Complexity matching in side-by-side walking

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Abstract:

Interpersonal coordination represents a very common phenomenon in daily-life activities. Three theoretical frameworks have been proposed to account for synchronization processes in such situations: the information processing approach, the coordination dynamics perspective, and the complexity matching effect. On the basis of a theoretical analysis of these frameworks, we propose three statistical tests that could allow to distinguish between these theoretical hypotheses: the first one is based on multifractal analyses, the second and the third ones on cross-correlation analyses. We applied these tests on series collected in an experiment where participants were instructed to walk in synchrony. We contrasted three conditions: independent walking, side-by-side walking, and arm-in-arm walking. The results are consistent with the complexity matching hypothesis.

Key-words: synchronized walking, complexity matching, multifractals, cross-correlation.

Introduction

Interpersonal synchronization represents a very common phenomenon in daily life activities, for example when people walk together, dance, play music, etc. However, the processes that sustain this kind of coordination are still poorly understood, and several theoretical frameworks are in competition for explaining how interpersonal synchronization occurs.

In the present paper we focus on a very usual activity, side-by-side walking. The final goal of this line of research is concerned by rehabilitation purposes, and this point will be developed in the concluding section. The main aim of the current paper is to enrich the theoretical approach of the alternative frameworks that compete in this domain, and to propose a statistical strategy for disentangling these different points of view. We then apply this theoretical and statistical background in an experimental study on side-by-side walking. In a first step it seems necessary to shortly introduce the theoretical paradigms that have been proposed in the study of interpersonal synchronization.

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The information-processing approach

The first framework suggests that interpersonal synchronization is based on cognitive, representational processes of anticipation. This information-processing paradigm originates in the analysis of sensorimotor synchronization (SMS), focusing at the experimental level on the synchronization of simple movements (e.g., finger tapping) with a regular metronome (Repp, 2005; Repp & Su, 2013). A number of studies suggested that in such tasks synchronization is achieved by a systematic correction of the current inter-tap interval, on the basis of the last asynchronies (Pressing & Jolley-Rogers, 1997; Torre & Delignières, 2008; Vorberg & Wing, 1996). This corrective process can be expressed as follows:

$$ITI_n = ITIth_n - \alpha ASYN_{n-1} + \gamma\varepsilon_n \quad (1)$$

where ITI_n represents the inter-tap interval produced by the participant at the n^{th} tap, and $ASYN_n$ the asynchrony between the n^{th} tap and the n^{th} onset of the metronome. $ITIth_n$ represents the inter-tap interval that should be intrinsically produced. $ITIth_n$ is supposedly produced by an internal timekeeper, and is corrected by a fraction of the preceding asynchrony. Finally ε_n is a white noise process.

In order to account for synchronization with more realistic environments, this paradigm has been extended to the study of synchronization with non-isochronous metronomes. The first studies focused on metronomes with regularly modulated deviations around the basic tempo (Madison & Merker, 2005; Thaut, Tian, & Azimi-Sadjadi, 1998). More recently a number of studies analyzed synchronization with metronomes presenting fractal variabilities, which are supposed to represent more closely the kind of fluctuations one encounters with natural situations, and especially with human partners (Delignières & Marmelat, 2014; Hunt, McGrath, & Stergiou, 2014; Kaipust, McGrath, Mukherjee, & Stergiou, 2013; Marmelat, Torre, Beek, & Daffertshofer, 2014; Rankin & Limb, 2014; Torre, Varlet, & Marmelat, 2013). These experiments generally showed that individuals tracked the timing variations of the sequence at a lag of one event (Delignières & Marmelat, 2014; Thaut et al., 1998; Torre et al., 2013). This tracking behavior is essentially similar to that supposed by the basic model proposed in Eq. (1).

This information processing approach to sensorimotor synchronization has been extended to interpersonal synchronization, especially in the study of dyadic finger tapping tasks (Konvalinka, Vuust, Roepstorff, & Frith, 2010; Nowicki, Prinz, Grosjean, Repp, & Keller, 2013; Pecenka & Keller, 2011). These experiments and their results will be presented and discussed latter in this paper.

The coordination dynamics perspective

A second theoretical framework has been proposed by the coordination dynamics perspective (Schmidt, Carello, & Turvey, 1990). This approach was initially developed in the analysis of bimanual coordination, and promoted a phenomenological model based on a continuous coupling between oscillators (Haken, Kelso, & Bunz, 1985; Schöner, Haken, & Kelso, 1986):

$$\begin{cases} \ddot{x}_1 + \delta\dot{x}_1 + \lambda\dot{x}_1^3 + \gamma x_1^2\dot{x}_1 + \omega^2 x_1 = (\dot{x}_1 - \dot{x}_2)[a + b(x_1 - x_2)^2] \\ \ddot{x}_2 + \delta\dot{x}_2 + \lambda\dot{x}_2^3 + \gamma x_2^2\dot{x}_2 + \omega^2 x_2 = (\dot{x}_2 - \dot{x}_1)[a + b(x_2 - x_1)^2] \end{cases} \quad (2)$$

where x_i is the position of oscillator i, and the dot notation represents derivation with respect to time. The left side of the equations represents the limit cycle dynamics of each oscillator determined by a linear stiffness parameter (ω) and damping parameters (δ, λ, and γ), and the right side represents the coupling function determined by parameters a and b. This model has been proven to adequately account for most empirical features in bimanual coordination tasks, such as the differential stability of in-phase and anti-phase coordination modes, and the

transition from anti-phase to in-phase coordination with the increase of oscillation frequency (Haken et al., 1985; Schöner et al., 1986).

Schmidt et al. (1990), in a series of experiments in which two seated participants were asked to visually coordinate their lower legs, showed that interpersonal coordination presents strong similarities with bimanual coordination: anti-phase and in-phase coordination patterns also emerged as intrinsically stable behaviors, with anti-phase being less stable than in-phase coordination, and spontaneous transitions from anti-phase to in-phase coordination were also observed with increasing frequency. Similar results were obtained in diverse interpersonal tasks, such as rocking side-by-side in rocking chairs (Richardson, Marsh, Isenhower, Goodman, & Schmidt, 2007), or swinging pendulums together (Schmidt, Bienvenu, Fitzpatrick, & Amazeen, 1998). Some important predictions of the original model, such as the effect of a difference between the uncoupled eigenfrequencies of the two oscillators, were also evidenced in interpersonal coordination tasks (Schmidt et al., 1998).

