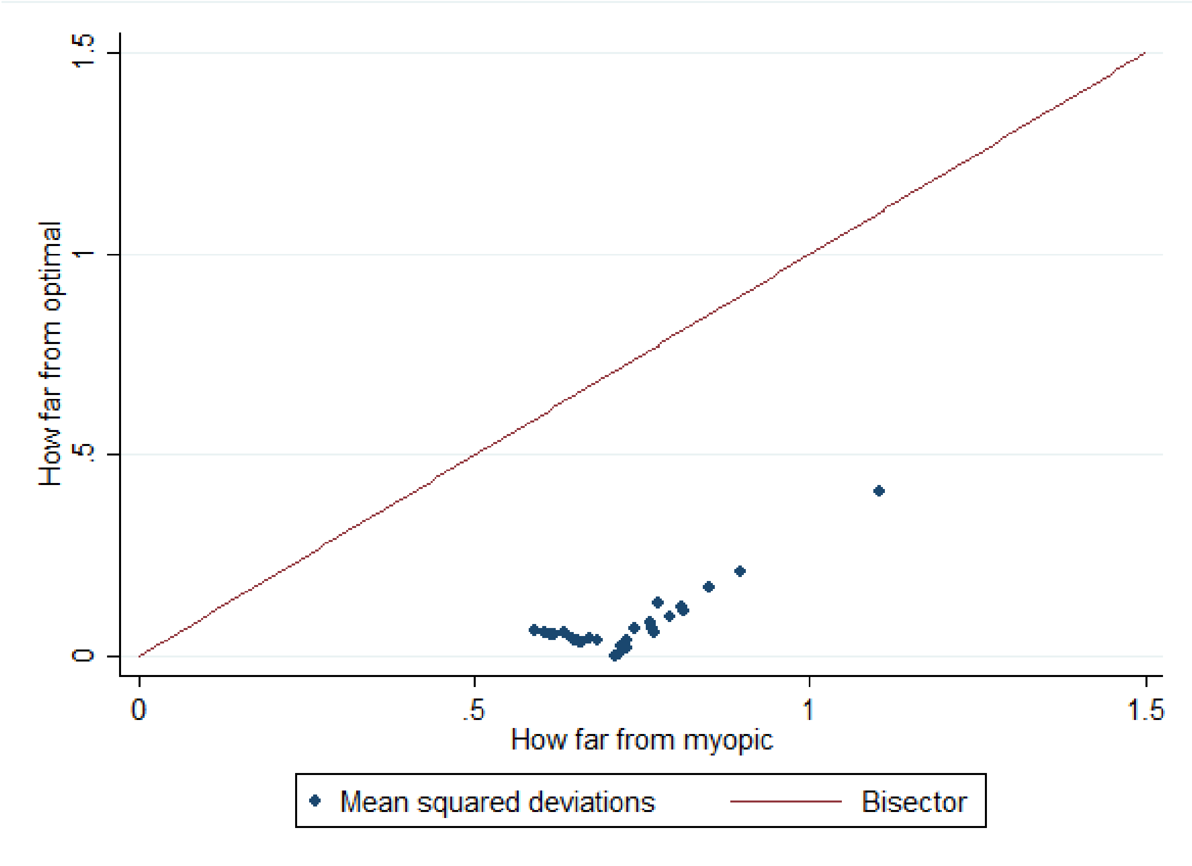

Figure 4 – MSD position of optimal agents in the optimal control

To summarize, almost half of the studied agents are able to optimally manage the resource when there is no externality related to the extraction of additional agents. We observe no myopic behavior that would have led to a depletion of the groundwater.

6.2 The Case of Multiple Agents: Game

In the game, as can be seen in Figure 5, agents are closer to one or the other conditional extraction path (less agents high in the diagonal), i.e., they have either a relatively small MSD^c_{op} or a relatively small MSD^c_{my}. The MSD classification indicates that we are observing 49 optimal, 3 myopic and 18 feedback agents. However, applying the regression filter leaves us with 26 optimal, 2 myopic, 4 feedback, and 38 undetermined agents. Again, notice that 38 agents would have been incorrectly classified without the regression filter we propose.

