The urban toll revenue recycling : what is the optimal share distributed towards mass transit system?

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Abstract

The paper examines the question of the redistribution of toll revenue as seen in a bottleneck congestion model. Our objective is to analyse the impact of this redistribution on total cost and on modal split between railroad and road. Following Tabuchi's two-mode model (J. Urban Econ. 34 (1993) 414), we integrate a redistribution of toll revenue between mass transit (share α of the revenue) and public budget (share $(1-\alpha)$ of the revenue) . This analysis is new in literature relative to queing models (Arnott et al. 1993). A very interesting result is shown in our model : in a pricing regime with a fine toll and a mass transit fare based on the average cost, the optimal redistribution towards mass transit users allows to obtain an equilibrium similar to the benchmark optimal situation with marginal cost fare.
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1 Introduction

In order to regulate auto traffic, public authorities can use various methods inspired by the Pigovian tax (Pigou, 1920). Thus, congestion pricing would allow the reduction of negative externalities caused by traffic congestion. A wide empirical and theoretical literature exists on the efficiency of urban road toll. In recent papers, this literature focuses on the technical constraints supported for the implementation of optimal road toll (De Palma and Lindsey 2011) that force the public authorities to implement second-best or third-best toll in order to reduce inefficiencies.

The recent debates on urban road tolls focus on the question concerning acceptability and equity. More precisely, the question concerning acceptability deals with the related measures that would make the toll more widely accepted. In this framework, it is very relevant to evaluate the role of revenue use in the acceptability of transport pricing policies (Schuitema and Steg 2008).

In examining multiple toll case studies, King et al. (2007) explain that toll revenues must be allocated towards city budgets to ensure the feasibility of this policy. In our paper, in the same way of Goodwin (1989)[1], we propose channeling a part of toll revenues towards mass transit and public budget so as to allow a better acceptability, to encourage modal transfer in order to reduce externalities due to automobile use (pollution, congestion, noise...). Along this line of thought, Kidokoro (2005) developed a model that explicitly uses revenue recycling to deal with cordon area congestion tax, as seen in London, and analyses its effects. In other models with two routes for commuting, some papers (Adler and Cetin 2001, Kidokoro 2010) analyze the toll revenue collection from one route and the revenue recycling used in particular to subsidize commuting in the other route.

Here we examine the question of the redistribution of toll revenue as seen in a bottleneck congestion model (see Arnott et al. 1990, 1993; Vickrey, 1963, 1969). It is the first time that redistribution of tolls revenue is introducted in the framework of Arnott De Palma Lindsey bottleneck models. Our

[1] Goodwin (1989) proposes that a third of the toll revenue be used to improve the effectiveness of public transport, that a third be used for new road infrastructure and maintenance, and that a third be used for general city funds.

objective is to analyse the impact of this redistribution model on total cost and on modal split between railroad and road. Following Tabuchi's model (1993) and, by extension, that of Danielis and Marcucci (2002), Mirabel and Reymond (2011) discussed the situation where the toll revenue is totally used to subsidize mass transit. In this framework, we extend this analysis assuming that toll revenue is allocated between mass transit (share α towards mass transit) and public budget (share $(1-\alpha)$ distributed for public budget).

In this context, we study the impact of a redistribution of toll revenues along a fine toll regime (time-varying road pricing) that would allow the elimination of waiting lanes thus streamlining traffic.

In Section 2, we present our model based on Tabuchi's which consists of a fine toll with public transport marginal cost pricing; into this model we integrate a redistribution of toll revenue between mass transit and collectivity. This scenario shows a very interesting result : the higher is the part of toll revenue distributed towards mass transit, the lower is the number of motorists and the lower is the level of toll revenue. In Section 3, we enlarge this analysis integrating an other pricing regime with public transport average cost pricing. In this scenario, we evaluate the optimal allocation of revenue accrued from congestion pricing, between the public transportation and public budget. In this pricing regime, the optimal redistribution towards mass transit users allows to obtain an equilibrium similar to the benchmark optimal situation with marginal cost fare. Section 4 concludes the paper and emphasizes ways for further analyses.