Complexity matching

Complexity matching represents a third, alternative framework that has been recently proposed for accounting for interpersonal coordination processes (Abney, Paxton, Dale, & Kello, 2014; Delignières & Marmelat, 2014; Marmelat & Delignières, 2012). The concept of complexity matching, introduced by West et al. (2008), states that the exchange of information between two complex networks is maximized when their complexities are similar. The response of a complex network to the stimulation of another network is a function of the matching of their complexities. This property requires that both networks generate $1/f$ fluctuations, and has been interpreted as a kind of "$1/f$ resonance" (Aquino, Bologna, West, & Grigolini, 2011)

An interesting conjecture exploiting the complexity matching effect supposes that two coupled complex systems tend to attune their complexities in order to optimize information exchange. This conjecture suggests a close matching between the scaling exponents characterizing the series produced by the coupled systems. Such results have been evidenced by Marmelat and Delignières (2012) in an inter-personal coordination task where participants oscillated pendulums in synchrony, and by Abney, Paxton, Dale, and Kello (2014), in the analysis of speech signals during dyadic conversations.

The processes that underlain this tailoring of fluctuations remain not fully understood. Stephen and Dixon (2011) propose an interesting hypothesis, which explains this attunement as a case of multifractal cascade dynamics in which perceptual-motor fluctuations are coordinated across multiple time scales. This coordination among multiple time scales could support the apparently predictive aspects of behavior without requiring an internal model.

These three theoretical frameworks have been jointly considered in a series of papers dedicated to the analysis of anticipation processes, and distinguishing several forms of anticipation (Dubois, 2003; Stephen & Dixon, 2011; Stepp & Turvey, 2010). Dubois (2003) considered that synchronization with fluctuating environments was based on a kind of "prediction" of its upcoming behavior (Delignières & Marmelat, 2014; Marmelat & Delignières, 2012; Stephen & Dixon, 2011; Stephen, Stepp, Dixon, & Turvey, 2008). Dubois suggested that a first form of anticipation was based on representational processes, allowing to predict the future of the environment with which the systems has to coordinate. The information-processing approach we previously presented corresponds to this kind of processes. Dubois (2003) proposed to refer this form of anticipation to as ''weak'' anticipation.

The author proposed a ''strong'' alternative that does not rely on internal models. Strong anticipation suggests that the organism is embedded within its environment. This embedding asserts lawful constraints upon both the actions of the organism and the environmental effects on those actions, and anticipation emerges as a lawful regularity of the organism–environment system.

Stephen and Dixon (2011) argued that two approaches to strong anticipation have to be distinguished. The first one suggests that strong anticipation results from an appropriate coupling between the organism and its environment. An interesting example was presented by Voss (2000), who showed that during the synchronization between a slave and a master systems, the presence of time delays in the master system yields the slave system to synchronize with future states of the master. The models of coupled oscillators proposed by the coordination dynamics perspective clearly refer to this kind of *local* strong anticipation processes. This conception supposes that anticipation is based on local time scales, and the quality of anticipation is supposed to be closely related to the strength of coupling between the two systems (Stepp & Turvey, 2010).

A second approach supposes that strong anticipation is based on a more global coordination between the organism and its environment. Stephen et al. (2008) were the first to evidence this kind of strong anticipation in an experiment which analyzed synchronization with a chaotic metronome. In such a situation, local predictions seem difficultly conceivable, because of the intrinsically unpredictable nature of the external pacing signal. Despite this unpredictability, the authors reported a quite acceptable synchronization with the metronome. They also observed a close matching between the fractal exponents of the chaotic signals and those of the corresponding inter-tap interval series. Such *global* strong anticipation corresponds to the previously presented complexity matching effect.

These three theoretical frameworks have received considerable supports in their respective fields of emergence, including interpersonal coordination tasks. We are not sure, however, that these frameworks represent alternative hypotheses for accounting for similar phenomena. Depending on the nature and the constraints of the situation, different synchronization processes could be at work, and each framework could offer satisfying accounts in specific tasks. The information processing approach seems particularly relevant for accounting for situations where one has to synchronize discrete movements (e.g., tapping) with series of discrete signals (Konvalinka et al., 2010; Repp, 2005). The coordination dynamics perspective was essentially developed for accounting for the coordination of continuous, oscillatory movements (Schmidt et al., 1990). The scope of complexity matching remains to define, but it has been previously applied to very diverse situations, including non periodic interactions between complex systems (e.g., Abney et al., 2014).

In order to test the relevance of these frameworks in specific situations, we need statistical signatures that could be able to unambiguously identify the processes at work in interpersonal coordination. In the following parts we present three possible tests: the first one is based on multifractal analyses, and has been proposed by Delignières et al (2016), the second and the third exploit cross-correlation analyses.

Multifractal signatures

Most experiments seeking to evidence a complexity matching effect tried to reveal a close attunement of the (mono)fractal properties of the series produced by the coordinated systems. Typically, the authors showed close correlations between scaling exponents (Delignières & Marmelat, 2014; Marmelat & Delignières, 2012; Marmelat, Delignières, Torre, Beek, & Daffertshofer, 2014; Stephen et al., 2008).

However, Delignières, Almurad, Roume and Marmelat (2016) claimed that the matching of scaling exponents could not be considered an unambiguous signature of complexity matching. They proposed to distinguish between *statistical matching* (i.e., the convergence of scaling exponents) and genuine complexity matching effect (i.e., the attunement of complexities). Some recent papers showed that the matching of scaling exponents could result from local, short-term adjustments or corrections (Delignières & Marmelat, 2014; Fine et al., 2015; Torre et al , 2013). For example, Delignières and Marmelat (2014) analyzed series of stride durations

produced by participants attempting to walk in synchrony with a fractal metronome. They evidenced a close correlation between the scaling exponents of the series of stride durations produced by the participants and those of the series of inter-onset intervals of the corresponding metronomes. The authors tried to simulate their empirical results by means of a model based on local corrections of asynchronies, and showed that this model was able to adequately reproduce the statistical matching observed in experimental series. The authors concluded that walking in synchrony with a fractal metronome could essentially involve short-term correction processes, and that the close correlation observed between scaling exponents could in such a case just represent the consequence of these local corrections.