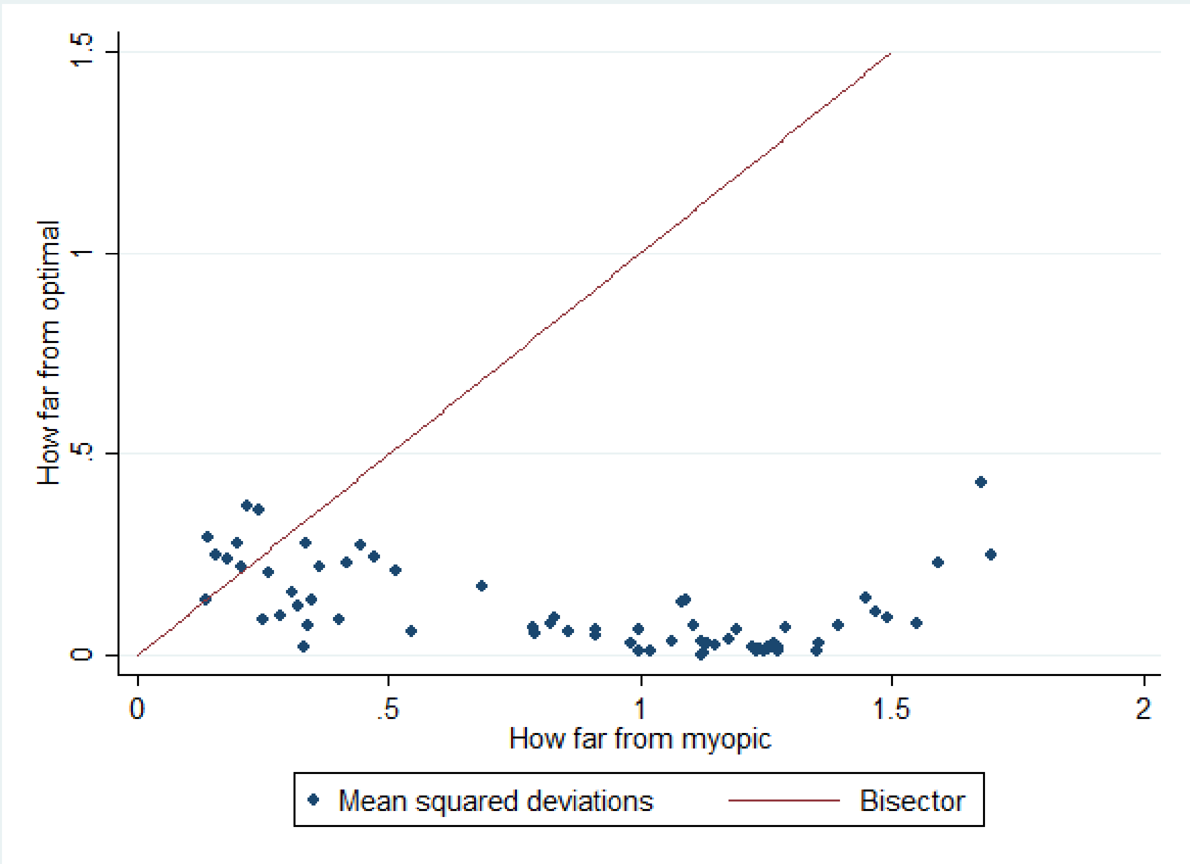

Figure 5 – MSD position of agents in the game

According to **prediction 1**, the fact that agents must now coordinate to optimally extract the groundwater should result in more myopic behavior. The presence of two myopic agents is bad news for the commons, since it moves towards an extinction of the resource and the tragedy of the commons. However, we do not observe the dramatic surge in the number of myopic agents we might have expected, due to coordination failures and the negative externality from the extraction of the other agent. A possible explanation is the fact that agents had the chance to manage the resource alone before playing the game, which allowed them to understand the optimal management of the resource. Finally, notice that we still observe 26 agents behaving optimally in the game, even in the presence of strategic interactions, which is good news for the sustainability of the resource.

Figure 6 and 7 shows, in line with ***prediction 2***, that the probability of being classified as optimal in the game is higher when the agent was optimal in the optimal control. Indeed, 61% of the agents classified as optimal in the game were classified as optimal in the optimal control while only 18% of the undetermined agents in the optimal control became optimal in the game.[20] Moreover, Figure 6 shows, in line with

[20]Percentages are calculated using the information presented in Figure 6 and 7: $\frac{9+10}{31} * 100 = 61\%$, $\frac{3+4}{39} * 100 = 18\%$.

our ***prediction 3***, that the type of pair that exhibits the highest share of optimal behavior in the game is the optimal-optimal pair (65% of the 14 agents that were paired with another optimal agent remained optimal in the game).

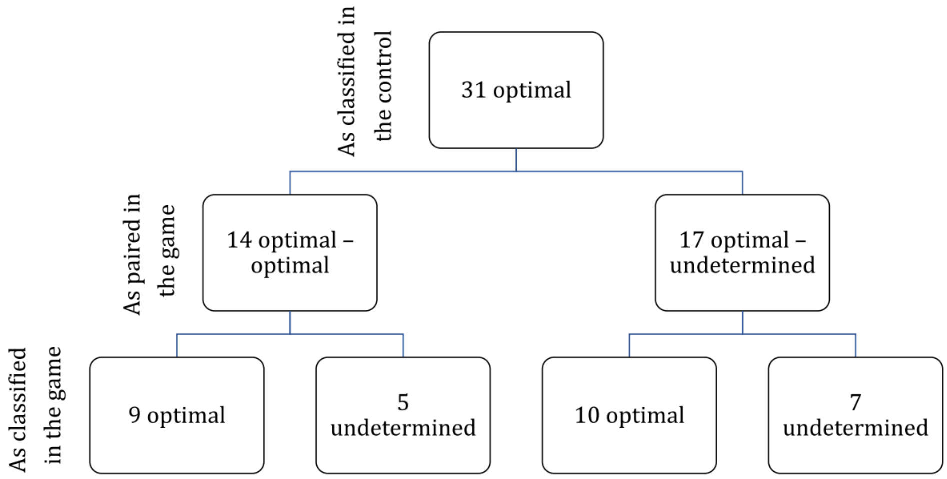

Figure 6 – The becoming of optimal agents in the game

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Prediction 4 isn't contradicted by our data, as we note that an undetermined agent in the optimal control is more likely to become optimal in the game if he plays with an optimal agent. As shown by Figure 7, 24% of the undetermined agents that were paired with an optimal agent became optimal in the game, while only 14% of the undetermined agents that were paired with another undetermined agent became optimal in the game. Moreover, we note that other types of behavior, such as feedback and myopic, appear only to a small extent in the case of a match between two undetermined agents. For instance, the share of agents classified as myopic in the game when the two agents were undetermined in the optimal control is 9%.

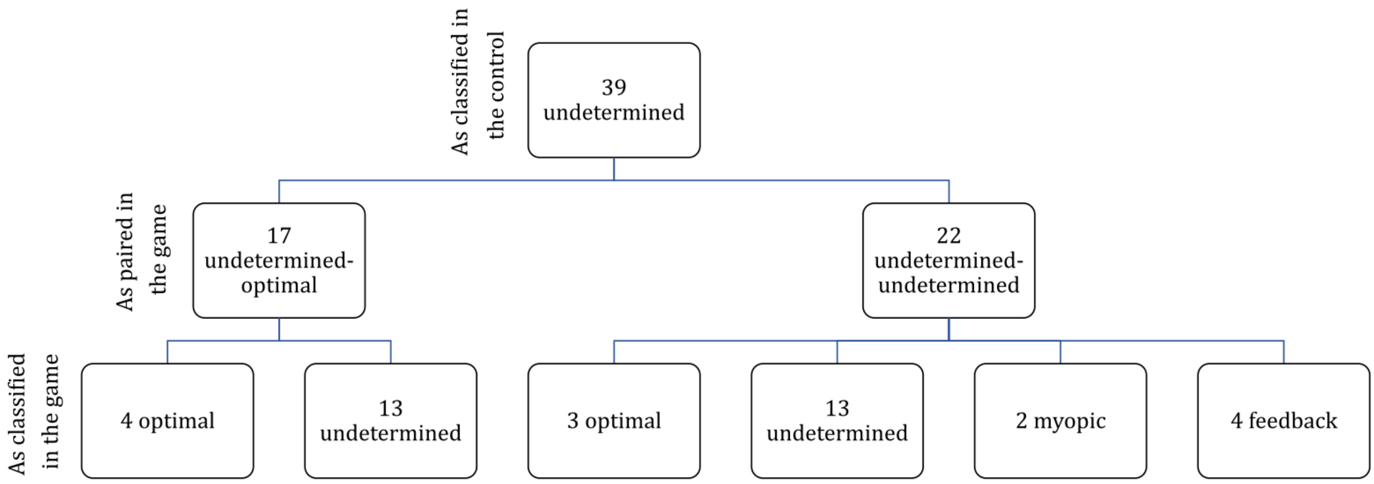

Figure 7 – The becoming of undetermined agents in the game

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6.3 Efficiency Analysis

To conclude our results analysis, it is interesting to look at the efficiency of the observed extraction patterns. To do this, we follow a very common procedure in the literature (Herr et al., 1997; Suter et al., 2012; Noussair et al., 2015; Tasneem et al., 2017), and compute efficiency as the ratio of an agent's observed payoff to the payoff that would have been achieved by a perfectly optimal agent (215 ECU).[23]