2 The model

2.1 Bottleneck congestion and modal split

First we present Tabuchi's model (1993) as the starting point of our analysis.

Tabuchi has developed a two-mode model, comprising a road with a bottleneck and a railroad between a residential area and a central business district. In his analysis, N commuters[2] split into N_a and N_b (motorists and public transport users, respectively). In this case the average travel cost of a motorist is rewritten as: $C_a = \frac{\delta N_a}{s}$ (where $\delta = \frac{\beta\gamma}{\beta+\gamma}$, with β the unit cost of an early arrival at the workplace and γ the unit cost of a late arrival, and

[2] Here we assume N to be inelastic, which means that we assume total travel demand to be constant.

where s is the road capacity), and city authorities set the railroad fare at marginal cost ($p = c$). The travel cost of public transport is expressed by: $C_b = c$

At equilibrium, commuters are indifferent to the mode of transport that they use, because time costs of driving and taking public transport are the same (Wardrop's equilibrium, 1952)[3].

Equilibrium can thus be defined as follows:

$$C_a = C_b \Longrightarrow \frac{\delta N_a}{s} = c \Longrightarrow \delta\left(N - N_b\right) - cs = 0 \tag{1}$$

In this case, $N_b > 0$ if $N > N^\dagger$ for $N^\dagger = cs/\delta$. However if $N \leq N^\dagger$, $N_a = N$ and $N_b = 0$. $N^\dagger$ may thus be considered as the threshold below which there is no transit users[4]. In this case, below $N^\dagger$, it is not beneficial to construct public transport networks as $N_b = 0$. Under this condition, Tabuchi obtains the modal split at equilibrium associated with price reference system[5]:

$$\begin{aligned} \left(N_a^m, N_b^m\right) &= \left(\frac{cs}{\delta}, N - \frac{cs}{\delta}\right) \text{ for } N > N^\dagger \\ &= (N, 0) \qquad \text{for } N \leq N^\dagger \end{aligned} \tag{2}$$

Total travel cost is expressed by:

$$\begin{aligned} TC^m &= C_a N_a^m + C_b N_b^m + F = cN + F \text{ for } N > N^\dagger \\ &= C_a N = \frac{\delta N^2}{s} \qquad \text{for } N \leq N^\dagger \end{aligned} \tag{3}$$

Where F represents fixed public transportation costs.

Here the temporal division of commuters at equilibrium causes a significant loss of time. Public authorities can intervene to eliminate waiting lines during rush hour periods by encouraging commuters to stagger the time they leave home.

[3] In other words, when individuals seek to optimise their route, they are confronted with a situation at equilibrium which prevents any single user from improving their travel time by unilaterally altering their route.

[4] This threshold (the "city size" in the Tabuchi's analysis) is calculated by solving the former equation.

[5] Where m represents the scenario at marginal cost, with no-toll.

2.2 Implementation of a fine toll

The implementation of tolls permits the elimination of the queue and thus allows the reduction of social cost incurred by all individuals. When city authorities set the railroad fare at marginal cost ($p = c$), this pricing regime represents our benchmark that induces the lowest total cost. With the implementation of the fine toll (τ^f)[6], the true travel cost incurred by each driver would be $C_a = \delta N_a/s$. This is the same cost as the cost borne by individuals at equilibrium without a toll (1), but the internal "structure" of this cost is not the same. The travel time cost disappears. The true travel cost incurred by each driver is *on average* equal to the total of the schedule delay cost ($\delta N_a/2s$) and the financial cost of the toll ($\delta N_a/2s$).