Delignières et al. (2016) proposed a more binding method for distinguishing genuine complexity matching from local corrective processes. They first suggested to base the analysis of statistical matching on a multifractal approach, rather than the monofractal analyses previously employed. This choice was motivated by the point developed by Stephen and Dixon (2011), arguing the tailoring of fluctuations that is typical of complexity matching could be considered as the product of multifractality, and also by the fact that multifractals allow for a more detailed picture of the complexity of time series.

Multifractal processes present more complex fluctuations than monofractal series, and cannot be characterized by a single scaling exponent. In multifractal series subsets with small and large fluctuations scale differently, and their description requires a hierarchy of scaling exponents (Podobnik & Stanley, 2008). Delignières et al. (2016) proposed to assess the statistical matching through the point-by-point correlation function between the sets of scaling exponents that characterize the coordinated series.

The authors used the Multifractal Detrended Fluctuation Analysis (MFDFA, see Method section), which is based in its first steps on the analysis of the evolution of average statistical moments with the length of the intervals over which these moments are computed. This method allows to choose the range of interval lengths that is taken into account. The authors proposed to estimate the set of multifractal exponents in first over the entire range of available intervals (i.e., from 8 to $N/2$, N representing the length of the series), and then over more restricted ranges, progressively excluding the shortest intervals (i.e., from 16 to $N/2$, from 32 to $N/2$, and then from 64 to $N/2$). They then computed the point-by-point correlation functions characterizing the four ranges of interval lengths considered.

The authors supposed that if synchronization is just based on local corrections, the statistical matching in long intervals is just the consequence of the short-term, local coupling between the two systems. As local corrections between unpredictable systems remains approximate, correlations should dramatically decrease when intervals of shorter durations are taken into consideration. In contrast, in the case of genuine complexity matching, the synchronization between systems is supposed to emerge from interactions across multiple scales. The authors hypothesized to find in this case close correlations, even when considering the entire range of intervals, from the shortest to the longest.

We present in Figure 1 the results obtained by the authors in three experiments. The first one analyzed the series of periods produced by the two hands of participants performing in-phase bimanual coordination. The correlation functions obtained remained close to one, whatever the range of intervals considered (Figure 1, left panel). The second one was an interpersonal coordination task in which participants were instructed to oscillate pendulums in phase (Figure 1, central panel). In this experiment the correlation functions remained significant, while a little bit lesser than in the first example. In the third experiment participants had to walk in synchrony with a fractal metronome (Figure 1, right panel). In that case a close-to-one correlation function was only obtained when the longest intervals were considered (i.e., from 64 to $N/2$). When widest ranges of intervals were considered, correlation functions lose statistical significance.

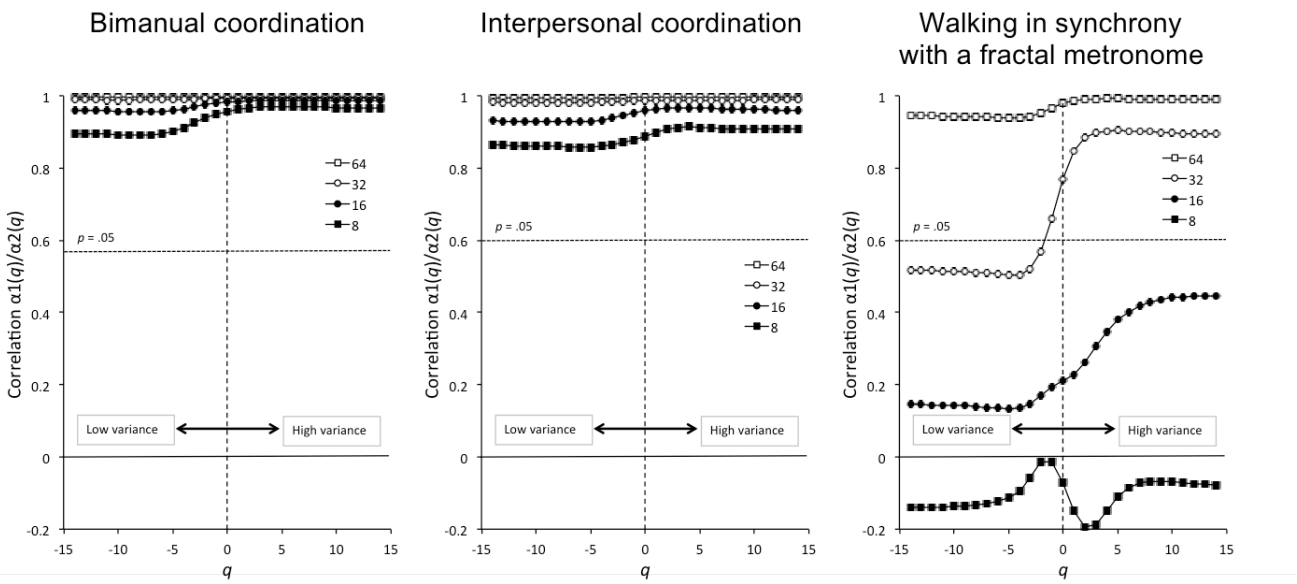

Figure 1: Correlation functions, for the four ranges of intervals considered (8 to $N/2$, 16 to $N/2$, 32 to $N/2$, 64 to $N/2$), for bimanual coordination (left), interpersonal coordination (middle), and walking in synchrony with a fractal metronome (right). From Delignières et al. (2016).

The authors concluded that in bimanual coordination and in interpersonal coordination, the statistical matching resulted from a genuine complexity matching between systems. In contrast during walking in synchrony with a fractal metronome, the apparent statistical matching was just the result of local adjustments.

This multifractal approach allows to clearly distinguishing between weak anticipation processes (i.e. local discrete correction) and strong anticipation processes. However, it seems unable to distinguish between the local and global forms of strong anticipation (Delignières et al., 2016).

Cross-correlation peaks

A second kind of signatures can be obtained from cross-correlation analyses. As previously evoked, a number of recent studies analyzed synchronization with non-isochronous metronomes, and especially metronomes presenting fractal fluctuations. These studies showed that synchronization in such situations was sustained by local corrections of the recent asynchronies, as expected from Eq. (1). Such behavior is typically revealed by a positive peak of cross-correlation at lag -1, between the series of asynchronies and the series of periods produced by the participant, or between the series of periods produced by the participant and that produced by the metronome. Note that some more complicated models have been proposed, involving corrective processes taking into account more previous asynchronies (Pressing & Jolley-Rogers, 1997). For example Delignières and Marmelat (2014), in an experiment where participants had to walk in synchrony with a fractal metronome, evidenced positive peaks of cross-correlations at lag -2 and lag -1 between the series of asynchronies and the series of step durations.

