Coordination problems and the control of epidemics affecting fruit trees

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A complex management problem

- production by private owners distributed within a landscape
- economic losses due to the infection outbreak
- diffusion of pathogens intra and inter-patch
- finite horizon, multi-year production
- treatment by (partially inefficient) detection and removal of infected trees, (discrete binary choice)

**Figure:** Sharka example
Objective: Understand the decentralized problem

Problem often studied under the centralized perspective.

**Our objective:** understand better the decentralized behavior. Emerging literature: [Atallah et al., 2017], [Fenichel et al., 2014], [Costello et al., 2017]

We analyze classical questions... **coordination issues, inefficiency characterization**... with specific modeling constraints
Modeling: infection diffusion within a period

Management options: $\rho_i \in \{0, \rho_{max}\}; \ 0 < \rho_{max} < 1$

State variables:
$I_i$ Quantity of infected in patch $i$.
$S_i$ Quantity of uninfected trees.

Growth and diffusion of the infection: $r_{ij}$

Evolutionary law (discrete time model), with $I << S$:

$$(I_i^{t+1}, S_i^{t+1}) = f(S^t, I^t, \rho^t)$$

$$I_i^{t+1} = I_i^t(1 - \rho_i) + \sum_{j=1}^{N} I_j^t(1 - \rho_j)r_{ji}$$

$$S_i^{t+1} = S_i^t - \sum_{j=1}^{N} I_j^t(1 - \rho_j)r_{ji}$$
Diffusion in a two patches model

\[ I_{i}^{t+1} = I_{i}^{t}(1 - \rho_{i}) + \sum_{j=1}^{N} I_{j}^{t}(1 - \rho_{j})r_{ji} \]

patch 1

\[ r_{11}I_{1}^{t}(1 - \rho_{1}) \]

\[ r_{12}I_{1}^{t}(1 - \rho_{1}) \rightarrow \]

\[ r_{21}I_{2}^{t}(1 - \rho_{2}) \leftarrow \]

partch 2

\[ r_{22}I_{2}^{t}(1 - \rho_{2}) \]
Economic model: profit function

$$\pi^t_i(l^t, s^t, \rho^t) = \left( s^{t+1}_i v_i + l^{t+1}_i u_i - \frac{\rho_i^t}{\rho_{\text{max}}} (c_a + c_h A_i) \right)$$

subject to:

$$(l^{t+1}, s^{t+1}) = f(s^t, l^t, \rho^t).$$

$v_i$: production value by an uninfected tree in patch $i$

$u_i$: production value by an infected tree $i$

$c_a$: access cost

$c_h$: per ha$^{-1}$ inspection cost

$A_i$: patch $i$ surface
Conceptual framework

Framework

\[ V_i^T (\rho^0, \rho^1, I^0, S^0) = \pi^0_i (I^0, S^0, \rho^0) + \beta \pi^1_i (I^1, S^1, \rho^1) \]

\[ I^{t+1}, S^{t+1} = f(I^t, S^t, \rho^t). \]

Resolution for the closed loop feedback-Nash equilibrium concept.
Comparison with the Pareto optimum.
Impact of the initial condition in the 2 patches 2 steps model:

- An example of analytical result: zone where $\left(\rho_{\text{max}}, \rho_{\text{max}}, \rho_{\text{max}}, \rho_{\text{max}}\right)$ is the unique FNE
- Multiplicity of FNE
- Characterization of inefficiency
Maximal effort as a FNE

**Proposition:** Within the initial condition state space, there is a zone where initial infection is sufficiently high so that both players do maximal effort:

$$(\rho_{\text{max}}, \rho_{\text{max}}, \rho_{\text{max}}, \rho_{\text{max}})$$ is the unique Nash equilibrium if and only if $$(I_1^0, I_2^0) \in \Delta_{\text{max}}$$, where $\Delta_{\text{max}}$ is defined by the set of inequalities:

$$\begin{cases}
I_2^0 > \frac{\alpha_1 - I_1^0 (1-\rho_{\text{max}})(1+r_{11})}{(1-\rho_{\text{max}}) r_{21}} \\
I_2^0 > \frac{\alpha_2 - I_1^0 (1-\rho_{\text{max}}) r_{12}}{(1-\rho_{\text{max}})(1+r_{22})} \\
I_2^0 > k_2 \\
I_1^0 > k_1
\end{cases}$$

where $\alpha_i \equiv \frac{1}{F_i} (c_a + c_h \frac{1}{\rho_{\text{max}}} A_i)$ where $F_i \equiv (v_i - u_i) r_{ii} - u_i$, and $k_1$ and $k_2$ are some constants.
Private efficiency in the case of maximal effort

\[
\alpha_i \equiv \frac{1}{F_i(c_a + c_h \rho_{\text{max}} A_i)}
\]

\[
(\rho_1^0, \rho_2^0, \rho_1^1, \rho_2^1)^* = (\rho_{\text{max}}, \rho_{\text{max}}, \rho_{\text{max}}, \rho_{\text{max}})
\]
Illustration for $\rho_{\text{max}}$ as a unique Nash equilibrium

\[ \beta_i \equiv \frac{1}{D_i}(c_a + c_h\frac{1}{\rho_{\text{max}}}A_i) \quad \text{where} \quad D_i \equiv (v_i - u_i)r_{ii} + (v_j - u_j)r_{ij} - u_i \]

\[ \alpha_i \equiv \frac{1}{F_i}(c_a + c_h\frac{1}{\rho_{\text{max}}}A_i) \quad \text{where} \quad F_i \equiv (v_i - u_i)r_{ii} - u_i \]
**Proposition:** Multiplicity might arise... even in a symmetric case (proof using an example).
Example, Nash equilibria according to the initial condition
Symmetric example, inefficiency

**Figure:** Pareto optimum, symmetric example

**Figure:** Nash equilibria, symmetric example
Symmetric example, inefficiency
Conclusion

Main results
When infection is still small, \((I \ll S)\), and detection imperfect, and given parameters \((T, R, U, V\ldots)\)

- Nash feedback resolution of the game shows equilibria depending on the initial infection level
- Geometric characterization of efficiency and inefficiency zones as a function of the initial infectious state
- Coordination issues: multiplicity of equilibria for some \((I_1^0, I_2^0)\)
Conclusion

Perspectives

- Introduce asymmetry in the case study, look at the impact of other parameters
- Study de-synchronization of production cycles and longer time horizons
- Apply this framework to analyze real life problems (find some data); question large scale management programs using known parameters
- Work on the modeling: $SI$ model, probabilistic framework...
Thanks for listening!


Temporal structure

BEGINNING OF THE GAME

decision

\[(\rho_1^0, \rho_1^0)\]

\[\downarrow\]

\[t = 0\]

\[\rightarrow\]

first season payoffs

\[\pi_1^0(I^0, \rho^0) = g(I_1^0)\]

\[\pi_2^0(I^0, \rho^0) = g(I_2^0)\]

\[\rightarrow\]

END OF THE GAME

decision

\[(\rho_1^1, \rho_2^1)\]

\[\downarrow\]

\[t = 1\]

\[\rightarrow\]

\[t = 2\]

periods

\[\rightarrow\]

second season payoffs

\[\pi_1^1(I^1, \rho^1) = g(I_1^1)\]

\[\pi_2^1(I^1, \rho^1) = g(I_2^1)\]