In the control case, two main observations can be made by looking at Figure 8. First, gains are heterogeneous. Second, the 31 optimal agents, in green, obtained higher payoffs on average than undetermined agents, which again confirms the validity of the empirical strategy we have proposed. Figure 8 displays a distribution of efficiency very similar to Tasneem et al. (2019). Moreover, average efficiency in Tasneem et al. (2019) is 85% while ours is 83%. We observe a smaller efficiency than in the optimal control of Suter et al. (2012)'s experiment (83% versus 95%). A possible explanation is that they implemented a discrete time experiment, which is easier to understand for the agents, while this paper and that of Tasneem *et al.* implement an experiment mimicking continuous time.

[23]Remember that the total payoff is the sum of the discounted payoff at each second, plus the infinite payoff that depends on the last extraction (see Section 4.2).

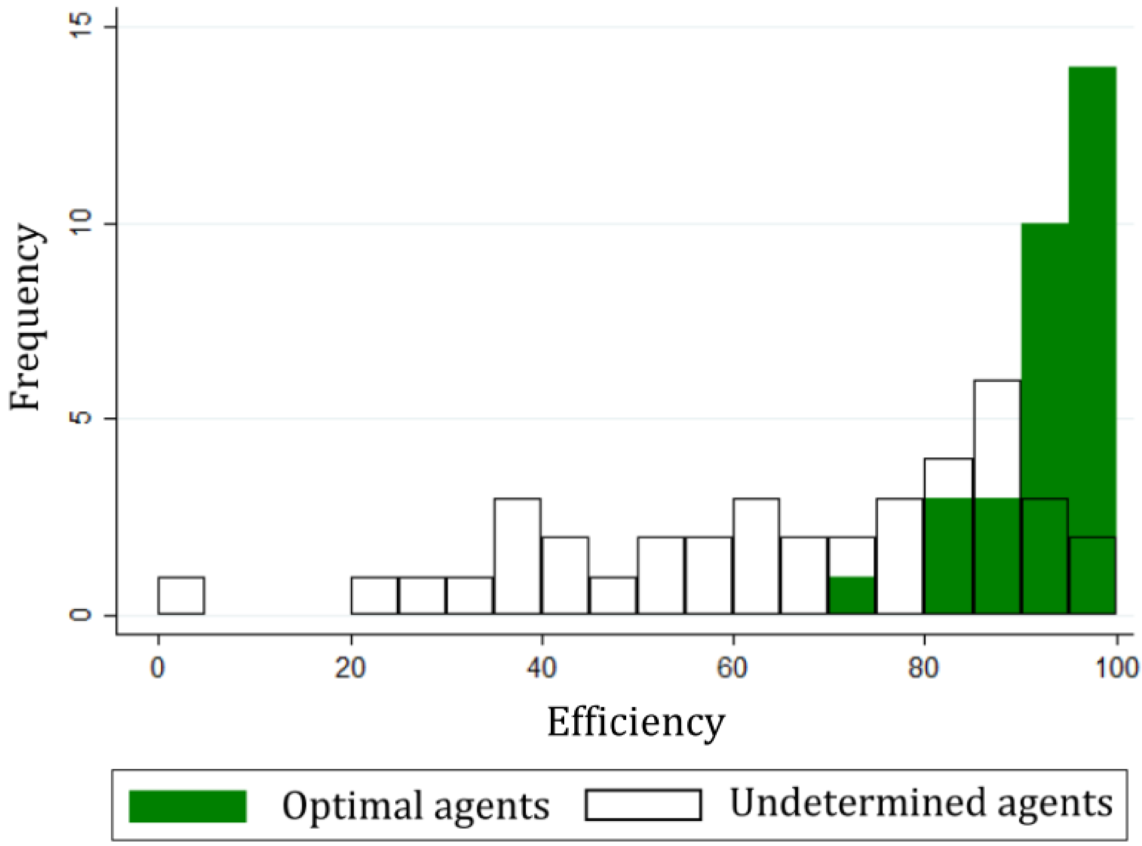

Figure 8 – Efficiency of individual payoffs in the optimal control

Regarding the game, the maximum payoff for each group is 240 ECU. We compare individual efficiency to 120 ECU, but it is possible to get "more than your own share". Obviously, if one of the two members of the pair extracts a very small amount of groundwater, the other member can obtain more than 50% of the total maximum benefits.[24]

Figure 9 shows, as expected, that optimal players get closer on average to their maximum payoffs.[25] However, as expected, due to the introduction of strategic interaction and the coordination problems that come along with it, the distribution of efficiency is more heterogeneous than in the optimal control and lower on average, as also found by Suter et al. (2012) (see standard deviations in the descriptive statistics provided in Appendix F).

[24] Descriptive statistics on the efficiency ratio by behavior type are available in Appendix F.

[25] It could be interesting to look at other categories, but it would be difficult to draw conclusions because of the small number of players classified as myopic and feedback.