The principle of phase correction can also be applied to interpersonal synchronization. When two individuals perform a rhythmic task in synchrony (e.g., tapping), phase correction suggests that participant A adapts his/her current inter-tap interval on the basis on the last asynchrony he/she perceived with his/her partner, and conversely for participant B. This mutual phase correction process could be modeled as follows:

$$\begin{cases} ITI_{A,n} = ITIth_{A,n} - \alpha ASYN_{A-B,n-1} + \gamma\varepsilon_{A,n} \\ ITI_{B,n} = ITIth_{B,n} - \alpha ASYN_{B-A,n-1} + \gamma\varepsilon_{B,n} \end{cases} \quad (3)$$

where $ITI_{A,n}$ represents the inter-tap interval produced by participant A at the n^{th} tap, $ASYN_{A\text{-}B,\,n}$ the asynchrony between the n^{th} tap of participant A and the n^{th} tap of participant B (hence, $ASYN_{A\text{-}B,\,n} = -\,ASYN_{B\text{-}A,\,n}$). As in the previous model (Eq. (1)), $ITIth_{A,n}$ is a long-range correlated series with Hurst exponent H, mean M and variance σ^2, representing the series of taps that should be intrinsically produced by participant A, and $\varepsilon_{A,n}$ is a white noise process with zero mean and unit variance. We generated 12 sets of coupled series with this simple model, with the following parameters: $H = 0.9$, $\alpha = 0.3$, $M = 1000$, $\sigma^2 = 400$, and $\gamma = 300$. We then computed the cross-correlation function, from lag -10 to lag +10, between the obtained *ITI* series ($ITI_{A,n}$ and $ITI_{B,n}$). We present in Figure 2 the averaged cross-correlation function. This model typically produces positive lag -1 and lag +1 cross-correlations, and a negative lag 0 cross-correlation. The positive lag -1 and lag +1 cross-correlations reflect the correction of asynchronies, and the negative correlation at lag 0 results from this mutual tendency of each participant to adapt towards the previous *ITI* of the other. This typical pattern of cross-correlation was evidenced by Konvalinka et al. (2010), in an experiment where participants had to synchronize their taps, and by Delignières and Marmelat (2014) in an experiment where each participant in a dyad swung a hand-held pendulum, and were instructed to swing in synchrony.

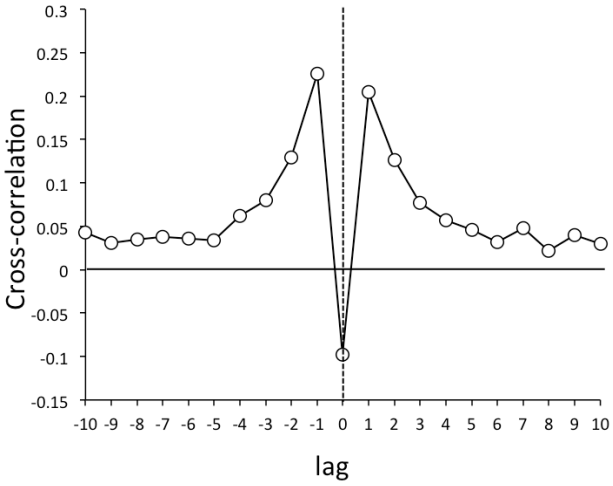

Figure 2: Averaged cross-correlation function obtained from series simulated with Eq. (3).

In contrast, both coupled oscillators models and complexity matching are likely to result in a unique, positive peak of cross-correlation, located at lag 0. Indeed, coupled oscillators models suggest a local, continuous coupling within the limit cycle, and the oscillators are clearly expected to synchronize their frequencies. Generally the authors working on coordination dynamics focus on the stability of relative phase, and ignore the possible serial dependencies between the series produced by the two oscillators. However, Delignières and Marmelat (2014) and Coey et al (2016) clearly evidenced a peak of cross-correlation at lag 0 between the two limbs in bimanual coordination. Complexity matching does not suggest such local coupling, but, rather, a global and multiscale coordination between systems (Stephen & Dixon, 2011). This should induce a close tailoring of fluctuations, which should also be expected to result in a peak at lag 0 in the cross-correlation function.

Then the location of the peak(s) of cross-correlation between the series produced by the two members of the dyad, could allow to distinguish between weak and strong anticipation processes, but not between the local and global forms of strong anticipation. We suggest,

however, that the magnitude of the lag 0 cross-correlation peak could represent an interesting test for the respective relevancy of the two last competing models.

Lag 0 windowed cross-correlation

Cross-correlations are strongly affected by trends, which could spuriously increase the obtained values. In order to control these biases and to focus on local processes, one could compute the Windowed Detrended Cross-Correlation function (WDCC). In this method the series are divided into non-overlapping intervals of short length (e.g., 15 data points), and detrended within each interval. The local cross-correlation function is then computed within each interval, and averaged over all intervals (Coey et al., 2016; Delignières & Marmelat, 2014; Konvalinka et al., 2010).

Fine et al. (2015) suggested that the local and continuous coupling involved in coordination dynamics models could be at the origin of the strong statistical matching observed in interpersonal synchronization experiments. However, several recent studies showed that in such situations, statistical matching occurs despite a lack of substantial short-term cross-correlation, considered as evidence against the local coupling hypothesis (Abney et al., 2014; Marmelat & Delignières, 2012; Rhea, Kiefer, D'Andrea, Warren, & Aaron, 2014; Washburn, Kallen, Coey, Shockley, & Richardson, 2015).

Delignières et al. (2016) performed a simulation study based on the HKB model (Eq. (2)). In order to account for the presence of $1/f$ fluctuations in limb oscillations, they provided the stiffness parameters (ω^2) of both equations with independent fractal properties. They showed that this model required very high coupling parameters (i.e., a and b in Eq. (2)) for maintaining the stability of coordination patterns. As a consequence, the local coupling between oscillators was strong and the mean lag 0 WDCC, computed from these simulated series, was of about 0.84. In contrast, Coey et al (2016) and Delignières and Marmelat (2014) obtained lag 0 WDCCs of about 0.4 in bimanual coordination tasks, and Coey et al (2016) observed a value of about 0.2 in an interpersonal synchronization tapping task. On the basis of these results, one can consider that the lag 0 WDCC value could allow distinguishing between the alternative models of strong anticipation: Coupled oscillators dynamics should be revealed by a significant peak of WDCC at lag 0, but in the case of complexity matching this peak should remain non-significant.