Tabuchi finds the modal split obtained in the base case i.e.[7]:

$$\begin{aligned} \left(N_a^f, N_b^f\right) &= \left(\frac{cs}{\delta}, N - \frac{cs}{\delta}\right) \text{ for } N > N^\dagger \\ &= (N, 0) \qquad \text{for } N \leq N^\dagger \end{aligned} \tag{4}$$

The main difference from the previous situation is that currently, the implementation of a tariff setting on urban roads generates revenue that reduces the total cost incurred by the population. From the average toll, we can deduce the revenue:

$$R^f = \frac{\delta\left(N_a^f\right)^2}{2s} = \frac{\delta c^2 s^2}{2s\delta^2} = \frac{c^2 s}{2\delta} \text{ for } N > N^\dagger \tag{5}$$

As noted by Arnott et al. (1994, p. 144): *"The toll replaces queuing time as the rationing device for desired arrival time slots, leaving drivers' private costs inclusive of the toll unchanged, but overall welfare higher by the amount of toll revenue."* It is then possible to calculate the total net cost:

[6]The fine toll is determined by the railroad fare minus the schedule delay costs:

$$\begin{aligned} \tau^f(t) &= c - \beta(t - t_0) \text{ for } t \in \left(t_0, \tilde{t}\right] \\ &= c - \gamma(t_1 - t) \text{ for } t \in \left[\tilde{t}, t_1\right) \end{aligned}$$

As noted by Tabuchi (1993, p.426), *"the role of the fine toll is therefore to replace any queue time by the toll (...) and to allocate the number of users in each sector."*

[7]Where f represents the scenario at marginal cost with fine toll.

$$
\begin{aligned}
TC^f &= cN + F - \frac{c^2 s}{2\delta} \quad \text{for } N > N^\dagger \\
&= \frac{\delta N^2}{2s} \qquad \text{for } N \leq N^\dagger
\end{aligned}
\tag{6}
$$

2.3 Redistribution of toll revenue in the benchmark case with public transport marginal cost pricing

Our objective is to evaluate the effects of toll revenue redistribution. More precisely, we analyze how this redistribution modifies modal split and the total cost incurred by drivers and railroad users. We assume that the government has the possibility to allocate the toll revenue between mass transit (share α towards mass transit) and public budget (share $(1-\alpha)$ distributed for public budget). This last part $(1-\alpha)$ is allocated towards other economic sectors non included in our partial equilibrium model.

In this context, the cost supported by public transport users is reduced because a share α of toll revenue is used to subsidize mass transit sector:

$$
\widetilde{C}_b^f = c - \frac{\alpha \ \times \ recettes}{\widetilde{N}_b^f} = c - \frac{\alpha\delta\left(\widetilde{N}_a^f\right)^2}{2s(N - \widetilde{N}_a^f)}
$$

With $0 < \alpha < 1$.

The average travel cost of a motorist is the same and is written as: $\widetilde{C}_a^f = \frac{\delta \widetilde{N}_a^f}{s}$. In this case, the modal split is such that :

$\widetilde{C}_a^f = \widetilde{C}_b^f \qquad \Longleftrightarrow \qquad \frac{\delta \widetilde{N}_a^f}{s} = c - \frac{\alpha\delta\left(\widetilde{N}_a^f\right)^2}{2s(N-\widetilde{N}_a^f)} \qquad \Longleftrightarrow$

$2\delta\left(\widetilde{N}_a^f\right)^2 - 2(sc + \delta N)\widetilde{N}_a^f - \alpha\delta\left(\widetilde{N}_a^f\right)^2 + 2scN = 0$

In that case, we obtain the following solution for modal split at equilibrium :

$$\left(\widetilde{N}_a^f, \widetilde{N}_b^f\right) = \left(\frac{cs+\delta N - \sqrt{(cs-\delta N)^2 + 2cs\delta\alpha N}}{\delta(2-\alpha)}, \right.$$
$$\left. N - \frac{cs+\delta N - \sqrt{(cs-\delta N)^2 + 2cs\delta\alpha N}}{\delta(2-\alpha)}\right) \text{ for } N > 0 \quad (7)$$

Proposition 1 *The number of motorists and the toll revenue are decreasing with the share α : the higher is the part (α) of toll revenue distributed towards mass transit, the lower are the number of motorists $\widetilde{N}_a^f$ and the toll revenue $\widetilde{R}^f$ (see Proof on appendix A).*

In this context, we can highlight a very interesting result : in our model, the objective of toll revenue increase could be incompatible with the objectives of traffic reduction. This result is a good illustration of the conflict between revenue collection and traffic reduction. For example in Singapore, the implementation of a fine toll during the rush hours has induced a very high decrease of the road traffic but has entailed a strong reduction of toll revenue collected.