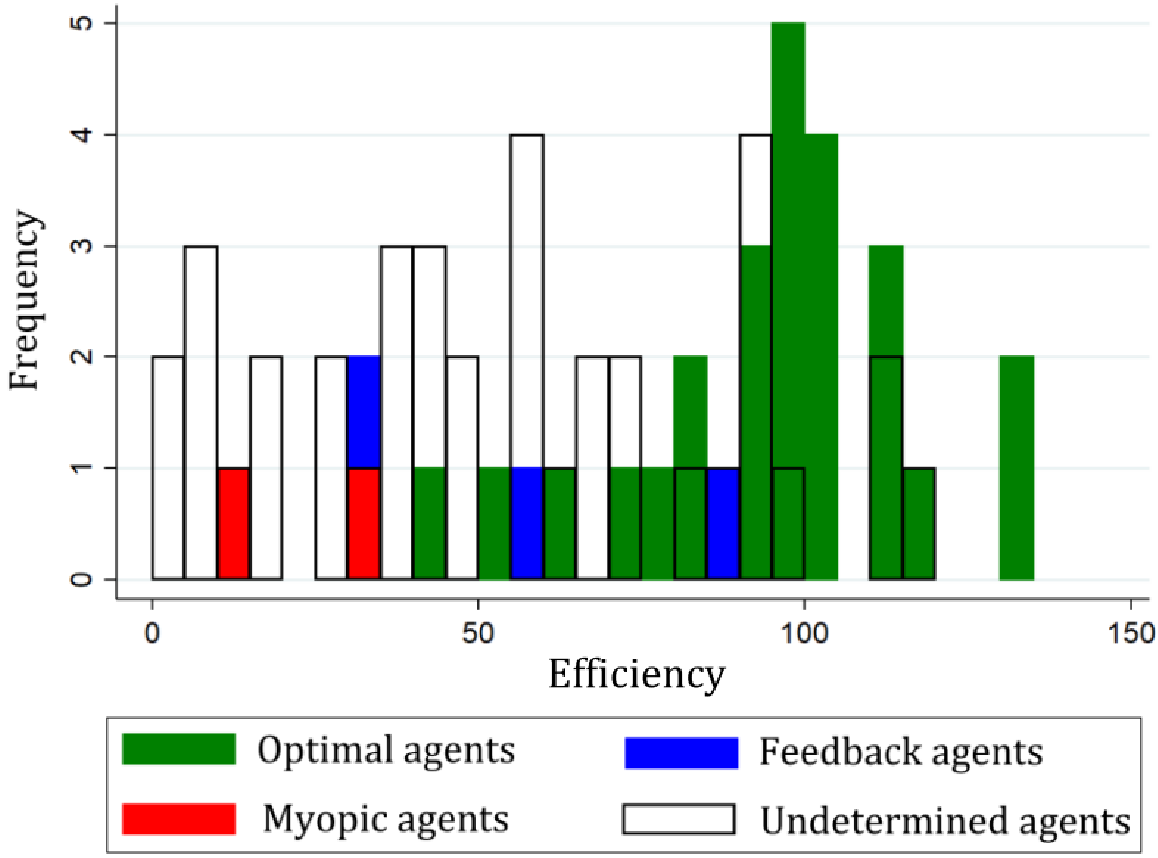

Figure 9 – Efficiency of individual payoffs in the game

7 Conclusion

In this paper, we start from a simple dynamic groundwater model to create a benchmark, that theoretically, experimentally, and econometrically, will be able to meet several challenges faced by the existing literature. To our knowledge, this is the first experimental study on differential games comparing the behavior of experimental subjects according to theoretical predictions.

Our theoretical model emphasizes the importance of the cost function, providing great details on the implications of the positivity of the marginal cost. This constraint led us to use a "LambertW" specification in the feedback solution's derivation. The model also allows to deduce different resource dynamics, depending on the type of behavior: social optimum, feedback and constrained myopic. The experiment takes the challenge of implementing continuous time and infinite horizon by allowing participants to make extraction decisions at any time they wish, and using payoffs to simulate infinite horizon. All the data are updated every second and continuously shown to participants in graphical and text form. To ensure that subjects understand the underlying mechanism well, they play alone in a first part before playing in groups of two in a second part.

The empirical analysis of extraction behaviors combines Mean Squared deviation using conditional extractions and linear regressions to correctly classify participants' behavior. We find that about half of the participants adopt an optimal extraction path both in the optimal control and in the game. We also find two agents who display a myopic-type behavior and four who exhibit a feedback-type behavior. However, the rest of the participants follow behaviors that cannot be described by the three types of behavior characterized in most of the theoretic literature. The combination of linear regressions in addition to the MSD traditionally computed helps in discriminating between real optimal behavior and undetermined behavior. Avoiding misclassification is important when trying to predict the impact of public intervention in the extraction sector.

We hope our theoretical model, the solutions to experimental challenges and the empirical strategy implemented can serve as a benchmark for more complex frameworks in the study of dynamic common pool resources.

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Appendices

A The Single Agent Problem: Social Optimum Solution

To prove theorem 1, we first prove that under condition 1 it is not possible to have a steady state other than $\frac{c_0}{c_1}$. To do this, we separately consider the case where the optimal solution lies in the regime with the level of the groundwater, H, smaller than $\frac{c_0}{c_1}$ and the case with H greater than $\frac{c_0}{c_1}$. The two regimes are differentiated by the cost function.

Proposition 1 *: When $H(t) < \frac{c_0}{c_1}$ for all t, the steady state of the following problem*

$$\max_{w} \int_0^\infty e^{-rt} \left[aw - \frac{b}{2}w^2 - (c_0 - c_1 H)w \right] dt, \tag{14}$$

s.t

$$\begin{cases} \dot{H} = R - \alpha w, \\ H(0) = H_0 \end{cases}$$

is

$$H^\infty = \frac{R\alpha c_1 + Rbr - a\alpha r + \alpha c_0 r}{\alpha c_1 r}, \quad w^\infty = \frac{R}{\alpha}$$

Proof 1 *: The associated Hamiltonian is:*

$$Hamiltonian = aw - \frac{b}{2}w^2 - (c_0 - c_1 H)w + \lambda(R - \alpha w),$$

where λ is the adjoint variable and the result is given by first order conditions at the steady state.

Furthermore, this steady state cannot be a steady state of our problem because by condition 1 it is greater than $\frac{c_0}{c_1}$.

Proposition 2 *: There is no steady state in the regime $H(t) > \frac{c_0}{c_1}$*

Proof 2 *: Suppose a solution with $H(t) > \frac{c_0}{c_1}$ for all t. The maximization problem is:*

$$\max_{w} \int_0^\infty e^{-rt} \left[aw - \frac{b}{2}w^2 \right] dt, \tag{15}$$

s.t

$$\begin{cases} \dot{H} = R - \alpha w, \\ H(0) = H_0 \end{cases}$$

The associated Hamiltonian:

$$Hamiltonian = aw - \frac{b}{2}w^2 + \lambda(R - \alpha w),$$

where λ is the adjoint variable, gives by first order conditions:

$$w(t) = \frac{a - \alpha\lambda_0 e^{rt}}{b}$$

It is not possible to maintain the groundwater greater than $\frac{c_0}{c_1}$ if $\lambda_0 \leq 0$. Note that if $\lambda_0 = 0$, condition 1 gives $\dot{H} < 0$. It is not possible to have $w \geq 0$ if $\lambda_0 > 0$.