The aim of the present work was to clarify the nature of synchronization in side-by-side walking. We applied the three previously presented statistical tests to empirical series, and we hypothesized to evidence the typical signatures of complexity matching in this situation.

2. Methods

2.1. Participants

26 participants (16 male and 10 female, mean age: 28.07 yrs ± 8.88, mean weight: 68.65 kg ± 10.5, mean height: 172.92 cm ± 9.67) were involved in the experiment. Participants were paired into 13 dyads. The pairing procedure was performed in order to preserve the homogeneity of weights and heights within each dyad. Participants signed an informed consent approved by the local ethic committee and were not paid for their participation. All work was conducted in accordance with the 1964 Declaration of Helsinki.

2.2. Experimental procedure

The experiment was performed around an indoor running track (circumference 200m), and comprised three experimental conditions:

- Condition 1: Independent walking. Each participant walked individually at his/her preferred velocity

- Condition 2: Side-by-side walking. The two members of the dyad walked together, side-by-side. They were explicitly instructed to synchronize their steps during the whole trial.

- Condition 3: Arm-in-arm walking. The two members of the dyad walked together, arm-in-arm. They were explicitly instructed to synchronize their steps during the whole trial.

Each trial, in the three conditions, lasted 16 minutes. Participants had a resting period of at least 10 minutes between two successive trials. Independent walking was performed at first. The order of the two last conditions was counterbalanced within dyads.

2.3. Data collection

Data were recorded with two Mobility Lab systems (APDM, Inc), one for each member of the dyads. Two body-worn inertial sensors were attached on the shanks of each participant. Data were then wirelessly streamed to a laptop. The device performed automated analyses providing a set of raw series (stride duration, stride length, etc., for both limbs). In the present paper we focused on the series of right stride durations.

2.4. Statistical analyses

Multifractal Detrended Fluctuation analysis (MF-DFA)

We performed multifractal analyses with the MF-DFA method, initially introduced by Kantelhardt et al (2002). Consider the series x(i), i = 1, 2, …, N. In a first step the series is centered and integrated:

$$X(k)=\sum_{i=1}^{k}\left[x(i)-\frac{1}{N}\sum_{i=1}^{N}x(i)\right] \tag{4}$$

Next, the integrated series $X(k)$ is divided into N_n non-overlapping segments of length n, and in each segment s = 1, ..., N_n. Within each segment the local trend $X_{n,s}$(k) is estimated and subtracted from $X(k)$. The variance is calculated for each detrended segment:

$$F^2(n,s)=\frac{1}{n}\sum_{k=(s-1)n+1}^{sn}\left[X(k)-X_{n,s}(k)\right]^2 \tag{5}$$

and then averaged over all segments to obtain qth order fluctuation function

$$F_q(n)=\left\{\frac{1}{N_n}\sum_{s=1}^{N_n}\left[F^2(n,s)\right]^{q/2}\right\}^{1/q} \tag{6}$$

where q can take any real value except zero. In the present work we used integer values for q, from -15 to +15. Note that Eq. (6) cannot hold for q = 0. A logarithmic averaging procedure is used for this special case:

$$F_0(n)=\exp\left(\frac{1}{2N_n}\sum_{s=1}^{N_n}\ln\left[F^2(n,s)\right]\right) \tag{7}$$

This calculation is repeated for all lengths n (practically, one considers intervals from 8 or 10 data points, in order to allow a proper assessment of statistical moments, up to $N/4$ or $N/2$). If long-term correlations are present, $F_q(n)$ should increase with n according to a power law:

$$F_q(n)\propto n^{h(q)} \tag{8}$$

The scaling exponent $h(q)$ is obtained as the slope of the linear regression of log $F_q(n)$ versus log n. $h(q)$ is called the generalized Hurst exponent.

These results are then converted into the more classical multifractal formalism by simple transformations (Kantelhardt et al., 2002): first, generalized Hurst exponents $h(q)$ are converted into Renyi exponents $\tau(q)$ by:

$$\tau(q)=qh(q)-1 \tag{9}$$

The singularity spectrum $f(\alpha)$ is then derived through the Legendre transform:

$$\alpha(q) = \frac{d\tau(q)}{dq} \tag{10}$$

$$f(\alpha) = q\alpha - \tau(q) \tag{11}$$

where $f(\alpha)$ is the fractal dimension of the support of singularities in the measure with Lipschitz-Hölder exponent α.

Note that for avoiding to obtain "inversed" spectra, exhibiting a zig-zag shapes rather than the expected parabolic shape in the singularity spectrum, we applied the focus-based approach introduced by Mukli et al. (2015). This approach considers that the moment-wise scaling functions, for all q values, should theoretically converge toward a common limit value at the coarsest scale. Indeed, substituting signal length (N) to interval length (n) in Eq. (6) yields:

$$F_q(N) = \left\{ \frac{1}{N_N} \sum_{s=1}^{N_N} \left[F^2(N,s) \right]^{q/2} \right\}^{1/q} = \left\{ F^2(N,s)^{q/2} \right\}^{1/q} = F(N,s) \tag{12}$$

$F(N,s)$ can then be considered the theoretical focus of the scaling functions, and this focus is used as a guiding reference when regressing for $h(q)$ (Delignières et al., 2016; Mukli et al., 2015).

Correlation functions

Just as DFA, MF-DFA allows to select the range of intervals over which exponents are estimated. As previously indicated, usually authors consider intervals from 8 or 10 data points, up to $N/4$ or $N/2$. Quite often, however, series present different scaling regimes over the short and the long term, and authors perform separate estimates over different ranges of intervals (Delignières & Marmelat, 2014). Here we propose to estimate the set of multifractal exponents in first over the entire range of available intervals (i.e., from 8 to $N/2$), and then over more restricted ranges, progressively excluding the shortest intervals (i.e., from 16 to $N/2$, from 32 to $N/2$, and then from 64 to $N/2$). We then computed for each q value the correlation between the individual Lipschitz-Hölder exponents characterizing the two coordinated systems, $\alpha_1(q)$ and $\alpha_2(q)$, respectively, yielding a correlation function $r(q)$. As previously explained, we expected to find in all cases a correlation function close to 1, for all q values, when only the largest intervals were considered (i.e. 64 to $N/2$). Increasing the range of considered intervals should have a negligible impact on $r(q)$ when coordination is based on a complexity matching effect. In contrast, if coordination is based on local corrections, a decrease in $r(q)$ should be observed, as shorter and shorter intervals are considered.