Using relation (7), we can now evaluate average cost supported by the motorist or the mass transit user :

$$\widetilde{C}_a^f = \widetilde{C}_b^f = \frac{\delta \widetilde{N}_a^f}{s} =$$
$$\frac{cs+\delta N - \sqrt{(cs-\delta N)^2 + 2cs\delta\alpha N}}{(2-\alpha)s} \text{ for } N > 0 \quad (8)$$

In this scenario, the total cost at equilibrium is therefore equal to:

$$\widetilde{TC}^f = \frac{N\left(cs+\delta N - \sqrt{(cs-\delta N)^2 + 2cs\delta\alpha N}\right)}{(2-\alpha)s} -$$
$$\frac{\left(cs+\delta N - \sqrt{(cs-\delta N)^2 + 2cs\delta\alpha N}\right)^2}{2\delta s(\alpha-2)} + F \quad (9)$$

At equilibrium, we can note that the redistribution of toll revenue

naturally accumulates to the total cost due to the distorsion from price-marginal cost : $\widetilde{TC}^f > TC^f$. In that case, the use of the toll revenue as subsidy for public transport is welfare decreasing, when the price of public transport is initially set based on its marginal cost. Moreover, the level of individual cost supported by a motorist or a mass transit user is lower after redistribution of toll revenue : $\left(\widetilde{C}_a^f = \widetilde{C}_b^f\right) < \left(C_a^f = C_b^f\right)$

We can easy show that the total cost $\widetilde{TC}^f$ increases with the level of (α): $\frac{\partial \widetilde{TC}^f}{\partial \alpha} > 0$. In this case, the total cost is minimized with $\alpha = 0$ that corresponds to the initial case without redistribution.

When the toll revenue is totally distributed towards mass transit sector ($\alpha = 1$), distorsions are higher that induce a strong increase of the total cost (see Mirabel and Reymond, 2011). On the contrary,when the toll revenue is completly integrated into the public budget ($\alpha = 0$), there is no effect on the commuters behavior and the total cost does not change. It is thus important to underline that higher is the affectation of revenue towards mass transit and higher is the total cost because of the effects of distortion. However, according to the new objectives for public authorities, the policy of toll revenue redistribution towards mass transit allows to decrease the number of motorists in the city inducing reduction of environmental damages.

3 Redistribution of toll revenue and self-financing of the mass transit system

In this pricing regime, we suppose that mass transit fare is set equal to the average cost, $p = c + \frac{F}{N_b}$. This pricing regime allows taking into account increasing return to scale in mass transit sector. In the same way as the previous scenario, we integrate a redistribution of toll revenue between mass transit and public budget. We can write the new individual cost supported by a motorist and a mass transit user and after the redistribution of toll revenue towards mass transit sector (share α of the revenue)[8] :

[8]Where F represents the scenario at average cost with fine toll.

$$\begin{aligned}
\widetilde{C}_a^F &= \frac{\delta \widetilde{N}_a^F}{s} \\
\widetilde{C}_b^F &= c + \frac{F}{\widetilde{N}_b^F} - \alpha \frac{R_b^F}{\widetilde{N}_b^F} = c + \frac{F}{\widetilde{N}_b^F} - \alpha \frac{\delta \widetilde{N}_a^F}{2s\widetilde{N}_b^F}
\end{aligned} \tag{10}$$