These two propositions show that the steady state of the optimal problem is:

$$H_{op}^{\infty} = \frac{c_0}{c_1}, \quad w_{op}^{\infty} = \frac{R}{\alpha}$$

Now to obtain the complete path we must solve first order conditions considering the Hamiltonian of the problem and taking into account the constraints.

For $H_0 < \frac{c_0}{c_1}$, the Lagrangian of the problem is:

$$L = aw - \frac{b}{2}w^2 - (c_0 - c_1 H)w + \lambda(R - \alpha w) + \mu\left(\frac{c_0}{c_1} - H\right) + \nu w, \tag{16}$$

where λ is the adjoint variable and μ and ν the Lagrange multipliers associated to the constraints $H \leq \frac{c_0}{c_1}$ and $w \geq 0$, respectively.

For $H_0 > \frac{c_0}{c_1}$, the Lagrangian of the problem is:

$$L = aw - \frac{b}{2}w^2 + \lambda(R - \alpha w) + \mu\left(H - \frac{c_0}{c_1}\right) \tag{17}$$

The time of change of regime is obtained using the continuity of the adjoint variable, the state variable and the control variable.

B The Single Agent Problem: Constrained Myopic Solution

Considering the different possibilities for H ($H <>= \frac{c_0}{c_1}$), we obtain the constrained myopic extraction. We can see that if $H < \frac{c_0}{c_1}$, the resolution of the differential equation gives:

$$H(t) = H_{my}^{\infty} + (H_0 - H_{my}^{\infty})e^{-\frac{\alpha c_1}{b}t}, \tag{18}$$

with the steady state that is:

$$0 < H_{my}^{\infty} = \frac{b}{c_1}\left(\frac{R}{\alpha} - \frac{a - c_0}{b}\right) < \frac{c_0}{c_1}$$

by conditions 1 and 2. However, if $H > \frac{c_0}{c_1}$, as extraction is $\frac{a}{b}$, condition 1 implies that $\dot{H} < 0$ and then, in a finite time, the trajectory enters the regime where $H < \frac{c_0}{c_1}$ and the reasoning for that regime applies.

C The game

Proof of theorems 3 and 5 are similar to the case of a single agent. To prove theorem 4 we consider cases where $H_0 \leq \dfrac{c_0}{c_1}$ and $H_0 > \dfrac{c_0}{c_1}$.

When $H_0 \leq \dfrac{c_0}{c_1}$, condition 3 guarantees the positivity of the extraction path for all t and that the Nash feedback trajectory remains in the region where $H < \dfrac{c_0}{c_1}$. The Nash equilibrium can be found by solving the Hamilton–Jacobi–Bellman (HJB) equation:

$$rV^i_{R_1}(H) = max_{w_i}\left[(aw_i - \frac{b}{2}w_i^2 - (c_0 - c_1 H)w_i - (V^i_{R_1})'(H)(R - \alpha(w_i + w_j(H))\right]$$

By using the guessing method to guess a quadratic value function and a linear strategy, one can easily find the Nash feedback equilibrium. Thus, proposing:

$$\begin{cases} V^i_{R_1}(H) &= A_1 + A_2 H + \dfrac{A_3}{2}H^2 \\ w_j(H) &= a_i H + b_i \end{cases}$$

One can find A_1, A_2, A_3, a_i, b_i, where A_3 is obtained by solving the following equation:

$$-\frac{3\alpha^2}{2b}A_3^2 + \frac{rb + 4c_1\alpha}{2b}A_3 - \frac{c_1^2}{2b} = 0, \tag{19}$$

with the condition $-c_1 + \alpha A_3 < 0$, and we have:

$$\begin{cases} a_1 &= \dfrac{c_1 - \alpha A_3}{b}, \\ \\ b_1 &= \dfrac{a - c_0 - \alpha A_2}{b}, \\ \\ A_2 &= \dfrac{(a - c_0)(-c_1 + 2\alpha A_3) - RbA_3}{-rb - 2c_1\alpha + 3A_3\alpha^2}, \\ \\ A_1 &= \dfrac{3\alpha^2 A_2^2 + 2Rb - 4\alpha(a - c_0)A_2 + (a - c_0)^2}{2br} \end{cases}$$

The evolution of the water table for H_0 is also given by:

$$H(t) = e^{\dfrac{2\alpha(-c_1 + \alpha A_3)\,t}{b}}\left(H_0 - H_f^\infty\right) + H_f^\infty, \quad H_f^\infty = \frac{Rb + 2\alpha^2 A_2 - 2\alpha(a - c_0)}{2\alpha(c_1 - \alpha A_3)}.$$

When $H_0 > \dfrac{c_0}{c_1}$ the problem is a bit different, because the following facts must be taken into account: first, there is no stationary steady state in the regime where $H > \dfrac{c_0}{c_1}$. As a consequence, the Nash feedback solution will decrease from this regime to the steady state H_f^{∞}. Second, our problem is an autonomous problem, thus the solution in this case is also the solution of an HJB equation of the form:

$$rV^i_{R_2}(H) = max_{w_i}\left[(aw_i - \frac{b}{2}w_i^2 - (V^i_{R_2})'(H)(R - \alpha(w_i + w_j(H))\right] \tag{20}$$

For the first point, the solution of this last HJB equation is constrained to the condition:

$$V^i_{R_2}(\frac{c_0}{c_1}) = V^i_{R_1}(\frac{c_0}{c_1}) \tag{21}$$

The first order condition for equation (20) gives:

$$w_i(H) = \frac{a - \alpha(V^i_{R_2})'(H)}{b} \tag{22}$$

Replacing (22) in equation (20) and taking into account that $w_j(H) = w_i(H)$, we obtain the following differential equation for $V^i_{R_2}(H)$:

$$V^i_{R_2}(H) = \frac{C}{2}\ [(V^i_{R_2})'(H)]^2 + B\ (V^i_{R_2})'(H) + A, \tag{23}$$

where,

$$\begin{cases} A &= \dfrac{a^2}{2br}, \\ B &= \dfrac{Rb - 2a}{br}, \\ C &= \dfrac{-\alpha^2 + 4\alpha}{br} \end{cases}$$

Differentiating (23) with respect to H, one must finally solve:

$$U(H) = B\ U'(H) + C\ U(H)U'(H), \quad with \quad U(H) = (V^i_{R_2})'(H) \tag{24}$$

The solution of equation (24) is given by:

$$U(H) = e^{-\dfrac{-H + BLambertW(x) - cte}{B}}, \quad x = \frac{Ce^{\frac{H}{B} + \frac{cte}{B}}}{B}, \tag{25}$$

where $LambertW$ is the Lambert W function and the constant cte is found using (21).