Cross-correlation analyses

We first computed the cross-correlation function between the series produced by the two members of each dyad in each condition. Cross-correlations were computed for each dyad from lag -60 to lag +60, and the cross-correlation functions were point-by-point averaged.

In a second step we computed for each dyad WDCC functions, from lag-10 to lag 10, between the series produced by the two participants. WDCC were computed over non-overlapping windows of short length (15 data points), and data were linearly detrended within each window before the computation of cross-correlations. WDCC functions were then point-by-point averaged.

3. Results

The length of the collected stride series obviously depended of the walking speed of each dyad. For the independent walking condition, we deleted for each dyad the last points of the

longest series, in order to obtain two series of equal lengths. For the two other conditions, we occasionally deleted some short segments, which presented synchronization errors, either at the beginning of the trial (due to difficulties to enter in synchronization) or at the end of the trial (due to fatigue or boredom). The resulting series lengths ranged from 801 to 990 data points for independent walking, from 716 to 1004 data points for side-by-side walking, and from 650 to 990 data points for arm-in-arm walking.

We present in Figure 3 (upper panel) two example stride intervals series recorded in a representative dyad in the arm-in-arm condition. This first graph shows how medium- or long-term fluctuations are synchronized within the dyad. The bottom panel of Figure 3 represents a focus of the previous series (one hundred strides). This graph suggests in contrast a quite poor synchronization on local scales. We analyze these points more deeply in the following sections.

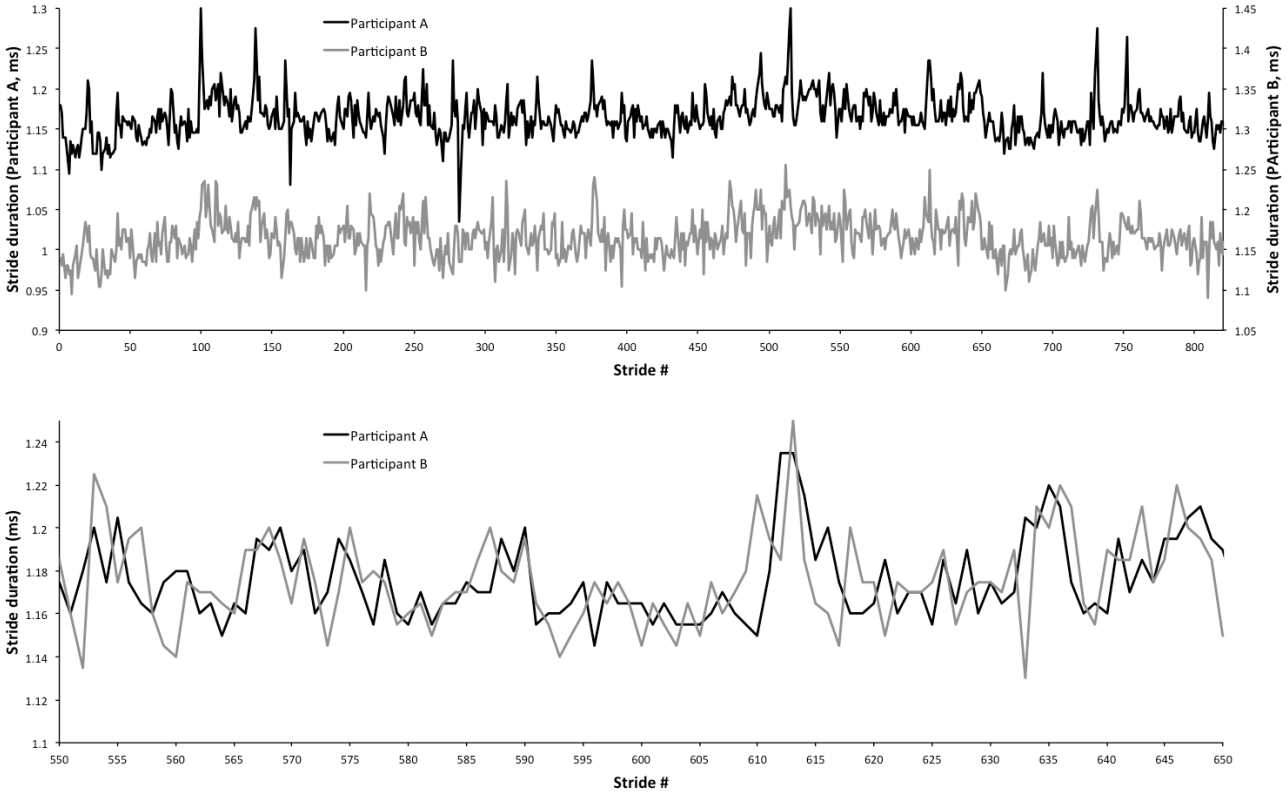

Figure 3: Upper panel: Two example stride intervals series recorded in a representative dyad in the arm-in-arm condition. For a better readability, the series are vertically shifted by 0.15 ms. Bottom panel: A focus on the previous series, between strides #550 and #650.

Multifractal analysis

We present in Figure 4 the correlation functions $r(q)$ between the multifractal spectra, for the three experimental conditions. Correlation coefficients are plotted against their corresponding q values. Four correlation functions are displayed, according to the shortest interval length considered during the analysis (8, 16, 32, or 64). For the independent walking condition (left panel), the correlation functions remained non-significant, whatever the considered range of intervals. In contrast, the correlation functions were systematically above the threshold of significance, whatever the range of interval considered, for side-by-side walking (middle panel) and for arm-in-arm walking (right panel). The correlation functions were close to one in the arm-in-arm condition, when the shortest ranges of intervals were considered (i.e. 32 to $N/2$ and 64 to $N/2$). They appeared a little bit lower in the side-by-side

condition, where the correlation functions for the same interval ranges were on average around 0.9 for positive q values, and around 0.82 for negative q values.