Logically, the level of mass transit fare is decreasing with the number of mass transit users. In this case, the modal split is such that $\widetilde{C}_a^F = \widetilde{C}_b^F$. Using this condition and expression (10), the modal split and the level of toll revenue can be derived at equilibrium:

$$\begin{aligned}
\left(\widetilde{N}_a^F, \widetilde{N}_b^F\right) &= \begin{pmatrix} \frac{cs+\delta N-\sqrt{(cs-\delta N)^2+2cs\delta\alpha N-4\delta Fs+2\alpha\delta Fs}}{\delta(2-\alpha)}, \\ \frac{\delta N(1-\alpha)-cs+\sqrt{(cs-\delta N)^2+2cs\delta\alpha N-4\delta Fs+2\alpha\delta Fs}}{\delta(2-\alpha)} \end{pmatrix} \text{ if } N > \widetilde{N}_s^F \\
\left(\widetilde{N}_a^F, \widetilde{N}_b^F\right) &= (N, 0) \text{ if } N \leq \widetilde{N}_s^F \qquad (11) \\
\widetilde{R}^F &= \frac{\delta \widetilde{N}_a^F}{2s} = \frac{\left(cs + \delta N - \sqrt{(cs-\delta N)^2 + 2cs\delta\alpha N - 4\delta Fs + 2\alpha\delta Fs}\right)^2}{2\delta s(2-\alpha)^2}
\end{aligned}$$

Using expression (11), we can write the following proposition :

Proposition 2 *In the pricing regime with fine toll and mass transit fare based on the average cost, the higher is the part* (α) *of toll revenue distributed towards mass transit, the lower are the number of motorists* $\widetilde{N}_a^F$ *and the toll revenue* $\widetilde{R}^F$.

According to the previous scenario, this result is a good illustration of the conflict between revenue collection and traffic reduction. As seen in the previous scenario, we evaluate the level of individual cost using expressions (10) and (11):

$$\widetilde{C}_a^F = \widetilde{C}_b^F = \frac{cs + \delta N - \sqrt{(cs-\delta N)^2 + 2cs\delta\alpha N - 4\delta Fs + 2\alpha\delta Fs}}{(2-\alpha)s} \tag{12}$$

Using this expression, we obtain finally the total cost supported by all users integrating the redistribution of toll revenue towards the public budget (share $(1-\alpha)$ distributed):

$$\begin{aligned} \widetilde{TC}^F &= N\widetilde{C}_a^F - (1-\alpha)\widetilde{R}^F \\ \widetilde{TC}^F &= N\frac{cs+\delta N-\sqrt{(cs-\delta N)^2+2cs\delta\alpha N-4\delta Fs+2\alpha\delta Fs}}{(2-\alpha)s} - \\ &\quad (1-\alpha)\frac{\left(cs+\delta N-\sqrt{(cs-\delta N)^2+2cs\delta\alpha N-4\delta Fs+2\alpha\delta Fs}\right)^2}{2\delta s(2-\alpha)^2} \end{aligned} \tag{13}$$

With this expression of total cost, we can highlight a very interesting result : when we compare the level of total cost TC^F before redistribution (see annex B) and the level of this cost $\widetilde{TC}^F$ after redistribution, we obtain the following result : if the fixed cost of mass transit is higher than the threshold $\left(\widehat{F}^F\right)$[9], the redistribution of toll revenue induces a decrease of total cost so that : $\widetilde{TC}^F < TC^F$. This result can be easily explained : the redistribution of toll revenue towards urban mass transit users induces a decrease of the fare; it entails therefore an increase of mass transit users, allowing a better amortization of the fixed cost that is profitable for a high level of fixed cost.

Using the expression (13), we can evaluate the optimal share $\alpha^*$of toll revenue that is distributed towards mass transit users. We can show that the total cost is minimized for the value of α equal to $\alpha^* = \frac{2\delta F}{c^2 s}$[10]. In that case, the optimum level of redistribution is such that $\alpha^* = \min\left(\frac{2\delta F}{c^2 s}, 1\right) > 0$. We obtain then the characterization of equilibrium in this pricing regime after

[9] $F > \widehat{F}^F = \frac{5\delta Ncs-\delta^2N^2+(6\delta N-2cs)\sqrt{(\delta N)^2-\delta Ncs}}{\delta N}$

[10] If $\alpha < \widehat{\alpha}$, the function $\widetilde{TC}^F$ is convex that allows to attain the minimum level for the total cost.