To finally prove the link between the steady states of the 3 types of behavior studied in the game, as shown by equation (9), condition 4 gives:

$$H_f^\infty = \frac{Rb + 2\alpha^2 A_2 - 2\alpha(a - c_0)}{2\alpha(c_1 - \alpha A_3)}$$

$$= \frac{Rb - 2\alpha(a - c_0)}{2\alpha c_1 - \alpha^2 A_3} + \frac{2\alpha^2 A_2}{2\alpha c_1 - \alpha^2 A_3}$$

$$= \left[\frac{Rb - 2\alpha(a - c_0)}{2\alpha c_1} \times \frac{1}{1 - \dfrac{\alpha A_3}{c_1}}\right] + \frac{2\alpha^2 A_2}{2\alpha(c_1 - \alpha A_3)}$$

$$H_f^\infty = \left[H_{my}^\infty \times \frac{1}{1 - \dfrac{\alpha A_3}{c_1}}\right] + \frac{2\alpha^2 A_2}{2\alpha(c_1 - \alpha A_3)}$$

Thanks to the condition $-c_1 + \alpha A_3 < 0$, we can deduce that $c_1 - \alpha A_3 > 0$. Moreover, we have $A_2 > A_3$, so that:

$$\frac{2\alpha^2 A_2}{2\alpha(c_1 - \alpha A_3)} > 0, \quad \textit{and} \quad \frac{2\alpha^2 A_2}{2\alpha(c_1 - \alpha A_3)} > \left[H_{my}^\infty \times \frac{1}{1 - \dfrac{\alpha A_3}{c_1}}\right]$$

Thus, one can say that $H_f^\infty > H_{my}^\infty$, and we can finally conclude that, by conditions 3 and 4:

$$H_{my}^\infty < H_f^\infty < H_{op}^\infty \tag{26}$$

D Figures in the experiment

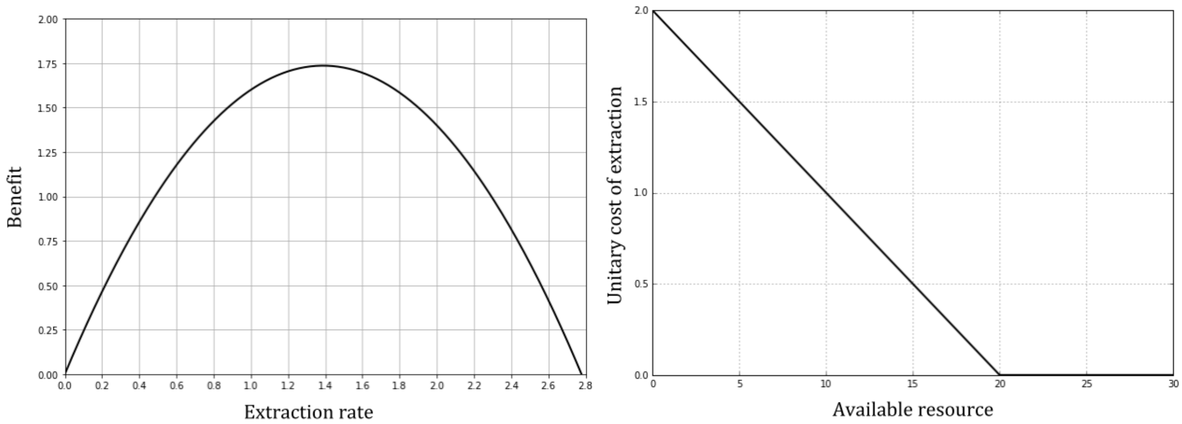

Figure 10 – Farmer's benefit and cost functions

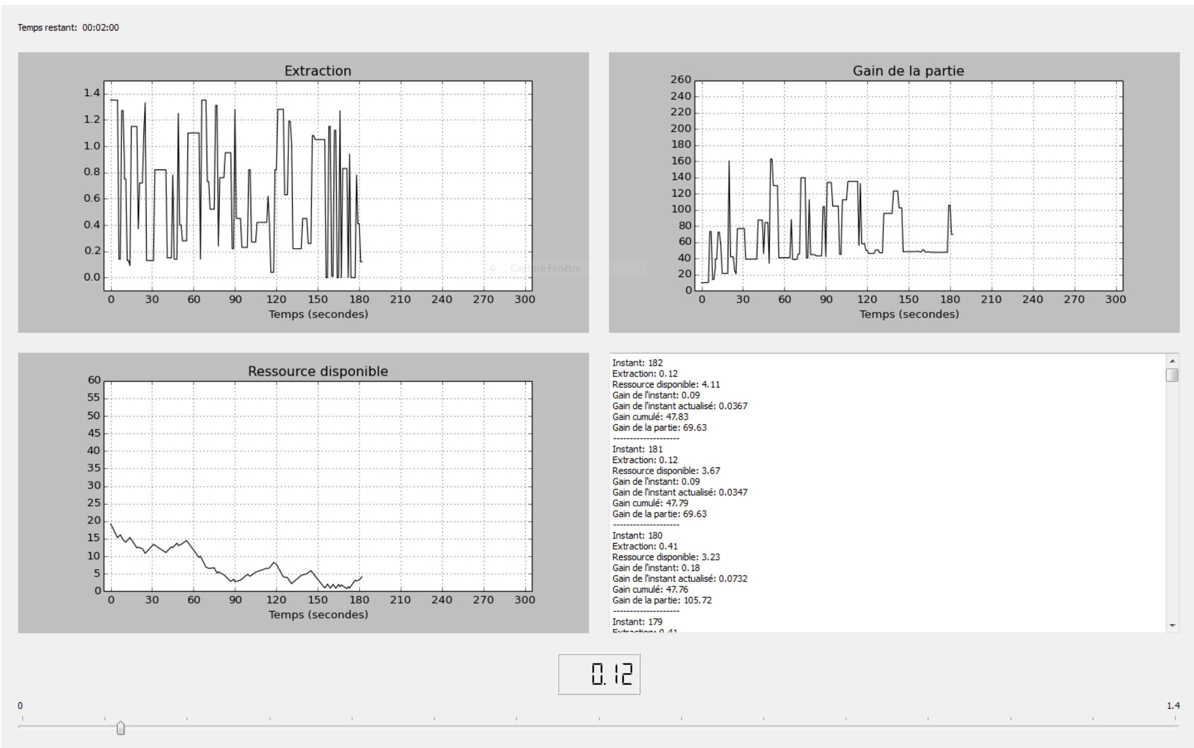

Figure 11 – The single agent screenshot

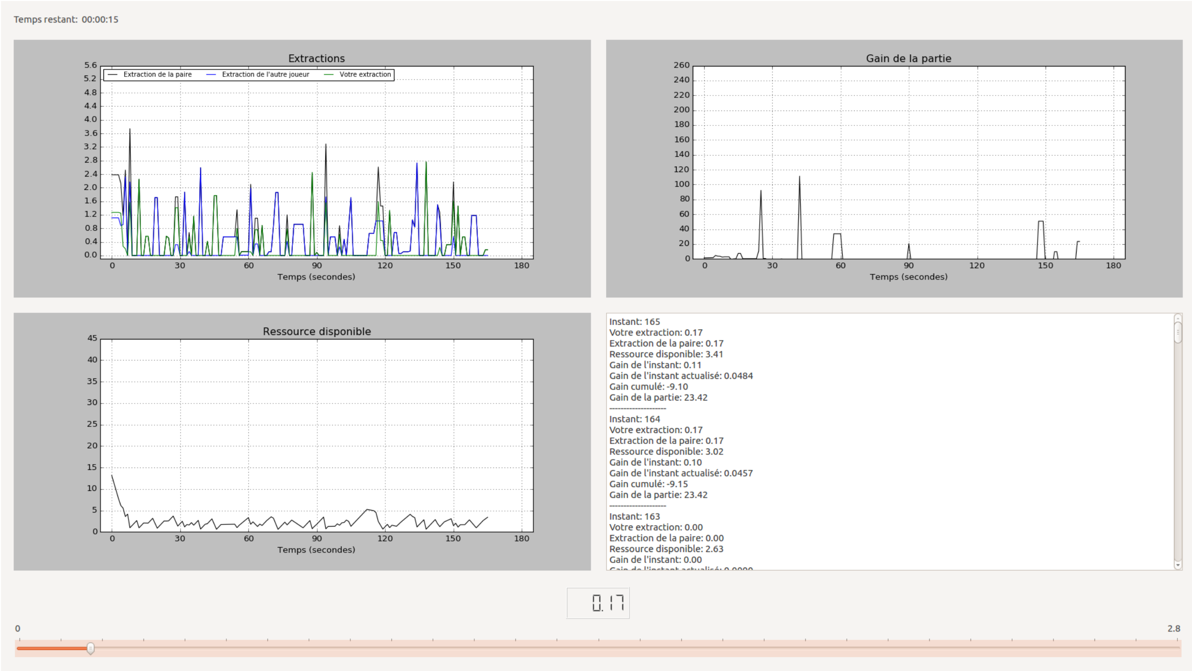

Figure 12 – The game screenshot

E An example of how the empirical strategy works

The purpose of this appendix is to provide a precise example of an application of our empirical strategy. We follow agent 58 and show all intermediate results.

Step 1: We compute the conditional MSDs in the optimal control. This gives us:

$$\begin{aligned} MSD^c_{my} &= 0.71818382 \\ MSD^c_{op} &= 0.01072926 \end{aligned} \tag{27}$$

MSD^c_{op} is the smallest. Extraction and conditional extraction paths of agent 58 are then shown by Figure 13.

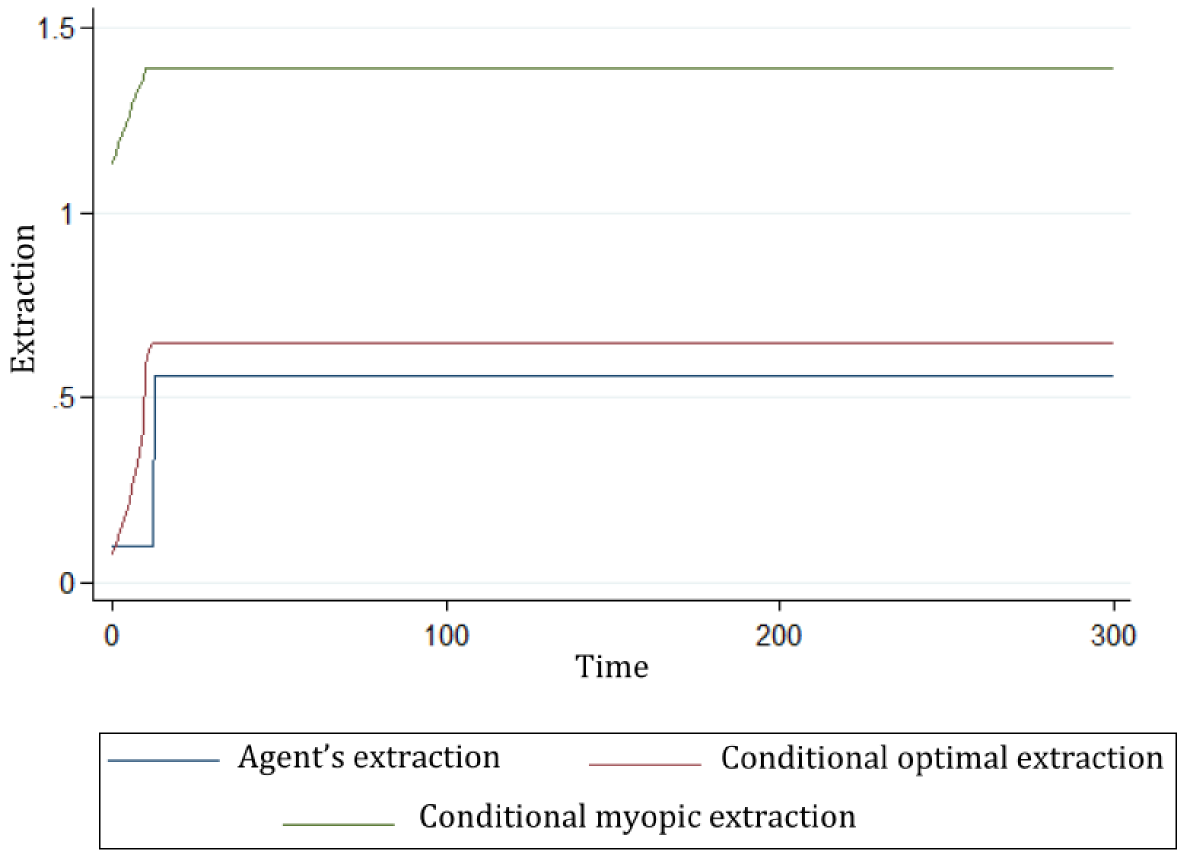

Figure 13 – Agent 58's extraction path versus the conditional extraction path for the optimal control

Visual inspection confirms that agent 58 is closer to the conditional optimal extraction path than to the conditional myopic extraction path.