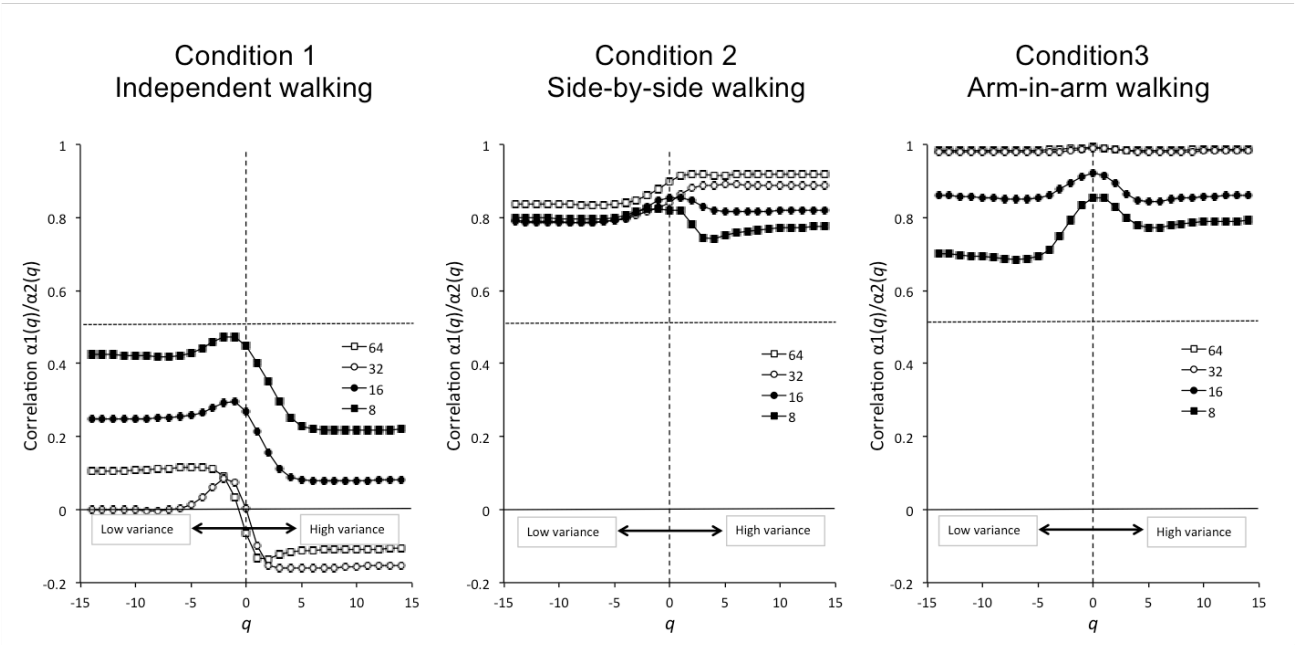

Figure 4: Correlation functions r(q), for the four ranges of intervals considered (8 to $N/2$, 16 to $N/2$, 32 to $N/2$, and 64 to $N/2$) , for independent walking (left), side-by-side walking (middle) and arm-in arm walking (right). q represents the set of orders over which the MF-DFA algorithm was applied.

Cross-correlation analyses

We present in Figure 5 (left panel) the averaged cross-correlation functions in the three conditions. In the first condition (independent walking), no correlation was observed over the investigated range of lags. In contrast, in the two conditions of synchronized walking cross-correlation functions were organized around a marked peak at lag 0, with an average lag 0 coefficient of about .45 in condition 2, and .57 in condition 3. Cross-correlations remained significant up to the negative and positive extrema of the investigated range. Finally, cross-correlations were systematically higher in condition 3, showing the effectiveness of the reinforcement of coupling in arm-in-arm walking, as compared with simple side-by-side walking.

The averaged WDCC functions are reported in Figure 5 (right panel). These functions present a peak at lag 0 for side-by-side and arm-in-arm walking. However in both cases these peaks did not present significant values (0.16 and 0.24, respectively). Note that in contrast with the previous analysis, the decay of cross-correlations was very fast, in both negative and positive directions.

4. Discussion

These results present strong evidence for the presence of a complexity matching effect in synchronized walking. The first analysis focused on multifractal correlation functions, and the results gave strong support for strong anticipation processes in both side-by-side and arm-in-arm walking. Whatever the range of intervals considered, correlation functions remained above the threshold of significance in both conditions. Note that we expected to find stronger correlations in arm-in-arm walking, whatever the considered range on intervals considered. This was observed for the shortest ranges, focusing on long-term intervals (i.e. 32 to $N/2$ and 64 to $N/2$): the correlation functions were in both cases consistently close to one, while they

were between 0.8 and 0.9 for side-by side walking (see Figure 4). When the widest range of intervals was considered (8 to $N/2$), however, the correlation function presents somewhat lesser values in arm-in-arm walking, especially for negative q values. We confess that we have no satisfying explanation for this result. Obviously, we did not obtain any significant correlation in the independent walking condition.