$\widehat{\alpha} = \frac{\left(2Nc\delta+4\delta F-2c^2s\right)\sqrt{c^4s^2+7\delta^2N^2c^2-8c^3sN+22cFN\delta^2-10\delta Fsc^2+16\delta^2F^2}}{9c^2s\delta(cN+F)}$
$+\frac{\delta\left(19c\delta NF+c^2N^2\delta+Nc^3s+16\delta F^2+5c^2sF\right)-c^4s^2}{9c^2s\delta(cN+F)}$

redistribution :

$$\begin{aligned}
\left(\widetilde{N}_a^F, \widetilde{N}_b^F\right) &= \left(\frac{cs}{\delta}, N - \frac{cs}{\delta}\right) \text{ if } N > \widetilde{N}_s^F \\
\left(\widetilde{N}_a^F, \widetilde{N}_b^F\right) &= (N, 0) \text{ if } N \leq \widetilde{N}_s^F \\
\widetilde{N}_a^F &= \widetilde{N}_b^F = c \\
\widetilde{R}^F &= \frac{c^2 s}{2\delta} \\
\widetilde{TC}^F &= cN + F - \frac{c^2 s}{2\delta} \qquad (14)
\end{aligned}$$

Logically, we obtain the lowest level of total cost similar to the scenario with public transport marginal cost pricing without redistribution. This result can be intuitively explained : α^* is such that the toll revenue redistributed towards mass transit sector allows covering fixed cost F so that : $\alpha^* \widetilde{R}^F = F \iff \alpha^* = \frac{F}{\widetilde{R}^F} = \frac{2\delta F}{c^2 s}$. An interior solution $\alpha^* < 1$ is obtained if and only if $\frac{2\delta F}{c^2 s} < 1 \iff F < \frac{c^2 s}{2\delta}$. If the fixed cost is superior to this value, the toll revenue is not sufficient to cover the fixed cost. In that case, even if the toll revenue is entirely distributed towards mass transit users ($\alpha = 1$), the level of the fare remains superior to the level of marginal cost pricing ; the total cost is higher due to lack of toll revenue in order to cover the fixed cost.

Proposition 3 *In a pricing regime with a fine toll and a mass transit fare based on the average cost, the optimal redistribution towards mass transit users is such that :*

- *The total cost is minimized and corresponds to the level of total cost supported in the benchmark situation with marginal cost fare*
- *The optimal share α^* distributed towards mass transit users is increasing with the level of the fixed cost (F) and is decreasing with the level of road capacity (s).*

In this context, it is important to make three relevant remarks:

- First, the minimization of total cost can be obtained with a mass transit fare based on the average cost according to a fine tuning redistribution of toll revenue. We enlarge and moderate therefore the results of Tabuchi who wrote in his article (Tabuchi 1993 p420): *"Under these circumstances (average cost fare), it is well known that the average cost pricing never attains an optimum because the total social cost is not minimized when both modes are in use"*.

- Second, in this scenario with redistribution, the mass transit sector is self-financed. Moreover, the redistribution of toll revenue allows subsidizing the mass transit fare that entails better acceptability of urban toll.

- Third, the modal split is corresponding to the benchmark situation with the highest level of mass transit users. This scenario is socially profitable since it induces a decrease of the pollution level in urban areas due to the reduction of traffic.

From a transport policy perspective, this analysis could be a relevant framework to set the level of urban toll and the redistribution of toll revenue towards mass transit and public budget. The model put on light the relevance of a fine-tuning policy in order to choose the best pricing regime and revenue redistribution scenario. In that context, public authorities could then determine the level of the fine toll and the optimal share of toll revenue distributed toward mass transit. With a self-financing mass transit sector (the fare is equal to the average cost), public authorities could then determine the level of toll revenue redistribution which allows to subsidize the mass transit fare until the marginal cost that entails to lessen the social cost. Moreover, public authorities could use the road capacity (s) as an additional instrument for transport policy. For example, since the optimal share distributed towards mass transit is equal to $\alpha^* = \frac{2\delta F}{c^2 s}$, the reduction of road capacity[11] justify an increase of the toll revenue share distributed towards mass transit users.