Step 2: Next, we regress agent 58's extractions from time $t = 0$ to $t = 300$ over its conditional optimal extraction path in the optimal control. Results are shown in Table 2.

Table 2 – Agent 58's extraction in the optimal control

	(1)
	$w(t)$
$w(t)^c_{op}$	1.016***
	(8.91)
Constant	-0.102
	(-1.35)
Observations	301

Newey-West standard errors with 5-period lags.

t statistics in parentheses.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The coefficient is positive (1.016) and significant at 0.1%. Therefore, we consider agent 58 as being significantly optimal in the optimal control.

Step 3: We compute the conditional MSDs in the game. This gives us:

$$\begin{aligned} MSD^c_{my} &= 1.2330157 \\ MSD^c_{fb} &= 0.2808204 \\ MSD^c_{op} &= 0.01470975 \end{aligned} \tag{28}$$

MSD^c_{op} is the smallest. Extraction and conditional extraction paths of agent 58 are shown by Figure 14.

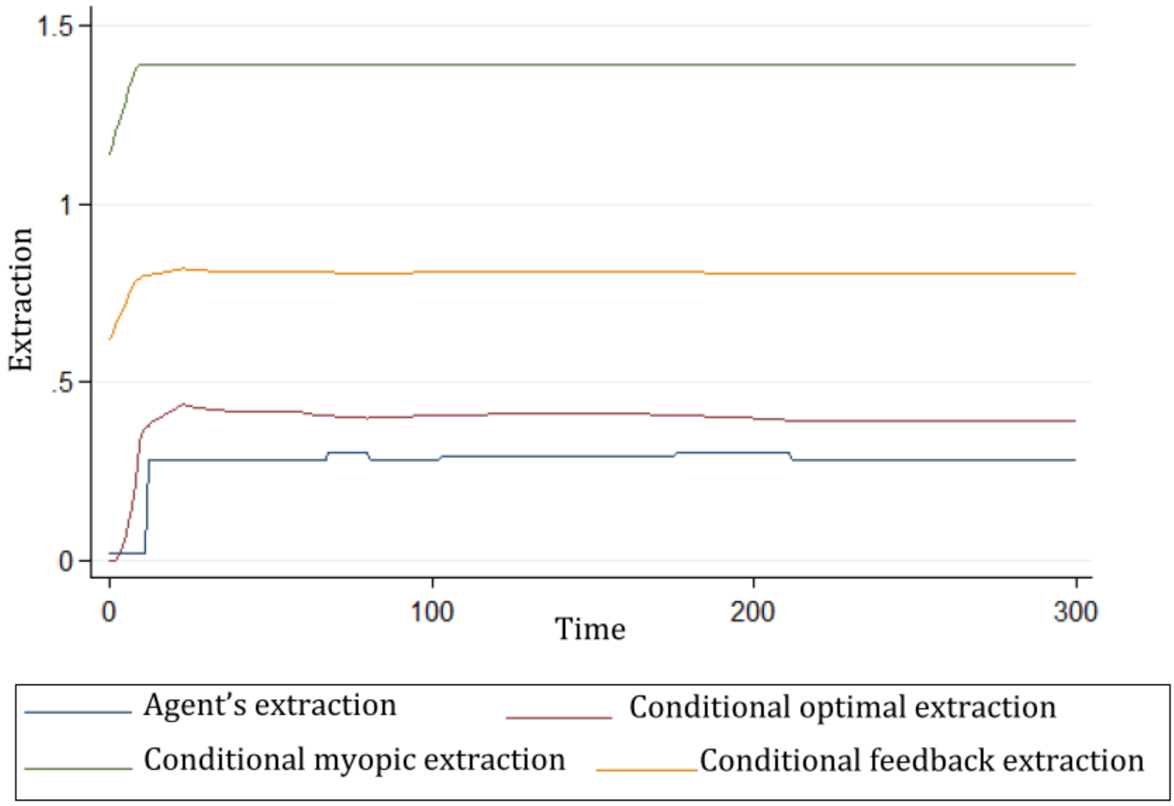

Figure 14 – Agent 58's extraction path versus conditional extraction path for the game

Visual inspection confirms that agent 58 is closer to the conditional optimal extraction path than to any other path.

Step 4: Next, we regress agent 58's extractions from time $t = 0$ to $t = 300$ over their conditional optimal extraction path in the game. Results are shown in Table 3.

Table 3 – Agent 58's extraction in the game

	(1)
	w(t)
$w(t)^c_{op}$	0.760^{***}
	(9.60)
Constant	-0.0232
	(-0.70)
Observations	301

Newey-west standard errors with 5-period lags.

t statistics in parentheses.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The coefficient is positive (0.760) and significant at 0.1%. Therefore, we consider agent 58 as being significantly optimal in the game.

F Efficiency

Tables 4 and 5 show descriptive statistics on the efficiency ratio.

Table 4 – Statistics on efficiency by agent's type in the control

	Payoff				
Agent type	Obs	Mean	Std. Dev.	Min	Max
Optimal	31	92.952	6.085	71.570	99.518
Undetermined	39	65.687	23.876	0	97.605
Total	70	77.762	22.718	0	99.518

Table 5 – Statistics on efficiency by agent type in the game

	Payoff				
Agent type	Obs	Mean	Std. Dev.	Min	Max
Optimal	26	94.769	20.530	43.861	131.343
Feedback	4	52.336	26.871	31.455	89.185
Myopic	2	21.471	15.915	10.217	32.724
Undetermined	38	53.638	33.387	0	116.192
Total	70	67.922	35.300	0	131.343

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