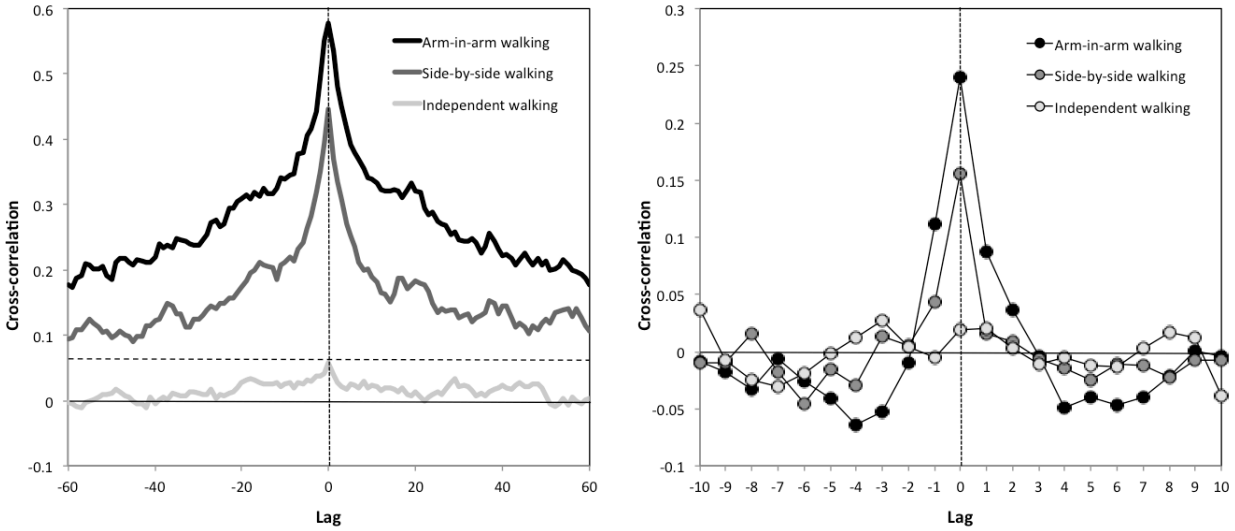

Figure 5: Left panel: Averaged cross-correlation functions, from lag -60 to lag 60, for independent walking (light grey), side-by-side walking (dark grey), and arm-in-arm walking (black). The horizontal dashed line indicates the significance threshold (p=.05). Right panel: Averaged windowed detrended cross-correlation functions, from lag -10 to lag 10, for independent walking (light grey circles), side-by-side walking (dark grey circles), and arm-in-arm walking (black circles).

Cross-correlation analyses confirmed these first results. The averaged cross-correlation functions in the first condition presented non-significant values over the whole range of investigated lags, a result which was obviously expected from independent series. In contrast, a unique and sharp peak was observed at lag 0 for both side-by-side and arm-in-arm walking, clearly showing the absence of local cycle-to-cycle adjustments. The second important observation is the persistence of cross-correlations, at least over the considered range, from lag -60 to lag 60. This kind of long-range cross-correlations could be interpreted as an evidence for complexity matching. Short-term adjustments are likely to produce a quicker, exponential-like decay in cross-correlations. However, this persistence of cross-correlations could also be due to the presence of common local trends in the synchronized series. Finally, we observed systematically higher cross-correlation coefficients in arm-in-arm walking, as compared to side-by-side. This shows that the experimental manipulation (side-by-side vs arm-in-arm) induced an effective difference in coupling strength between the two members of the dyads.

As evoked in the introduction of this paper, such cross-correlation analyses have also been applied in studies about synchronization in music, and especially for synchronization with expressively interpreted musical sequences (Dixon, Goebl, & Cambouropoulos, 2006; Rankin, Fink, & Large, 2014; Rankin, Large, & Fink, 2009; Repp, 1999, 2002, 2006). Repp (2002, 2006) showed that when participants were required to tap along with recordings of such expressively performed music, one observed a lag 0 peak of cross-correlation between the series of inter-tap intervals and the inter-onset intervals of the corresponding tones in the musical excerpt. In contrast, when participants were asked to tap along with a sequence of simple clicks reproducing the expressive timing pattern of a complex piece of music, the peak in the cross-

correlation function was shifted by one lag. Repp (2002) considered this latter result as evidence that participants tracked the timing variations of the sequence, at a lag of one event. In contrast, the author considered that the lag 0 peak of cross-correlation in the first condition showed that participants adjusted their inter-tap intervals on the basis of upcoming rather than preceding inter-onset intervals in the music. In other words, they seem able to anticipate or predict ongoing timing fluctuations.

Analyzing the perfect synchronization to musical sequences in terms of prediction is obviously consistent with the representational point of view of the author. However, considering that musical sequences, when expressively interpreted by expert musicians, present fractal fluctuations (Rankin et al., 2009), one could consider such synchronization as a typical case of strong, non representational anticipation. Note also that these results confirm that artificial signals mimicking natural variability do not allow strong anticipation to occur (Delignières et al., 2016; Delignières & Marmelat, 2014).

The two first analyses clearly discarded the hypothesis of local error corrections (or weak anticipation). was to distinguish between the two remaining theoretical accounts: coordination dynamics and complexity matching. The windowed detrended cross-correlation analysis confirmed the presence of a unique peak of cross-correlation at lag 0, but showed that local synchronization remained weak, and on average non significant. This result is consistent with the graphical example we presented in Figure 2. This represents in our mind, in conjunction with previous results, a strong argument for complexity matching. The weakness of short-term cross-correlation has been considered in several previous studies as evidence discarding the local coordination account and favoring the complexity matching hypothesis (Abney et al., 2014; Marmelat & Delignières, 2012; Rhea et al., 2014; Washburn et al., 2015).

Conclusion

The complexity matching effect could appear a quite strange phenomenon, and it could certainly hurts common conceptions and models. However, this framework clearly proposes innovative and fruitful ways of thinking about coordination between living systems. The information-processing and the coordination dynamics approaches have been supported by a number of (strongly controlled) experimental protocols, but their relevancy could be limited to these restricted and artificial contexts. The analysis of more complex, daily-life like situations, suggests that coordination between living systems relies on other kinds of processes, which could be accounted for by the complexity matching effect. We propose in the present paper a set of statistical tests that aim to distinguish genuine complexity matching from other kinds of synchronization processes that could mimic some aspects of the complexity matching effect.

Evidencing the presence of a complexity matching effect in side-by-side or arm-in-arm synchronized walking could have important implication, especially for rehabilitation purposes. The presence of fractal fluctuations in stride duration series have been evidenced for a long time, suggesting the complexity of the locomotor system (Hausdorff, Peng, Ladin, Wei, & Goldberger, 1995). However, Haussdorf et al. (1997) evidenced a typical extinguishing of fractal scaling in elderly and patients suffering from neurodegenerative diseases. Additionally, they showed that the level of fractality in stride duration series was predictive of fall propension. These results were consistent with the hypothesis of the loss of complexity with age and disease (Goldberger et al., 2002). This raises a central question, from a rehabilitation perspective: could it be possible to restore complexity in a deficient system?

The complexity matching effect could offer some interesting perspectives in this regard. If a deficient (simplified) system is entrained by a healthy (complex) system, one could suppose that the complexity matching effect should result in a momentary attunement of complexities among systems, and especially an increase of the complexity of the former. In other words, if an elderly person is invited to walk in synchrony with a young and healthy companion, one could

expect to observe (at least temporarily) a restoration of complexity. We currently try to test this hypothesis, and future work will aim to analyze the long-term effects of a prolonged training in such situation.

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