4 Conclusion

In this article, we assume in an innovative way that the toll revenue is allocated towards public transport and public budget as an instrument of public policy. The revenue redistribution towards public transport results in a distortion of travel demand (there is a strong tendency to commute with public transport rather than driving) inducing an increase in the total cost. However, this total cost does not take into account external effects related to the environment, and a reduction of automobile use would allow e reduction of city traffic jams.

[11] For example, a transfert of road capacity for other uses : walking, cycling, railroad,...

Section 2 focuses on the case where the railroad fare is set equal to marginal cost. The subsidies corresponding to fixed costs are integrated into the total cost *ex post*. The total cost is always higher after redistribution towards mass transit sector and public budget. And the higher is the part of toll revenue distributed towards mass transit, the lower is the number of motorists.

Section 3 enlarges the analysis integrating a new pricing regime with the mass transit fare based on the average cost. In this pricing regime, the distribution of toll revenue towards mass transit and public budget allows to minimize the total cost and therefore to obtain the benchmark optimal situation with marginal cost fare.

From a transport policy perspective, the redistribution of toll revenues that was integrated in our model would induce benefits for the collectivity. Firstly, the number of user transit is higher when revenue is redistributed. The decline in the number of drivers allows the reduction of environmental externalities related to city automobile use. Secondly, redistribution of revenue allows a better acceptability: when citizens are aware that revenue is assigned towards public transport, toll acceptability is greater than when it is not redistributed. This point has already been highlighted on various occasions (See Reymond 2004, Schade and Baum, 2007). In order to extend our new analysis, other pricing regimes will be integrated for further researches. It will be so relevant to compare the impact of theses pricing regimes on the total cost and modal split. Finally, it will allow to evaluate the best splitting of toll revenue between mass transit and public budget in order to make some recommandations for a fine-tuning redistribution of the toll revenue.

Appendix A

Sign of $\frac{\partial \widetilde{N}_a^f}{\partial \alpha}$.

Evaluate the value of this derivate

$$\frac{\partial \widetilde{N}_a^f}{\partial \alpha} = \frac{-cs\delta\alpha N - c^2s^2 - \delta^2N^2 + \sqrt{(cs-\delta N)^2 + 2cs\delta\alpha N}\,(cs+\delta N)}{\delta\,(\alpha-2)^2\sqrt{c^2s^2 - 2cs\delta N + \delta^2N^2 + 2cs\delta\alpha N}}$$

In order to sign this derivate, we sign the numerator of the expression using the assumption about the value of N:

$N > \frac{cs}{\delta} \Rightarrow\; N = r\frac{cs}{\delta}$ with $r > 1$. In that case, the denominator is positive and the numerator writes (after simplifications):

$$Num = -\alpha r + (1+r)\sqrt{(1-r)^2 + 2\alpha r} - 1 - r^2 \quad \text{with } r > 1 \;\text{ and } 0 < \alpha < 1$$

Evaluate the value of this expression for $\alpha = 0$ and $\alpha = 1$:

$Num_{\alpha=0} = (1+r)\sqrt{(1-r)^2} - 1 - r^2 = (r+1)(r-1) - 1 - r^2 = r^2 - 1 - r^2 = -1 < 0$

$Num_{\alpha=1} = -r + (1+r)\sqrt{(1-r)^2 + 2r} - 1 - r^2 = -(1+r^2+r) + \sqrt{1+r^2} < 0$

If $r > 1$, the expression Num of the numerator is defined and continuous on the interval $\alpha \in [0,1]$. Moreover, we can show that $Num = 0$ for $\alpha = 2$. In that context, whatever the value of $\alpha \in [0,1]$, $Num < 0$

In that case, the sign of $\frac{\partial \widetilde{R}^f}{\partial \alpha}$ is similar to the sign of $\frac{\partial \widetilde{N}_a^f}{\partial \alpha} < 0$ since toll revenue writes $\widetilde{R^f} = \frac{\delta\left(\widetilde{N}_a^f\right)^2}{2s}$ $\square$

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